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Properties of Eco-colonies

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Abstract

Eco-colonies are new grammar systems with very simple grammars called agents placed in a common dynamic environment. Every agent generates its own finite language, all agents cooperate on the shared environment. The environment is developing not only by the action of agents, but also using its own developmental rules.

The generative power of eco-colonies was discussed in several papers, eco-colonies were compared especially with various types of colonies, but not all relations were proved. In this paper we summarize previous results and present some new results about the generative power of eco-colonies.

1 Introduction

Colonies were introduced in [5] as collections of simple grammars (called components) working on a common environment. A component is specified by its start symbol and by its finite language. This language determines actions to do with the start symbol, it is usually a list of words, the component substitutes its start symbol by some of these words. The environment is static itself, only the components can modify it.

There exist several variants of colonies with various types of derivation. The original model was sequential (only one component works in one derivation step), the other basic types of derivation are sequential with parallely working components or parallel. Parallel colonies were introduced in [4], the parallel behavior of a colony means the working of all the components that can work (the components whose start symbols are in the environment and no other component is occupying this symbol for the actual derivation step), one component processes one occurrence of its start symbol.

Eco-colonies were first studied in [10], their E0L form in [11] and [12]. Eco-colonies are colonies with developing environment. The concept of developing of the environment is inspired by another type of grammar systems, eco-grammar systems ([3]). The environment of eco-colonies is specified not only by its alphabets but as 0L or E0L scheme. Every symbol of the environment not processed by agents (components) is overwritten by some of the developing rules of this scheme.

In [1] there is defined a related system, *e-colonies* (extended colonies). Similarly as eco-colonies are based on parallel colonies and their environment is 0L- or E0L-scheme, e-colonies in [1] are based on sequential colonies and their environment is T0L-scheme.

The presented paper consists of four parts. In Section 2 preliminaries are mentioned, than in Section 3 we introduce eco-colonies with two different derivation modes and illustrate these systems on the examples.

In Section 4 we deal with the derivation power of eco-colonies. We compare them mutually, and we compare the generative power of various types of colonies with the generative power of the both types of eco-colonies. We will discuss the systems with single alphabet and also the systems with terminal alphabets.

Section 5 is devoted to the conclusions.

2 Preliminaries

In this section we define colonies and the types of derivation in colonies, and we preface lemmas used in the next sections. For other prerequisites from the theory of formal languages and grammars we refer to [9], related information about theory of grammar systems can be found in [2]. L-systems, 0L-, E0L-, ET0L- and T0L-systems are defined in [8]. For definitions of some properties of languages (e.g. logarithmically clustered, pump-generated) see the paper [7].

In this paper we denote by $|w|_S$ the number of occurrences of S in w for a word w and a symbol S.

Definition 1 *A colony is a (n+3)-tuple* $\mathcal{C} = (V, T, A_1, \ldots A_n, w_0)$*, where*

- V *is a total (finite and non-empty) alphabet of the colony,*
- T *is a non-empty terminal alphabet of the colony,* $T \subset V$*,*
- $A_i = (S_i, F_i),\ 1 \le i \le n$*, is a component, where*
  - $S_i \in V$ *is the start symbol of the component,*
  - $F_i \subseteq (V - \{S_i\})^*$*,* F_i *is the finite language of this component,*
- w_0 *is the axiom.*

The derivations for colonies were introduced in several ways. Basic of them are following modes:

***b*-mode** is *sequential* type of derivation, one component is active in one derivation step, the active component replaces one occurrence of its start symbol by some word of its finite language F,

***t*-mode** is *sequentially-parallel* – one component is active in one derivation step and this component rewrites all occurrences of its start symbol by words of its language, *wp*-**mode** is *parallel* mode, where every component which can work must work in the following sense: each component rewrites at most one occurrence of its start symbol, a component is active if its start symbol is in the environment and no other component with the same start symbol occupies this occurrence of the symbol,

sp-**mode** is *parallel* mode similar to *wp*, but if there is an occurrence of a symbol in the environment, every component with this start symbol has to be active – if all occurrences of this symbol are occupied by another components with the same start symbol, the derivation is blocked.

Definition 2 *We define a basic derivation step (b mode) in a colony* $\mathcal{C}$, $\mathcal{C} = (V, T, A_1, \ldots, A_n, w_0)$ *as the relation* $\stackrel{b}{\Rightarrow}$ *–* α *directly derives* β *in b mode of derivation (written as* $\alpha \stackrel{b}{\Rightarrow} \beta$*) if*

- $\alpha = v_1 S v_2$, $\beta = v_1 f v_2$, *where* $v_1, v_2 \in V^*$, $S \in V$, $f \in (V - \{S\})^*$,
- *there exists a component* (S, F) *in* $\mathcal{C}$ *such as* $f \in F$.

Definition 3 *We define a terminal derivation step (t mode) in a colony* $\mathcal{C}$, $\mathcal{C} = (V, T, A_1, \ldots, A_n, w_0)$ *as the relation* $\stackrel{t}{\Rightarrow}$ *–* α *directly derives* β *in t mode of derivation (written as* $\alpha \stackrel{t}{\Rightarrow} \beta$*) if*

- $\alpha = v_0 S v_1 S v_2 \ldots v_{n-1} S v_k$,
- $\beta = v_0 f_1 v_1 f_2 v_2 \ldots v_{n-1} f_k v_k$,
- *where* $v_i \in (V - \{S\})^*$, $0 \le i \le k$, $S \in V$, $f_i \in (V - \{S\})^*$,
- *there exists a component* (S, F) *in* $\mathcal{C}$ *such as for all strings* f_i, $1 \le i \le k$, *is* $f_i \in F$.

Definition 4 *We define a strongly parallel derivation step (sp mode) in a colony* $\mathcal{C} = (V, T, A_1, \ldots, A_n, w_0)$ *as the relation* $\stackrel{sp}{\Longrightarrow}$ *–* α *directly derives* β *in sp mode of derivation (written as* $\alpha \stackrel{sp}{\Longrightarrow} \beta$*) if*

- $\alpha = v_0 S_{i_1} v_1 S_{i_2} v_2 \ldots v_{k-1} S_{i_k} v_k$,
- $\beta = v_0 f_{i_1} v_1 f_{i_2} v_2 \ldots v_{k-1} f_{i_k} v_k$,
- *where* $v_j \in V^*$, $0 \le j \le k$, $S_{i_j} \in V$, $1 \le j \le k$, $f_{i_j} \in \left(V - \{S_{i_j}\}\right)^*$, $1 \le j \le k$,
- *there exist components* (S_{i_j}, F_{i_j}) *in* $\mathcal{C}$ *such as* $f_{i_j} \in F_{i_j}$, $1 \le j \le k$,
- $i_t \ne i_s$ *for all* $t \ne s$, $1 \le t, s \le k$ *(one component can rewrite at most one occurrence of its start symbol),*
- *if* $|\alpha|_S > 0$ *for some symbol* $S \in V$, *then for every component* (S_t, F), *where* $S_t = S$, *is* $t = i_j$ *for some* j, $1 \le j \le k$ *(if some symbol occurs in environment, then all components with this symbol as the start symbol must work).*

Definition 5 *We define a weakly parallel derivation step (wp mode) in a colony* $\mathcal{C} = (V, T, A_1, \ldots, A_n, w_0)$ *as the relation* $\stackrel{wp}{\Longrightarrow}$ *–* α *directly derives* β *in wp mode of derivation (written as* $\alpha \stackrel{wp}{\Longrightarrow} \beta$*) if*

- $\alpha = v_0 S_{i_1} v_1 S_{i_2} v_2 \dots v_{k-1} S_{i_k} v_k$,
- $\beta = v_0 f_{i_1} v_1 f_{i_2} v_2 \dots v_{k-1} f_{i_k} v_k$,
- *where* $v_j \in V^*,\ 0 \leq j \leq k,\ S_{i_j} \in V,\ 1 \leq j \leq k,\ f_{i_j} \in \big(V - \{S_{i_j}\}\big)^*,\ 1 \leq j \leq k$,
- *there exist components* (S_{i_j}, F_{i_j}) *in* $\mathcal{C}$ *such as* $f_{i_j} \in F_{i_j},\ 1 \leq j \leq k$,
- $i_t \neq i_s$ *for all* $t \neq s,\ 1 \leq t, s \leq k$ *(one component can rewrite at most one occurrence of its start symbol),*
- *for every* $S \in V$*, if the number of agents with the start symbol* S *is denoted by* t*, then*

$$\sum_{\substack{j=1 \\ S_{i_j}=S}}^{r} |\alpha|_{S_{i_j}} = min\,(|\alpha|_S, t)$$

(all components which can work – their start symbol is in the environment and some of the occurrences of this symbol is not occupied by any other agent – they must work; the left side of the equation means the number of components with the start symbol S *which work in the given derivation step).*

The formal definitions of an eco-grammar system and its type of derivation are in [3].

For all the relations $\stackrel{x}{\Longrightarrow},\ x \in \{b, t, wp, sp\}$, we define the reflexive and transitive closure $\stackrel{x}{\Longrightarrow}^*$.

Definition 6 *Let* $\mathcal{C}$ *be a colony and* $\mathcal{C} = (V, T, A_1, \dots A_n, w_0)$*. The language generated by the derivation step* $x,\ x \in \{b, t, wp, sp\}$ *in* $\mathcal{C}$ *is*

$$L(\mathcal{C}, x) = \{w \in T^* \ :\ w_0 \stackrel{x}{\Longrightarrow}^* w\}.$$

For more information about languages of colonies see [6].

We use the notations for colonies with various types of derivation:

COL_x for class of languages generated by colonies with x type of derivation, $x \in \{b, t, wp, sp\}$,

COL_x^T for class of languages generated by colonies with $T = V$ and x type of derivation, $x \in \{b, t, wp, sp\}$.

Lemma 1

$$COL_x^T \subseteq COL_x$$

where $x \in \{b, t, wp, sp\}$*.*

Proof. Colonies generating the class COL_x^T are colonies with only one alphabet ($T = V$), it is a special type of colonies generating COL_x. □

Let $\mathcal{C}$ be a colony, $\mathcal{C} = (V, T, A_1, \dots A_n, w_0)$, with n components. Denote by m the length of the longest word in the languages of components, over the all components $A_1, \dots, A_n$:

$$m = max\,\{|u| \ :\ u \in F_i,\ A_i = (S_i, F_i),\ 1 \leq i \leq n\}\,.$$

Lemma 2 (Pumping lemma for parallel colonies) *Let L be an infinite language generated by a colony $\mathcal{C}$ with $x \in \{wp, sp\}$ derivation mode. Then the length set of L contains infinite linearly dependent subsets, i.e.*

$$\{a \cdot i + b \ : \ i \geq 0\} \subseteq \{|w| \ : \ w \in L\}$$

for some natural numbers $a, b > 0$.

Proof. Let $\mathcal{C} = (V, T, A_1, \ldots A_n, w_0)$ be a colony with $x \in \{wp, sp\}$ derivation mode and $L(\mathcal{C}, x) = L$ for some infinite language L. Let m be length of the longest word in the languages of components $A_1, \ldots, A_n$,

$$m = max\{|u| \ : \ u \in F_i, \ A_i = (S_i, F_i), \ 1 \leq i \leq n\}.$$

Let us choose some w in L, $|w| \geq |w_0| \cdot m \cdot n \cdot 2^n$, the derivation of word w from the axiom consists of at least 2^n steps. Since in one derivation step $w_i \overset{x}{\Longrightarrow} w_{i+1}$ we have $|w_{i+1}| - |w_i| \leq m \cdot n$. Therefore there are indices $i, j, \ i < j$, such that the same set of agents is active in the derivation steps $w_i \overset{x}{\Longrightarrow} w_{i+1}$ and $w_j \overset{x}{\Longrightarrow} w_{j+1}$.

We split this derivation to the parts

$$w_0 \overset{x}{\Longrightarrow}^* w_i \overset{x}{\Longrightarrow}^* w_j \overset{x}{\Longrightarrow}^* w$$

Denote by

- n_0 number of terminal symbols generated in the subderivation $w_0 \overset{x}{\Longrightarrow}^* w_i$, which are not rewritten in any next derivation step,
- n_i the same for the subderivation $w_i \overset{x}{\Longrightarrow}^* w_j$,
- n_j the same for the subderivation $w_j \overset{x}{\Longrightarrow}^* w$.

Now we transform the derivation as follows:

- in the derivation step $w_j \overset{x}{\Longrightarrow} \ldots$ we use the same components and words of languages of these components as in the derivation step $w_i \overset{x}{\Longrightarrow} \ldots$,
- in this way we link up a copy of processing the sets of symbols from the subderivation $w_i \overset{x}{\Longrightarrow}^* w$ to the subderivation $w_j \overset{x}{\Longrightarrow} \ldots$ (we link up only the way of rewriting symbols, the other symbols stay in the word),
- we apply the previous operation z-times, $z \geq 0$,
- the word derived using the described method of "pumping" the derivation is denoted by w'_z.

The described derivations for the numbers z generate the words with the following length:

$$|w| = |w'_1| = n_0 + n_i + n_j, \quad (1)$$
$$|w'_z| = n_0 + z \cdot n_i + n_j. \quad (2)$$

We can construct the derivation of w'_z for any $z \geq 0$, so $w'_z \in L$. Linear dependence is obvious. □

The following theorems are used in the proof of Theorem 7.

Theorem 1 ([7]) *If K is an infinite $ET0L_{[1]}$ language*[1] *then either K contains an infinite logarithmically clustered language or K contains a pump-generated language.*

Theorem 2 ([4])

$$COL_t = ET0L_{[1]}.$$

3 Eco-Colonies

In this section we define two types of eco-colonies and then two types of derivation in eco-colonies.

Definition 7 *An E0L eco-colony of degree n, $n \geq 1$, is an $(n+2)$-tuple $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$, where*

- *$E = (V, T, P)$ is E0L scheme, where*
  - *V is an alphabet,*
  - *T is a terminal alphabet, $T \subseteq V$,*
  - *P is a finite set of E0L rewriting rules over V,*
- *$A_i = (S_i, F_i)$, $1 \leq i \leq n$, is the i-th agent, where*
  - *$S_i \in V$ is the start symbol of the agent,*
  - *$F_i \subseteq (V - \{S_i\})^*$ is a finite set of action rules of the agent (the language of the agent),*
- *w_0 is the axiom.*

An *0L eco-colony* is defined similarly, the environment is 0L scheme $E = (V, P)$, P is a finite set of 0L rewriting rules over V.

As we can see, agents are defined in the same way as components in colonies, an environment is determined by the alphabets in colonies, and by E0L or 0L scheme in eco-colonies.

We define two derivation modes for eco-colonies – the first one, *wp*, is inspired by the *wp* mode for colonies, we only add the possibility of developing for the environment. In every derivation step each agent (S, F) looks for its start symbol S. If it finds some occurrence of this symbol not occupied by any other agent, the agent becomes active, occupies this symbol and rewrites it by some of words of its language F.

Definition 8 *We define a weakly competitive parallel derivation step in an eco-colony $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$ as the relation $\stackrel{wp}{\Longrightarrow}$ – α directly derives β in wp mode of derivation (written as $\alpha \stackrel{wp}{\Longrightarrow} \beta$) if*

[1] $ET0L_{[1]}$ languages are languages generated by 1-restricted $ET0L$ systems: 1-restricted $ET0L$ system is $ET0L$ system $G = (\Sigma, \mathcal{P}, S, \Delta)$ such that for every table $P \in \mathcal{P}$ there exists a letter $b \in \Sigma$ such that if $c \in \Sigma - \{b\}$ and $(c \to \alpha) \in P$ then $\alpha = c$ (in every table only one rule is not static).

- $\alpha = v_0 S_{i_1} v_1 S_{i_2} v_2 \ldots v_{r-1} S_{i_r} v_r,\ r > 0,$
- $\beta = v_0' f_{i_1} v_1' f_{i_2} v_2' \ldots v_{r-1}' f_{i_r} v_r',$ *for* $A_{i_k} = (S_{i_k}, F_{i_k}),\ f_{i_k} \in F_{i_k},\ 1 \le k \le r,$
- $i_k \neq i_m$ *for every* $k \neq m,\ 1 \le k, m \le r$ *(the agent* A_{i_k} *is* active *in this derivation step),*
- $\{i_1, i_2, \ldots, i_r\} \subseteq \{1, 2, \ldots, n\},$
- *for every* $S \in V$*, if the number of agents with the start symbol* S *is denoted by* t*, then*

$$\sum_{\substack{j=1 \\ S_{i_j}=S}}^{r} |\alpha|_{S_{i_j}} = min\,(|\alpha|_S, t)$$

  (all agents which can work – their start symbol is in the environment and some of the occurrences of this symbol is not occupied by any other agent – they must work; the left side of the equation means the number of agents with the start symbol S *which work in the given derivation step),*

- $v_k \overset{E}{\Longrightarrow} v_k',\ v_k \in V^*,\ 0 \le k \le r,$ *is the derivation step of the scheme* E*.*

The second type of derivation step, *ap*, means that all agents must work in every derivation step and if some agent is not able to work (there is not any free occurrence of its start symbol), the derivation is blocked. This type of derivation is inspired by the basic type of derivation in eco-grammar systems.

Definition 9 *We define a derivation step ap (all are working parallely) in an eco-colony* $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$ *as the relation* $\overset{ap}{\Longrightarrow}$ *–* α *directly derives* β *in ap mode of derivation (written as* $\alpha \overset{ap}{\Longrightarrow} \beta$*) if*

- $\alpha = v_0 S_{i_1} v_1 S_{i_2} v_2 \ldots v_{n-1} S_{i_n} v_n,$
- $\beta = v_0' f_{i_1} v_1' f_{i_2} v_2' \ldots v_{n-1}' f_{i_n} v_n',$ *for* $A_{i_k} = (S_{i_k}, F_{i_k}),\ f_{i_k} \in F_{i_k},\ 1 \le k \le n,$
- $\{i_1, i_2, \ldots, i_n\} = \{1, 2, \ldots, n\}$ *(every agent works in every derivation step),*
- $v_k \overset{E}{\Longrightarrow} v_k',\ v_k \in V^*,\ 0 \le k \le n,$ *is the derivation step of the scheme* E*.*

For the relations $\overset{x}{\Longrightarrow}$, $x \in \{wp, ap\}$, we define the reflexive and transitive closure $\overset{x}{\Longrightarrow}^*$.

Definition 10 *Let* Σ *be an 0L eco-colony,* $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$*. The language generated by the derivation step* x*,* $x \in \{wp, ap\}$*, in* Σ *is*

$$L(\Sigma, x) = \{w \in V^* \ :\ w_0 \overset{x}{\Longrightarrow}^* w\}.$$

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$$L(\Sigma, x) = \{w \in T^* \ :\ w_0 \overset{x}{\Longrightarrow}^* w\}.$$

Example 1 Let $\Sigma = (E, A_1, A_2, AbB)$ be an E0L eco-colony, where

$$E = (\{A, B, a, b\}, \{a, b\}, \{a \to a, b \to bb, A \to A, B \to B\}),$$

$$A_1 = (A, \{aB, \varepsilon\}), \quad A_2 = (B, \{aA, \varepsilon\}).$$

Let us construct derivations with *ap* and *wp* types of derivations:

$$AbB \overset{ap}{\Longrightarrow} aBb^2aA \overset{ap}{\Longrightarrow} a^2Ab^4a^2B \overset{ap}{\Longrightarrow} \ldots \overset{ap}{\Longrightarrow} a^nAb^{2^n}a^nB \overset{ap}{\Longrightarrow} a^nb^{2^{(n+1)}}a^n,$$

$$AbB \overset{wp}{\Longrightarrow} aBb^2aA \overset{wp}{\Longrightarrow} a^2Ab^4a^2B \overset{wp}{\Longrightarrow} a^2b^8a^3A \overset{wp}{\Longrightarrow} a^2b^{16}a^4B \overset{wp}{\Longrightarrow} \ldots$$

The *wp* derivation allows "resting" of non-active agents. If we use the *ap* type of derivation, a terminal word is generated only if the both agents use the ε-rule in the same derivation step, otherwise the derivation is blocked without creating the final word.

The generated languages are:

$$\begin{aligned} L(\Sigma, ap) &= \left\{a^nb^{2^{(n+1)}}a^n \ : \ n \geq 0\right\}, \\ L(\Sigma, wp) &= \left\{a^ib^{2^n}a^j \ : \ 0 \leq i, j < n\right\}. \end{aligned}$$

4 Generative Power of Eco-Colonies

We compare the generative power of eco-colonies and colonies, for systems with terminal alphabets as well as for special systems with the terminal alphabet equal to the alphabet of the system. For eco-colonies we use the notations:

$0EC_x$ for the class of languages generated by 0L eco-colonies with x type of derivation, $x \in \{wp, ap\}$,

EEC_x for the class of languages generated by E0L eco-colonies with x type of derivation, $x \in \{wp, ap\}$.

Theorem 3

$$COL_{wp} \subset EEC_{wp}. \tag{3}$$

Proof. The relation $COL_{wp} \subseteq EEC_{wp}$ is trivial, colonies with *wp* derivation are a special version of E0L eco-colonies with a static environment (with rules $a \to a$ for every letter from V). To prove the proper inclusion we use the language

$$L_1 = \left\{a^{2^n} \ : \ n \geq 0\right\}.$$

The language L_1 is generated by the eco-colony $\Sigma = (E, A, b)$, where

$$E = (\{a, b\},\ \{a\},\ \{a \to aa,\ b \to b\}), \quad A = (b,\ \{a\}).$$

The language L_1 does not include infinite subsets of words with linearly dependent length so according to Lemma 2 there is no colony C with *wp* derivation which generates the language L_1. □

Corollary 1

$$COL_b \subset EEC_{wp}, \tag{4}$$
$$COL_b^T \subset EEC_{wp}, \tag{5}$$
$$COL_{wp}^T \subset EEC_{wp}. \tag{6}$$

Proof. Equation (4) follows from $COL_b \subset COL_{wp}$ ([4]) and from Equation (3). Equations (5) and (6) follow from Lemma 1 and from Equations (3) and (4). □

Theorem 4

$$0EC_{wp} \subset EEC_{wp}. \tag{7}$$

Proof. The relation $0EC_{wp} \subseteq EEC_{wp}$ is trivial, 0L eco-colonies are the special type of E0L eco-colonies with the terminal alphabet $T = V$.

We can find a language $L_2 \in EEC_{wp} - 0EC_{wp}$:

$$L_2 = \left\{ a^{2^i} \ : \ i \geq 0 \right\} \cup \left\{ b^{3^i} \ : \ i \geq 0 \right\}.$$

This language is generated by the E0L eco-colony $\Sigma = (E, A, S)$, where

$$\begin{aligned} E &= (\{S, a, b\},\ \{a, b\},\ \{a \to aa,\ b \to bbb,\ S \to S\}), \\ A &= (S, \{a, b\}) \text{ (this agent is active only in the first derivation step)}, \end{aligned}$$

$$S \overset{wp}{\Longrightarrow} a \overset{wp}{\Longrightarrow} a^2 \overset{wp}{\Longrightarrow} a^4 \overset{wp}{\Longrightarrow} a^8,$$
$$S \overset{wp}{\Longrightarrow} b \overset{wp}{\Longrightarrow} b^3 \overset{wp}{\Longrightarrow} b^9 \overset{wp}{\Longrightarrow} b^{27}.$$

Assume that some 0L eco-colony $\Sigma_0 = (E, A_1, A_2, \ldots, A_n, w_0)$, $E = (V, P)$, generates the language L_2. Every state in the environment including the axiom is one of the elements of the language of Σ_0.

Let the rule $a \to \varepsilon$ is in P (the case for b is analogous). If we have only the ε-rule for a there, the exponential growing would be carried by agents, but the agents work similarly to the components in colonies. Components are not able to ensure exponential growing (see Lemma 2), nor agents in this eco-colony.

If there are some non-ε-rules in the environment, the ε-rule is not allowed, because the random application of this rule would mean random disappearing of symbols in the environment, so some words not contained in L_2 could be generated. That is why the axiom is one of the two shortest words – a or b.

Suppose the axiom a. We need to generate every word of the language L_2 including the words b^{3^i}, so the rule $a \to b$ is in the language of some agent or it is a rule of the 0L scheme in the environment.

If this rule is used by some agent, the eco-colony can generate only the words $a \cup b^{3^i}$, because the agent must work whenever it can work. If some another agent rewrites symbols b to a, it is able to do it in the next derivation step, but every state of the environment belongs to the language generated by Σ_0, including the states before and after application of this derivation step. In this case only one derivation is possible, $a \overset{wp}{\Longrightarrow} b \overset{wp}{\Longrightarrow} a \overset{wp}{\Longrightarrow} b \overset{wp}{\Longrightarrow} \ldots$, it generates the language $\{a, b\}$.

The superior indexes 2 and 3 in the definition of L_2 have not any common divisor, so the alternate rewriting of all the symbols a to b and then b to a with the growing length of the words by the environment is not possible.

So if the rule $a \to b$ (or some rule rewriting a to more than one b) is in the 0L scheme of the environment and the 0L scheme is deterministic, the eco-colony is not able to generate any word of the form a^{2^i} longer than the power of the number of agents in this system, because the deterministic 0L scheme does not contain any rule rewriting a to a sequence of a. The rules rewriting b to a sequence of a are not usable as suggested in the previous paragraph.

If the 0L scheme is not deterministic, this situation allows to have more than one rule for rewriting the symbol a – one rule $a \to b$ and some rule rewriting a to a sequence of a. But in this case the eco-colony can generate some words containing both the symbols a and b, and these words are not elements of the language L_2.

The case of the axiom b can be solved similarly, so any 0L eco-colony cannot generate the language L_2. □

Theorem 5

$$0EC_{ap} \subset EEC_{ap}. \tag{8}$$

Proof. 0L eco-colonies are special types of E0L eco-colonies where $T = V$, so the relation $0EC_{ap} \subseteq EEC_{ap}$ is trivial.

To prove the proper subset we use language

$$L_3 = L_1 - \{a\} = \left\{a^{2^n} \ : \ n \geq 1\right\}.$$

This language is generated by the E0L eco-colony $\Sigma = (E, A_1, A_2, UVa)$, where $E = (\{a, U, V\}, \{a\}, \{a \to aa, U \to U, V \to V\})$, $A_1 = (U, \{V, \varepsilon\})$, $A_2 = (V, \{U, \varepsilon\})$.

$$UVa \overset{ap}{\Longrightarrow} VUa^2 \overset{ap}{\Longrightarrow} UVa^4 \overset{ap}{\Longrightarrow} \dots \overset{ap}{\Longrightarrow} UVa^{2^{n-1}} \overset{ap}{\Longrightarrow} a^{2^n}.$$

Each agent generates the empty word only and using the *ap* derivation agents A_1 and A_2 are active in every derivation step and they alternate symbols U, V until the terminal word is generated.

Suppose that the language L_3 can be generated by some 0L eco-colony Σ with *ap* derivation. Σ contains at least one agent, which is active in every derivation step. $V = \{a\}$, so the start symbol of each agent is a. The agent generates a finite language over $V - \{a\}$, so we have $A = (a, \{\varepsilon\})$ for each agent in Σ.

P is deterministic, it contains exactly one rule for a. (Otherwise the system generates an infinite set of pairs of words with the constant difference of their length and there is no such an infinite subset in L_3.)

The language generated with the 0L eco-colony where $P = \{a \to a^s\}$ with n agents $A = (a, \{\varepsilon\})$ using the *ap* mode from the axiom a^m is equal to $\left\{a^{2^n} \ : \ n \geq 1\right\}$ for no parameters m, n, s and $L_3 \notin EEC_{ap}$. □

Theorem 6 *The classes of languages* $0EC_{wp}$ *and* $0EC_{ap}$ *are incomparable.*

Proof. 1) $0EC_{wp} - 0EC_{ap} \neq \emptyset$:

In Theorem 5 we proved that the language $L_3 = \left\{a^{2^n} \;:\; n \geq 1\right\}$ is not generated by any 0L eco-colony with *ap* derivation. This language is generated by the following 0L eco-colony with *wp* derivation: $\Sigma = (E, A, a)$, where $E = (\{a, b\}, \{a\}, \{a \rightarrow aa, b \rightarrow b\})$, $A = (b, \{a\})$.

We need at least one agent, but using *wp* derivation this agent does not work if its start symbol is not in the environment.

2) $0EC_{ap} - 0EC_{wp} \neq \emptyset$:

To prove this we use language

$$\begin{aligned} L_4 &= \left\{a^{15-2n}b^n cb^n d \;:\; 0 \leq n < 7, n \text{ is even}\right\} \\ &\cup \left\{a^{15-2n}b^n db^n c \;:\; 0 < n \leq 7, n \text{ is odd}\right\}. \end{aligned}$$

This language is generated by the 0L eco-colony $\Sigma = (E, A_1, A_2, A_3, A_4, a^{15}cd)$ with *ap* derivation, where

$$E = (\{a, b, c, d\}, \{a \rightarrow a, b \rightarrow b, c \rightarrow c, d \rightarrow d\}),$$

$$A_1 = (a, \{\varepsilon\}),\; A_2 = (a, \{\varepsilon\}),\; A_3 = (c, \{bd\}),\; A_4 = (d, \{bc\}).$$

This language consists only of eight words derived as follows:

$$a^{15}cd \overset{ap}{\Longrightarrow} a^{13}bdbc \overset{ap}{\Longrightarrow} a^{11}b^2cb^2d \overset{ap}{\Longrightarrow} a^9b^3db^3c \overset{ap}{\Longrightarrow} a^7b^4cb^4d \overset{ap}{\Longrightarrow} a^5b^5db^5c \overset{ap}{\Longrightarrow}$$

$$\overset{ap}{\Longrightarrow} a^3b^6cb^6d \overset{ap}{\Longrightarrow} ab^7db^7c.$$

Assume that there exists an 0L eco-colony Σ_0 with *wp* derivation generating L_4. Suppose that the axiom is $a^{15-2i}b^icb^id$ for some i, $0 \leq i \leq 7$, i is even (the proof for the axiom with odd number i is analogous). Σ_0 generates all words of the language for $n > i$ and/or $n < i$.

a) Words for $n < i$: $\quad a^{15-2i}b^icb^id \overset{wp}{\Longrightarrow}^{+} a^{15}dc\ldots$

   The number of a-s increases, the number of b-s decreases. But with using the *wp* type of derivation the system is not able to stop growing of a-s, so it is possible to generate words not included in L_4 such as $a^{19}cd$.

b) Words for $n > i$: $\quad a^{15-2i}b^icb^id \overset{wp}{\Longrightarrow}^{+} a^{13-2i}b^{i+1}db^{i+1}c\ldots$

   The number of a-s decreases, the number of b-s increases. As in the previous part of this proof, the system is not able to stop growing of b-s, the words b^8cb^8d, etc. not included in L_4 are generated.

The outcome is identical for growing by agents as well as by the environment. □

Theorem 7

$$xEC_y - COL_z \neq \emptyset \tag{9}$$

where $x \in \{0, E\}$, $y \in \{wp, ap\}$, $z \in \{b, t, wp, sp\}$.

Proof. In this proof we use language

$$L_5 = \left\{ cda^{2^{2n}} b^{2^{2n}} \;:\; n \geq 0 \right\} \cup \left\{ dca^{2^{2n+1}} b^{2^{2n+1}} \;\middle|\; n \geq 0 \right\}.$$

This language can be generated by the eco-colony $\Sigma = (E, A_1, A_2, cdab)$, where $E = (\{a,b,c,d\}, \{a \to aa, b \to bb, c \to c, d \to d\})$, $A_1 = (c, \{d\})$, $A_2 = (d, \{c\})$.

$$cdab \Rightarrow dca^2b^2 \Rightarrow cda^4b^4 \Rightarrow dca^8b^8 \Rightarrow cda^{16}b^{16} \Rightarrow \dots$$

Considering $T = V$ this is 0L as well as E0L eco-colony. Both agents are active for all words, i.e. in every derivation step so wp and ap coincide in it.

The language L_5 is not context-free, so $L_5 \notin COL_b$ and it grows exponentially so $L_5 \notin COL_{wp}$ and $L_5 \notin COL_{sp}$ according to Lemma 2.

$L_5 \notin COL_t$ according to the results of Kleijn and Rozenberg, see Theorems 1 and 2 in Preliminaries. □

Corollary 2

$$xEC_y - COL_z^T \neq \emptyset \tag{10}$$

where $x \in \{0, E\}$, $y \in \{wp, ap\}$, $z \in \{b, t, wp, sp\}$.

Proof. Follows from Theorem 7 and Lemma 1. □

Corollary 3

$$COL_{wp}^T \subset 0EC_{wp}, \tag{11}$$

$$COL_b^T \subset 0EC_{wp}. \tag{12}$$

Proof. Colonies COL_{wp}^T are a special type of eco-colonies $0EC_{wp}$ with the static environment (only rules of type $a \to a$), so $COL_{wp}^T \subseteq 0EC_{wp}$. Equation (11) follows from this fact and from Corollary 2.

Colonies COL_b^T can be simulated by colonies COL_{wp}^T where every possible pair of components has different start symbols, so $COL_b^T \subseteq COL_{wp}^T$. This gives inclusion (12). □

Theorem 8

$$COL_x - 0EC_{wp} \neq \emptyset, \quad x \in \{b, t, wp, sp\}. \tag{13}$$

Proof. In Theorem 6 we proved that the language

$$\begin{aligned} L_4 \;=\; & \left\{ a^{15-2n} b^n c b^n d \;:\; 0 \leq n < 7, n \text{ is even} \right\} \\ \cup\; & \left\{ a^{15-2n} b^n d b^n c \;:\; 0 < n \leq 7, n \text{ is odd} \right\} \end{aligned}$$

is not in $0EC_{wp}$. It is a finite language, so $L_4 \in COL_x$ for $x \in \{b, t, wp, sp\}$. □

Theorem 9

$$COL_x - 0EC_{ap} \neq \emptyset, \quad x \in \{b, t, wp, sp\}. \tag{14}$$

Proof. The finite language

$$L_6 = \{a, aa\}$$

is produced by a colony with one component $(S, \{a, aa\})$ and axiom S for any derivation mode $x, x \in \{b, t, wp, sp\}$, therefore $L_6 \in COL_x$.

In an 0L eco-colony we have only one alphabet. So all active agents have the start symbol a and the form $(a, \{\varepsilon\})$. The axiom is one of the words of the language – a or aa.

Assume that the axiom is a. There exists at least one agent rewriting a to ε, so the generated language is $\{a, \varepsilon\}$. But the empty word $\varepsilon \notin L_6$.

Suppose that the axiom is aa. There exists some agent rewriting one of the both a-s to ε, so the word a can be generated. But this agent works in the next derivation step (or steps) too: $aa \overset{ap}{\Longrightarrow}^* a \overset{ap}{\Longrightarrow}^* \varepsilon$, and the word not contained in L_6 is generated. So $L_6 \notin 0EC_{ap}$. □

Theorem 10

$$COL_b \subset EEC_{ap}. \tag{15}$$

Proof. We have a colony with the b mode of derivation $\mathcal{C} = (V, T, A_1, \ldots A_n, w_0)$, and we create an equivalent E0L eco-colony $\Sigma = (E, A_1, A_2, BCw_0)$ with the ap derivation and agents $A_1' = (B, \{C, \varepsilon\})$, $A_2' = (C, \{B, \varepsilon\})$.

We create rules of the environment from the components $A_1, \ldots A_n$. We can suppose that all these components have different start symbols.

For each component $(a, \{\alpha_1, \alpha_2, \ldots, \alpha_k\})$ we create developing rules for the environment:

$$a \to a \mid \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_k$$

and for every symbol b which is not the start symbol in any component we create one rule $b \to b$.

So the environment simulates the action of the components in the colony. The simulation of a sequential derivation is possible using the identical rules rewriting symbol to itself for all but one letter of the word.

From the construction it follows that $w_0 \overset{b}{\Rightarrow}^* w$ implies $BCw_0 \overset{ap}{\Longrightarrow}^* w$ and $BCw_0 \overset{ap}{\Longrightarrow}^* w$ implies $w_0 \overset{b}{\Rightarrow}^* w$.

The proper subset comes from Theorem 5. □

Example 2 We demonstrate the construction of the proof on the colony generating the language

$$L_7 = \left\{ waw^R a^i \;:\; w \in \{0,1\}^*,\; i > 0 \right\}.$$

We have a colony $\mathcal{C} = (\{S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, A_1, A_2, A_3, A_4, A_5, S)$ generating the language L_7 where

$$\begin{array}{lll} A_1 = (S,\ \{HA\}), & A_2 = (H,\ \{0H'0, 1H'1, a\}), & A_4 = (A,\ \{aA', a\}), \\ & A_3 = (H',\ \{H\}), & A_5 = (A',\ \{A\}). \end{array}$$

Now we create an E0L eco-colony with ap derivation $\Sigma = (E, A_1, A_2, BCS)$, $E = (\{B, C, S, H, H', A, A', 0, 1, a\},\ \{0, 1, a\},\ P)$, $A_1 = (B, \{C, \varepsilon\})$, $A_2 = (C, \{B, \varepsilon\})$, the set of rules P in the environment is

$$P = \{\ H \to H \mid 0H'0 \mid 1H'1 \mid A, \quad H' \to H' \mid H, \quad 1 \to 1, \quad a \to a,$$
$$A \to A \mid aA' \mid a, \quad A' \to A' \mid A, \quad 0 \to 0, \quad S \to S \mid HA\ \}.$$

One of the derivations in $\mathcal{C}$:

$$S \stackrel{b}{\Rightarrow} HA \stackrel{b}{\Rightarrow} 1H'1A \stackrel{b}{\Rightarrow} 1H1A \stackrel{b}{\Rightarrow} 10H'01A \stackrel{b}{\Rightarrow} 10H01A \stackrel{b}{\Rightarrow} 10a01A \stackrel{b}{\Rightarrow}$$
$$\stackrel{b}{\Rightarrow} 10a01aA' \stackrel{b}{\Rightarrow} 10a01aA \stackrel{b}{\Rightarrow} 10a01aa.$$

Two of possible derivations of the same word in Σ:

$$BCS \stackrel{ap}{\Longrightarrow} CBHA \stackrel{ap}{\Longrightarrow} BC1H'1A \stackrel{ap}{\Longrightarrow} CB1H1A \stackrel{ap}{\Longrightarrow} BC10H'01A \stackrel{ap}{\Longrightarrow}$$
$$\stackrel{ap}{\Longrightarrow} CB10H01A \stackrel{ap}{\Longrightarrow} BC10a01A \stackrel{ap}{\Longrightarrow} CB10a01aA' \stackrel{ap}{\Longrightarrow} BC10a01aA \stackrel{ap}{\Longrightarrow}$$
$$\stackrel{ap}{\Longrightarrow} 10a01aa, \text{ and}$$

$$BCS \stackrel{ap}{\Longrightarrow} CBHA \stackrel{ap}{\Longrightarrow} BC1H'1aA' \stackrel{ap}{\Longrightarrow} CB1H1aA \stackrel{ap}{\Longrightarrow} BC10H'01aa \stackrel{ap}{\Longrightarrow}$$
$$\stackrel{ap}{\Longrightarrow} CB10H01aa \stackrel{ap}{\Longrightarrow} 10a01aa.$$

Corollary 4

$$COL_b^T \subset EEC_{ap}. \tag{16}$$

Proof. Follows from Theorem 10 and Lemma 1. □

5 Conclusions

In this paper we study the type of grammar systems, eco-colonies based on colonies and eco-grammar systems. We summarize the results in the table 1. The symbol *T<number>* means Theorem with the referred number, the symbol *C<number>* means Corollary with the referred number. The symbol ⚭ in the table means incomparable classes of languages.

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	$0EC_{wp}$	$0EC_{ap}$	EEC_{wp}	EEC_{ap}
COL_b	$\bigcirc\!\!\!\bigcirc$ $T7,T8$	$\bigcirc\!\!\!\bigcirc$ $T7,T9$	$\subset$ $C1$	$\subset$ $T10$
COL_t	$\bigcirc\!\!\!\bigcirc$ $T7,T8$	$\bigcirc\!\!\!\bigcirc$ $T7,T9$	$\not\supseteq$ $T7$	$\not\supseteq$ $T7$
COL_{wp}	$\bigcirc\!\!\!\bigcirc$ $T7,T8$	$\bigcirc\!\!\!\bigcirc$ $T7,T9$	$\subset$ $T3$	$\not\supseteq$ $T7$
COL_{sp}	$\bigcirc\!\!\!\bigcirc$ $T7,T8$	$\bigcirc\!\!\!\bigcirc$ $T7,T9$	$\not\supseteq$ $T7$	$\not\supseteq$ $T7$
COL_b^T	$\subset$ $C3$	$\not\supseteq$ $C2$	$\subset$ $C1$	$\subset$ $C4$
COL_t^T	$\not\supseteq$ $C2$	$\not\supseteq$ $C2$	$\not\supseteq$ $C2$	$\not\supseteq$ $C2$
COL_{wp}^T	$\subset$ $C3$	$\not\supseteq$ $C2$	$\subset$ $C1$	$\not\supseteq$ $C2$
COL_{sp}^T	$\not\supseteq$ $C2$	$\not\supseteq$ $C2$	$\not\supseteq$ $C2$	$\not\supseteq$ $C2$
$0EC_{wp}$	$=$	$\bigcirc\!\!\!\bigcirc$ $T6$	$\subset$ $T4$	
$0EC_{ap}$	$\bigcirc\!\!\!\bigcirc$ $T6$	$=$		$\subset$ $T5$
EEC_{wp}	$\supset$ $T4$		$=$	
EEC_{ap}		$\supset$ $T5$		$=$

Table 1: Results from theorems and corollaries

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