Proceedings of Grammar Systems Week 2004, edited by E. Csuhaj-Varjú and Gy. Vaszil, MTA SZTAKI, Budapest, pages 313 - 327, 2004.

On metalinear CD grammar systems

Bettina Sunckel
Institut für Informatik Johann Wolfgang Goethe-Universität
D-60054 Frankfurt am Main, Germany
sunckel@psc.informatik.uni-frankfurt.de

Abstract

Metalinear CD grammar systems are defined to be context-free CD grammar systems where each component consists of metalinear productions. The maximal number of nonterminals in a starting production is the width of a CD grammar system. It will be shown that the width of metalinear CD grammar systems induces an infinite hierarchy of language classes. In addition it is established that metalinear CD grammar systems of a certain width generate language classes that do not contain all context-free languages but contain some context-sensitive languages. The resulting language classes are closed under union, intersection with regular languages, homomorphism and inverse homomorphism. They are not closed under concatenation, Kleene closure, intersection and complement.

1 Introduction

There are many ways to combine the classical formal language theory with the concept of distribution. Grammar systems are a combination of formal grammars and distribution. A grammar systems consist of several components where each component is a grammar. Cooperating distributed (CD) grammar systems are a sequential model where at one point of time only one component (the active component) contributes to the derivation. A good overview of the topic can be found in [2] or in [4] and also a connection to artificial intelligence is given in [2].

From the language theoretic point of view it is desirable to find grammar models that combine the simplicity of context-free rules with the power of generating some context-sensitive languages. Context-free CD grammar systems consist of several context-free grammars and can be seen as a generalization of context-free grammars. The derivation mode defines how long productions of one component can be used and when the next component becomes active. Apart from the t-mode where each component contributes to the derivation as long as possible, we will consider the $(= k)$- and $(\geq k)$-mode where each component performs exactly k or $\geq k$ steps, respectively. In case of the t-mode the class of $ET0L$ languages is generated and using the $(= k)$ and$(\geq k)$-mode we obtain a subclass of the language class generated by matrix grammars [2]. Thus, the concept of distribution adds generative power to context-free productions.

However, context-free CD grammar systems are not as easy to handle as context-free grammars. For instance, it is not known whether a tool like a pumping lemma exists. Hence, it is difficult to exclude a certain language from a certain language class generated by CD grammar systems. It is therefore natural to find subclasses which are easier to use but still contain interesting non context-free languages. When considering context-free languages, the most simple form of a production is a linear production. But we will show that CD grammar systems with linear productions only generate linear context-free languages. Metalinear context-free grammars are only allowed to have more than one nonterminal in a starting production. In this way the derivation trees stay nearly as simple as linear derivation trees.

We define metalinear CD grammar systems to be CD grammar systems with metalinear context-free grammars as components and prove a pumping lemma similar to that for metalinear context-free languages. Then it is established that the width, which is the maximal number of nonterminals in a starting production, induces an infinite hierarchy of language classes. Also some interesting context-sensitive languages that can be generated by metalinear CD grammar systems are presented. However, it is not possible to generate all context-free languages. Furthermore, some nice closure properties are proven such as closure under homomorphism, inverse homomorphism, union and intersection with regular sets. The resulting language classes are not closed under under concatenation, Kleene closure intersection and complement.

2 Definitions

The basics of formal language theory can be found in [5]. Let A be a finite alphabet. We denote the empty string by ε and the Kleene closure of A by A^*. Let B be a finite set. With $|B|$ we refer to the cardinality of B and with B' we refer to $\{a' \mid a \in B\}$.

A context-free grammar $G = (N, T, P, S)$ is linear, if each rule is of the form $A \to uBv$ or $A \to u$ where $A, B \in N$ and $u, v \in T^*$. A language L is said to be linear if there exists a linear grammar G such that $L(G) = L$.

A context-free grammar $G = (N, T, P, S)$ is m-linear, if each rule is of the form $S \to A_1 \dots A_{m_0}$, $A \to uBv$ or $A \to u$ where $A, A_1, \dots, A_{m_0}, B \in (N \setminus \{S\})$, $m_0 \leq m$ and $u, v \in T^*$. A language L is m-linear, if there is an m-linear grammar G such that $L(G) = L$. If a grammar or language is m-linear for some $m \geq 2$, it is also called metalinear.

We denote the class of regular languages by REG, the class of context-free languages by CF and the class of linear languages by LIN. For the class of m-linear languages we will write $mLIN$ and $METALIN := \bigcup_{i \geq 1} iLIN$.

Definition 2.1. A CD grammar system is a $k+3$ tuple $\Gamma = (N, T, P_1, \dots, P_k, S)$, where N is a finite set of nonterminals, T is a finite set of terminals, P_i, $1 \leq i \leq k$ is a finite set of rules and $S \in N$ is the axiom.

In the following, we will write $\beta_1 X \beta_2 \Rightarrow_{P_i} \beta_1 \alpha \beta_2$ if $X \to \alpha$ is in P_i and β_1, β_2 are in $(N \cup T)^*$. We say that the production $X \to \alpha$ is applied to the derivation string $\beta_1 X \beta_2$. We denote reflexive and transitive closure of $\Rightarrow_{P_i}$ by $\Rightarrow^*_{P_i}$ and we write

$\alpha \Rightarrow_{P_i}^k \beta$, if α derives β in exactly k steps. We define the domain of a component as $Dom(P_i) := \{A \mid A \to \alpha \in P_i\}$.

Definition 2.2. Let Γ be a CD grammar system.

1. For each i, $1 \leq i \leq n$, a terminating derivation in the i-th component is $x \Rightarrow_{P_i}^t y$ iff $x \Rightarrow_{P_i}^* y$ and there is no $z \in (N \cup T)^*$ with $y \Rightarrow_{P_i} z$.
2. For each i, $1 \leq i \leq n$, a k-steps derivation in the i-th component is $x \Rightarrow_{P_i}^{=k} y$ iff there are $x_1, ..., x_{k+1} \in (N \cup T)^*$ such that $x = x_1, y = x_{k+1}$ and $x_j \Rightarrow_{P_i} x_{j+1}, 1 \leq j \leq k$.
3. For each i, $1 \leq i \leq n$, an at most k-steps derivation in the i-th component is $x \Rightarrow_{P_i}^{\leq k} y$ iff $x \Rightarrow_{P_i}^{=k'} y$ for some $k' \leq k$.
4. For each i, $1 \leq i \leq n$, an at least k-steps derivation in the i-th component is $x \Rightarrow_{P_i}^{\geq k} y$ iff $x \Rightarrow_{P_i}^{=k'} y$ for some $k' \geq k$.

Now we define metalinear CD grammar systems as CD grammar systems that consist of context-free metalinear grammars as components.

Definition 2.3. A CD grammar system is called m-linear for a fixed $m \geq 2$, if each production is formed as follows: $S \to A_1 \dots A_{m_0}$, $A \to uBv$, $A \to u$ with $A, B, A_1, \dots, A_{m_0} \in (N \setminus \{S\})$, $m_0 \geq m$ and $u, v \in T^*$. If a CD grammar system is m-linear for some $m \geq 2$, it is also called metalinear. We refer to m as the width of an m-linear CD grammar system.

We denote the class of m-linear CD grammar systems with l components by CD_l-*mLIN* and the class of metalinear CD grammar systems with l components by CD_l-*METALIN*. For the class of languages which are generated by grammars in CD_l-*mLIN* or CD_l-*METALIN* in derivation mode $f \in \{*, t, = k, \geq k, \leq k \mid k \geq 1\}$ we write $\mathcal{L}_f(CD_l\text{-}mLIN)$ or $\mathcal{L}_f(CD_l\text{-}METALIN)$, respectively. We omit the number of components, if no restrictions are made.

Definition 2.4. An $ET0L$ system is a (n+3)-tuple $G = (V, T, P_1, \dots, P_n, w)$ where V is the total alphabet, $T \subseteq V$ is the terminal alphabet, $P_i, 1 \leq i \leq n$ are the tables consisting of context-free rules, $w \in V^*$ is the start string.

Each table is complete, that is for all symbols $a \in V$ it contains at least one rule of the form $a \to x$. In the following rules of the form $a \to a$ are not explicitly written. A symbol a in an $ET0L$ system is called active if there is a table in G with a production $a \to x$ where $a \neq x$. The number of active symbols in the string $\alpha \in V$ is denoted by $\#_{A(G)}(\alpha)$.

Definition 2.5. An $ET0L$ system $G = (V, T, P_1, \dots, P_n, S)$ is called m-linear if and only if it has the following properties. $S \in V \setminus T$ does not appear at the right hand side of any production. If $S \Rightarrow_G \alpha$ then $\#_{A(G)}(\alpha) \leq k$ Every production whose left-hand side is not S is linear. An $ET0L$ system is called metalinear if and only if it is m-linear for some m.

Metalinear $ET0L$ systems are investigated in [6]. We will denote the class of m-linear ET0L systems with $ET0L_{mLIN}$, the class of metalinear ET0L systems with $ET0L_{METALIN}$ and the corresponding language classes with $\mathcal{L}(ET0L_{mLIN})$ and $\mathcal{L}(ET0L_{METALIN})$.

Definition 2.6. A context-free matrix grammar is a 4-Tuple $G = (N, T, M, S)$ where N is a set of nonterminals, T is a set of terminals, M a finite set of sequences $s : (r_1, r_2, \ldots, r_{n_s}), n_s \geq 1$ with $r_i \in N \times (T \cup N)^*$ and S is the axiom.

A derivation step of a matrix grammar consists of the sequential application of the rules $r_1, \ldots, r_{n_m}$ to the derivation string.

Definition 2.7. A matrix grammar $G = (N, T, M, S)$ is called m-linear if and only if each production is formed as follows: $S \rightarrow A_1 \ldots A_{m_0}$, $A \rightarrow uBv$, $A \rightarrow u$ with $A, B, A_1, \ldots, A_{m_0} \in (N \setminus \{S\})$, $m_0 \geq m$ and $u, v \in T^*$. An matrix grammar is called metalinear if and only if it is m-linear for some m.

Definition 2.8. Let Γ be a CD grammar system. A tree is a derivation tree of Γ iff the following holds. Every vertex has a label which is a symbol of $N \cup T \cup \{\varepsilon\}$. The label of the root is S. If a vertex is interior and has label A, then A must be in N. If n has label A and vertices $n_1, n_2, \ldots, n_k$ are sons of vertex n, in order from the left, with labels $X_1, X_2, \ldots X_k$, respectively, then $A \rightarrow X_1 X_2 \ldots X_k$ must occur in one of the components of Γ. If vertex n has label ε, then n is a leaf and is the only son of its father.

The labels of the leaves of a derivation tree read from left to right are called the *yield* of the tree. A *subtree* of a derivation tree is a particular vertex together with all its descendants. It should be noted that each derivation of a CD grammar system corresponds to a derivation tree, but there are also derivation trees which do not correspond to a derivation of the underlying CD grammar system. We call derivation trees which correspond to a derivation of Γ *valid.* If a valid derivation tree has a yield $w \in T^*$ we call it a *valid complete* derivation tree.

3 Examples

In this section we will show, that there are interesting non context-free languages which can be generated by m-linear CD grammar systems. Furthermore there are m-linear context-free languages which can be generated by $(m-1)$-linear CD grammar systems.

Example 3.1. Consider the language $L_1 = \{a_1^n a_2^n b_1^m b_2^m c_1^l c_2^l \mid l, m, n \geq 1\}$. The following 2-linear grammar system will generate L_1 using the t-mode. L_1 is known to be 3-linear context-free.

$$\begin{aligned}
\Gamma_1 &= \{\{S, A, C, B_1, B_2, B_1', B_2'\}, \{a, b, c\}, P_1, P_2, P_3, P_4, S\},\\
P_1 &= \{S \rightarrow B_1 B_2, B_1 \rightarrow B_1' b_1, B_2 \rightarrow b_2 B_2'\},\\
P_2 &= \{B_1' \rightarrow B_1, B_2' \rightarrow B_2\},\\
P_3 &= \{B_1 \rightarrow A b_1, B_2 \rightarrow b_2 C\},\\
P_4 &= \{A \rightarrow a_1 A a_2, C \rightarrow c_1 C c_2, A \rightarrow a_1 a_2, C \rightarrow c_1 c_2\}.
\end{aligned}$$

With the component P_1 the two nonterminals B_1 and B_2 are inserted. After that, the numbers of b_1s and b_2s will be increased with the components P_1 and P_2. The derivation of bs is finished with component P_3. Component P_4 inserts as and cs like a context-free grammar and completes the derivation.
$S \Rightarrow_{P_1} B_1' b_1 b_2 B_2' \Rightarrow_{P_2} B_1 b_1 b_2 B_2 \Rightarrow_{P_1} \ldots \Rightarrow_{P_2} B_1 b_1^{m-1} b_2^{m-1} B_2 \Rightarrow_{P_3} A b_1^m b_2^m C \Rightarrow_{P_4} a_1^n a_2^n b_1^m b_2^m c_1^l c_2^l$.

Example 3.2. The language $L_2 = \{ww \mid w \in \{a,b\}^+\}$ is generated by the following grammar system using the $(=2)$-mode:

$$\begin{aligned}
\Gamma_2 &= \{\{S, S', A, B, A', B'\}, \{a, b\}, P_1, P_2, P_3, S\},\\
P_1 &= \{S \to S', S' \to AB, A \to aA', B \to aB', A \to a, B \to a\},\\
P_2 &= \{A \to bA', B \to bB', A \to b, B \to b\},\\
P_3 &= \{A' \to A, B' \to B\}.
\end{aligned}$$

The following derivation shows how Γ_2 works.
$S \Rightarrow_{P_1} AB \Rightarrow_{P_1} aA'aB' \Rightarrow_{P_3} aAaB \Rightarrow_{P_2} abA'abB' \Rightarrow_{P_3} abAabB \Rightarrow_{P_1} abaaba$

Example 3.3. The language $L_3 = \{a^n b^n a^n b^n a^n b^n \mid n \geq 0\}$ is generated by the following grammar system using the $(=2)$-mode

$$\begin{aligned}
\Gamma_3 &= \{\{S, A, B, C, A', B', C', B'', B''', B''''\}, \{a, b\}\\
&\quad , P_1, P_2, P_3, P_4, P_5, P_6, P_7, S\}\\
P_1 &= \{S \to S', S' \to ABC\},\\
P_2 &= \{A \to aA'b, B \to aB'b\},\\
P_3 &= \{B' \to B'', C \to aC'b\},\\
P_4 &= \{A' \to A, B'' \to B'''\},\\
P_5 &= \{B''' \to B, C' \to C\},\\
P_6 &= \{A \to \varepsilon, B \to B''''\},\\
P_7 &= \{B'''' \to \varepsilon, C \to \varepsilon\}.
\end{aligned}$$

The derivations of Γ_3 are formed as follows:
$S \Rightarrow_{P_1} ABC \Rightarrow_{P_2} aA'baB'bC \Rightarrow_{P_3} aA'baB''baC'b \Rightarrow_{P_4}$
$aAbaB'''baC'b \Rightarrow_{P_5} aAbaBbaCb (\Rightarrow_{P_2} \ldots \Rightarrow_{P_3} \ldots \Rightarrow_{P_4} \ldots \Rightarrow_{P_5})^{n-1}$
$a^n A b^n a^n B b^n a^n C b^n \Rightarrow_{P_6} a^n b^n a^n B'''' b^n a^n C b^n \Rightarrow_{P_7} a^n b^n a^n b^n a^n b^n$

4 Generative Capacity

4.1 Pumping Lemma

Considering the generative capacity of languages we need the possibility to decide, if certain languages are included in a language class. For metalinear CD grammar systems it will be possible to obtain a pumping lemma. This will help us to establish a hierarchy of language classes consisting of languages generated by metalinear CD grammar systems.

Lemma 4.1. *Let $f \in \{t, = k, \geq k \mid k \geq 2\}$ and $L \in \mathcal{L}_f(CD_l\text{-}mLIN)$ be an infinite language. Then there is a $h \in \mathbb{N}$ and for all w with $|w| \geq h$ such that $w = c_1 \dots c_m$ and*

1.
   - $c_i = x_i u_i y_i v_i z_i$, $1 \leq i \leq m$,
   - $u_1 v_1 u_2 v_2 \dots u_m v_m \neq \varepsilon$,
   - *for all* i_0, $1 \leq i_0 \leq m$ *with* $|c_{i_0}| > h/m$ *it is* $u_{i_0} v_{i_0} \neq \varepsilon$ *and* $|x_{i_0} u_{i_0} v_{i_0} z_{i_0}| \leq h/m$,
   - *for all* $j, j \geq 0$ $x_1 u_1^j y_1 v_1^j z_1 x_2 u_2^j y_2 v_2^j z_2 \dots x_m u_m^j y_m v_m^j z_m \in L$.

2. *for all* i_0, $1 \leq i_0 \leq m$ *with* $|c_{i_0}| > h/m + s$ *and each substring* c'_{i_0} *with* $|c'_{i_0}| > h/m + s$ *there is a* γ *with*
   - $c'_{i_0}\gamma$ *is a substring of* c_{i_0},
   - $c'_{i_0}\gamma = xuyvz$ *and* xu *is a substring of the first* $h/m + s$ *terminals of* c'_{i_0} *and* $|u| > 0$,
   - *for all* $j, j \geq 0$ $\phi x u^j y v^j z \phi' \in L$.

   or

   - $\gamma c'_{i_0}$ *is a substring of* c_{i_0},
   - $\gamma c'_{i_0} = xuyvz$ *and* vz *is a substring of the last* $h/m + s$ *terminals of* c'_{i_0} *and* $|u| > 0$,
   - *for all* $j, j \geq 0$ $\phi x u^j y v^j z \phi' \in L$.

Proof. We will first prove the claim for the t-mode of derivation.
1. Let $L \in \mathcal{L}_t(CD_l\text{-}mLIN)$ an infinite language and $G \in CD_l\text{-}mLIN$ be a grammar system that generates L using the t-mode. Furthermore let s be the maximum number of terminals in one Production of G, p the maximum number of productions with terminals in one component and r the number of nonterminals in G. Now, consider a word $w \in L$ consisting of n terminals. Then there were at least n/s productions used in a derivation and the corresponding derivation tree of this word. Let $h = s \cdot (r \cdot l \cdot p + 1) \cdot m^2$ and $w \in L$ with $|w| \geq h$. Then w is divided in m parts $w = c_1 \dots c_m$ with $S \Rightarrow A_1 \dots A_m$ and $A_i \Rightarrow c_i$ and $1 \leq i \leq m$. There is at least one $c_{i_0} \geq s \cdot (r \cdot l \cdot p + 1)$. Now, consider the subtree with root A_{i_0} that yields c_{i_0}. Note that only linear productions are used in this subtree. We analyze the first $l \cdot r \cdot p + 1$ productions with terminals.
Case 1: When deriving c_{i_0} one of the components uses more than p productions that contain terminals. Then there are two vertices vt_1 and vt_2 satisfying the following conditions.

- Both vertices have the same label and the same productions applied to them.
- Vertex vt_1 is closer to the root than vertex vt_2.
- The potion of the path from vt_1 to vt_2 is of length at most p

In this case we have $x_h, u_h, v_h, z_h = \varepsilon,\ 1 \le h \le m,\ h \ne j$ and

$$\begin{aligned} w &= y_1 \dots x_j u_j y_j v_j z_j \dots y_m \\ &= x_1 u_1 y_1 v_1 z_1 \dots x_j u_j y_j v_j z_j \dots x_m u_m y_m v_m z_m. \end{aligned}$$

When the productions on the path from vt_1 to vt_2 are applied i times, we obtain the word

$$\begin{aligned} w' &= y_1 \dots x_j u_j^i y_j v_j^i z_j \dots y_m \\ &= x_1 u_1^i y_1 v_1^i z_1 \dots x_j u_j^i y_j v_j^i z_j \dots x_m u_m^i y_m v_m^i z_m. \end{aligned}$$

Furthermore $0 < |x_{j_0} u_{j_0} v_{j_0} z_{j_0}| \le l \cdot p + 1 \le h/m$.
Case 2: When deriving w none of the components uses more than p productions that contain terminals. Therefore there have to be at least $(r \cdot l + 1) \cdot m$ changes of components in the derivation.

Now, consider the leaves of the derivation tree of w when a component stops working. The leaves are a sequence of at least m nodes each labelled with a terminal or a nonterminal. Since terminals in the derivation tree can not be changed, they are represented as $\times$ in the sequence. Hence, there are only $(r+1) \cdot m$ different sequences for nonterminals at the leaves of the derivation tree. One of the sequences has therefore to occur twice while deriving w. Let

$$(\times_1, \dots, \times_{j_1-1}, B^1_{j_1}, \times_{j_1+1}, \dots, \times_{j_p-1}, B^1_{j_p}, \times_{j_p+1}, \dots, \times_m)$$

and

$$(\times_1, \dots, \times_{j_1-1}, B^2_{j_1}, \times_{j_1+1}, \dots, \times_{j_p-1}, B^2_{j_p}, \times_{j_p+1}, \dots, \times_m)$$

be these sequences where each position stands for one of the m branches of the derivation tree and $\times$ means that there is a terminal at the end of this branch. In this case $x_h, u_h, v_h, z_h = \varepsilon,\ 1 \le h \le m,\ h \ne j_1 \dots j_p$. We have

$$\begin{aligned} w &= y_1 \dots y_{j_1-1} x_{j_1} u_{j_1} y_{j_1} v_{j_1} z_{j_1} y_{j_1+1} \dots y_{j_p-1} x_{j_p} u_{j_p} y_{j_p} v_{j_p} z_{j_p} y_{j_p+1} \dots y_m \\ &= x_1 u_1 y_1 v_1 z_1 \dots x_{j_1-1} u_{j_1-1} y_{j_1-1} v_{j_1-1} z_{j_1-1} x_{j_1} u_{j_1} y_{j_1} v_{j_1} z_{j_1} \\ &\quad x_{j_1+1} u_{j_1+1} y_{j_1+1} v_{j_1+1} z_{j_1+1} \dots \\ &\quad x_{j_p-1} u_{j_p-1} y_{j_p-1} v_{j_p-1} z_{j_p-1} x_{j_p} u_{j_p} y_{j_p} v_{j_p} z_{j_p} \\ &\quad x_{j_p+1} u_{j_p+1} y_{j_p+1} v_{j_p+1} z_{j_p+1} \dots x_m u_m y_m v_m z_m. \end{aligned}$$

If the productions used to change the tree from configuration

$$(\times_1, \dots, \times_{j_1-1}, B^1_{j_1}, \times_{j_1+1}, \dots, \times_{j_p-1}, B^1_{j_p}, \times_{j_p+1}, \dots, \times_m)$$

to configuration

$$(\times_1, \dots, \times_{j_1-1}, B^2_{j_1}, \times_{j_1+1}, \dots, \times_{j_p-1}, B^2_{j_p}, \times_{j_p+1}, \dots, \times_m)$$

are applied i times one gets the following word

$$\begin{aligned} w' &= y_1 \dots y_{j_1-1} x_{j_1} u_{j_1}^i y_{j_1} v_{j_1}^i z_{j_1} y_{j_1+1} \dots y_{j_p-1} x_{j_p} u_{j_p}^i y_{j_p} v_{j_p}^i z_{j_p} y_{j_p+1} \dots y_m \\ &= x_1 u_1^i y_1 v_1^i z_1 \dots x_{j_1-1} u_{j_1-1}^i y_{j_1-1} v_{j_1-1}^i z_{j_1-1} x_{j_1} u_{j_1}^i y_{j_1} v_{j_1}^i z_{j_1} \\ & \quad x_{j_1+1} u_{j_1+1}^i y_{j_1+1} v_{j_1+1}^i z_{j_1+1} \dots \\ & \quad x_{j_p-1} u_{j_p-1}^i y_{j_p-1} v_{j_p-1}^i z_{j_p-1} x_{j_p} u_{j_p}^i y_{j_p} v_{j_p}^i z_{j_p} \\ & \quad x_{j_p+1} u_{j_p+1}^i y_{j_p+1} v_{j_p+1}^i z_{j_p+1} \dots x_m u_m^i y_m v_m^i z_m \end{aligned}$$

with $w' \in L$. Furthermore $0 < |x_{j_0} u_{j_0} v_{j_0} z_{j_0}| \leq l \cdot p + 1 \leq h/m$.
2. As shown in 1. c_{i_0} is the yield of a linear subtree t of the derivation tree of w. When we complete c'_{i_0} with the appropriate γ we obtain the yield of a subtree t'' of t'. Since $|c'_{i_0}| > h/m$, t'' consists of at most $h/m/s$ productions with terminals. We will only consider the case $yield(t'') = c'_{i_0}\gamma$ because the other one can be proven analogously. We have shown in 1. that in a subtree consisting of $h/m/s$ productions either one component has applied more than p productions (case 1) or there have been at least $(r \cdot l + 1) \cdot m$ changes of components (case 2). In both cases there is a vertex vt_1 and a vertex vt_2 of t'' labelled with the same nonterminal. The path from vt_1 to vt_2 is not longer than p or $(r \cdot l + 1) \cdot m$, respectively. Hence $c'_{i_0}\gamma = xuyvz$. When the productions from vt_1 to vt_2 are applied j times we obtain xu^jyv^jz. Since $c'_{i_0} > h/m + s$ we have xu is a substring of the first $h/m + s$ terminals of c'_{i_0} and $|u| > 0$. We have shown in 1. that it is possible to apply the productions from vt_1 to vt_2 j times (possibly together with productions in other subtrees) and obtain a word in L. Therefore there are ϕ and ϕ' and for all $j, j \geq 0$ $\phi xu^jyv^jz\phi' \in L$.

Now we consider the $= k$ mode of derivation. Since only k productions are applied in one component the proof of 1. and 2. is similar to case 2 of the proof of Lemma 4.1. Case 1 is not applicable, because one component has to use exactly k components and pumping within these components is not possible.

The proof for the $\geq k$ mode is similar to the proof for the t mode since here also both cases of pumping can occur. □

4.2 Generative Capacity Results

With the above pumping lemma we first show, that there are context-free languages that can not be generated by metalinear CD grammar systems.

Theorem 4.1. *$CF \not\subseteq \mathcal{L}_f(CD\text{-}mLIN)$ for all $m \geq 2$ and $f \in \{t, = k, \geq k \mid k \geq 2\}$.*

Proof. Consider the language $L = \{a^nb^n \mid n \geq 0\}^*$. This language is context-free. Now we assume that L is in $\mathcal{L}_f(CD_l\text{-}mLIN)$ for $f \in \{t, = k, \geq k \mid k \geq 2\}$ and an arbitrary $m \geq 2$. Consider the word $w = (a^hb^h)^{3m+1}$ where h is the constant from Lemma 4.1. Then $w = c_1 \dots c_m$. It follows that there is a c_{i_0} with $|c_{i_0}| > 6h$. According to Lemma 4.1 we choose a substring $c'_{i_0} = a^{h/m+s}b^ha^hb^{h/m+s}$ of c_{i_0}. Note that this is possible because of the length of c_{i_0}. Then c'_{i_0} is divided as follows $c'_{i_0} = xuyvz$.
Case 1: c'_{i_0} is continued to $c'_{i_0}\gamma$. Then $|xuvz| < h/m$, $|u| > 0$ and therefore only contains as from the first part of c'_{i_0}. It follows that $y > 4h$ and thus contains b^{2h} in the first half of c'_{i_0}. The word $w' = \phi xu^jyv^jz\phi'$ contains at least one pair $a^{2h+x}b^{2h}$

where the number of as is not equal to the number of bs. The word w' is not in L.
Case 2: c'_{i_0} is continued to $\gamma c'_{i_0}$. A word $w' = \phi x u^j y v^j z \phi'$ which contains at least one pair $a^{2h} b^{2h+x}$ where the number of as is not equal to the number of bs can be constructed similar as in case 1. The word w' is not in L.
It follows that L is not in $\mathcal{L}_f(CDmLIN)$ for $f \in \{t, = k, \geq k \mid k \geq 2\}$ and an arbitrary $m \geq 2$. □

Definition 4.1. For all $n \in \mathbb{N}$ let $L_n = \{(a^i b^i)^n \mid i \geq 1\}$.

Theorem 4.2. *Let $f \in \{t, = k, \geq k \mid k \geq 2\}$ and $n \geq 2$. The language L_n is in $\mathcal{L}_f$(CD-nLIN) but not in $\mathcal{L}_f$(CD-$(n-1)$LIN).*

Proof. The following grammar system in *CD-nLIN* can generate L_n in the t-mode: Let $G_n = (\{S, A, B\}, \{a, b\}, P_1 \ldots P_5, S)$ and

$$\begin{aligned} P_1 &= \{S \to A^n\}, \\ P_2 &= \{A \to aBb\}, \\ P_3 &= \{B \to aAb\}, \\ P_4 &= \{A \to ab\}, \\ P._5 &= \{B \to ab\} \end{aligned}$$

The components P_2 or P_3 add an equal number of as and bs two the n substrings derived from the n nonterminals A which are added with the starting production. Component P_4 or P_5 terminate a derivation.
We will now give a grammar system in *CD-nLIN* that generates L_n in the $(= 2)$- or (≥ 2)-mode. Let $G_n = (\{S, B, A_1, A'_1, A''_1, A'''_1, A''''_1, \ldots, A_n, A'_n, A''_n, A'''_n, A''''_n\}, \{a, b\}, P_1 \ldots P_{3n-2}, S)$ and

$$\begin{aligned} P_1 &= \{S \to BA_2 \ldots A_n, B \to A_1\}, \\ P_2 &= \{A_1 \to aA'_1 b, A_2 \to A''_2\}, \\ P_3 &= \{A''_2 \to aA'_2 b, A_3 \to A''_3\}, \\ &\ldots \\ P_{n-1} &= \{A''_{n-2} \to aA'_{n-2} b, A_{n-1} \to A''_{n-1}\}, \\ P_n &= \{A''_{n-1} \to aA'_{n-1} b, A_n \to aA'_n b\}, \\ P_{n+1} &= \{A'_n \to A_n, A'_{n-1} \to A'''_{n-1}\}, \\ P_{n+2} &= \{A'_{n-2} \to A'''_{n-2}, A'''_{n-1} \to A_{n-1}\}, \\ &\ldots \\ P_{2n-1} &= \{A'_1 \to A_1, A'''_2 \to A_2\}, \\ P_{2n} &= \{A_1 \to ab, A_2 \to A''''_2\}, \\ P_{2n+1} &= \{A''''_2 \to ab, A_3 \to A''''_3\}, \\ &\ldots \\ P_{3n-2} &= \{A''''_{n-1} \to ab, A_n \to ab\}. \end{aligned}$$

The derivation begins with P_1. After that P_2 to P_n add one a and one b to the n substrings derived from A_1 to A_n. Note that these components can only be applied in the given order. The resulting derivation string contains only primed nonterminals. Then the components P_{n+1} to P_{2n-1} change the nonterminals in the derivation string to unprimed nonterminals. These components can also only be applied in the given order. The derivation is finished with the components P_{2n} to P_{3n-1} which can again be only applied in the given order.
Now we assume that L_n is in $\mathcal{L}_f$*(CD-*$(n-1)$*LIN)*. Let h be the constant known from the pumping lemma. We choose a word w in L_n with $|w| > h$. Then $w = (a^i b^i)^n$. But we know from Lemma 4.1 that

$$w = x_1 u_1 y_1 v_1 z_1 x_2 u_2 y_2 v_2 z_2 \dots x_n u_n y_n v_n z_n$$

and the following holds: At least one of the u_j or v_j is not the empty word. All u_j and v_j only consist of as only consist of bs or else we can obtain a word not in L_n. There are at most $2(n-1)$ u_j and v_j. But if at least one and at most $2(n-1)$ of the substrings a^n or b^n are pumped to $a^{n'}$ or $b^{n'}$ there remain at least two substrings a^n or b^n and we also obtain a word that is not in L_n. Therefore $L_n \notin \mathcal{L}_f$*(CD-*$(n-1)$*LIN)*. □

Theorem 4.3. *Let* $f \in \{*, t, =k, \geq k, \leq k \mid k \geq 1\}$ *then* $\mathcal{L}_f(CD_l$*-1LIN)=LIN.*

Proof. Let L be a language in LIN then there is a linear CD grammar system Γ with one component that consists of the productions of the linear context-free grammar G with $L(G) = L$ and chain productions $A \to A$ for each nonterminal A in G. Γ generates L using an arbitrary mode in f.
Now, let Γ be a linear CD grammar system and $L = \mathcal{L}_f(\Gamma)$. If $f \in \{*, =1, \geq 1, \leq k \mid k \geq 1\}$ we can modify the proof in [2] for unrestricted CD grammar systems and context-free grammars and obtain a linear context-free grammar that generates L.
Now, let $f = t$ and let $\Gamma = (N, T, P_1, \dots P_l, S)$ be a linear CD grammar system that generates L in t-mode. The following linear context-free grammar G generates L: $G = (\{N_i \mid 1 \leq i \leq l\} \cup \{S'\}, T, P', S')$ and $P' = \{A_i \to \alpha B_i \beta \mid A \to \alpha B \beta \in P_i, 1 \leq i \leq l\} \cup \{A_i \to A_j \mid A \notin Dom(P_i), A \in Dom(P_j) 1 \leq i,j \leq l\} \cup \{S' \to S_i \mid S \in P_i, 1 \leq i \leq l\}$.
Next, let $f = (=k)$ or $f = (\geq k)$, $k \geq 2$ and et $\Gamma = (N, T, P_1, \dots P_l, S)$ be a linear CD grammar system that generates L in $=k$-mode. The following linear context-free grammar G generates L: $G = \{N_{i,j} \mid 1 \leq i \leq k, 1 \leq j \leq l\}, T, P', S')$ and $P' = \{A_{i,j} \to \alpha B_{i+1,j} \beta \mid A \to \alpha B \beta \in P_j, 1 \leq j \leq l, 1 \leq i \leq k-1\} \cup \{A_{k,j_1} \to A_{1,j_2} \mid A \in N, 1 \leq j_1, j_2 \leq l\} \cup \{S' \to S_{1,j} \mid 1 \leq j \leq l\}$. In a similar way we can construct a linear context free grammar if $f = (\geq k)$: $G = \{N_{i,j} \mid 1 \leq i \leq k, 1 \leq j \leq l\}, T, P', S')$ and $P' = \{A_{i,j} \to \alpha B_{i+1,j} \beta \mid A \to \alpha B \beta \in P_j, 1 \leq j \leq l, 1 \leq i \leq k-1\} \cup \{A_{i,j} \to \alpha B_{i,j} \beta \mid A \to \alpha B \beta \in P_j, 1 \leq j \leq l, 1 \leq i \leq k\} \cup \{A_{k,j_1} \to A_{1,j_2} \mid A \in N, 1 \leq j_1, j_2 \leq l\} \cup \{S' \to S_{1,j} \mid 1 \leq j \leq l\}$. □

Theorem 4.4. *Let* $f \in \{=k, \geq k \mid k \geq 2\}$ *then* $\mathcal{L}_f$*(CD-mLIN)*$\subseteq \mathcal{L}(MAT_{mLIN})$

Proof. We consider the proof in the general case from [2] theorem 3.13 and 3.14. When the construction is applied to a metalinear CD grammar system the structure of the productions is not changed and the resulting matrix grammar is metalinear and of the same width. □

With a proof related to that for the 2-normal form [3] for matrix grammars it will be possible to show that for all CD grammar systems of finite index or metalinear CD grammar systems there exists an equivalent CD grammar system working in $(=2)$-mode that is of finite index or metalinear, respectively [1]. Hence it is likely that there exists a pumping lemma for metalinear CD grammar systems working in one of the k-modes that is easier than the one shown in this paper.

Theorem 4.5. *For the t-mode the following holds:*

1. $\mathcal{L}_t(CD_3\text{-}mLIN) = \mathcal{L}_t(CD\text{-}mLIN)$
2. $\mathcal{L}_t(CD_1\text{-}mLIN) = \mathcal{L}_t(CD_2\text{-}mLIN)$

Proof. ¿From [4] Theorem 3.1 we have the proof for 1. and in [2] Theorem 3.10 is the proof for 2. in the general case. When we apply the constructions to a metalinear CD grammar system the structure of the productions is not changed. Thus the resulting CD grammar system in 1. has only three components and is still metalinear and of the same width. The resulting context-free grammar in 2. is m-linear. □

Since metalinear CD grammar systems working in the t-mode are equivalent to metalinear $ET0L$ systems the normal form and the pumping lemma from [6] also holds for metalinear CD grammar systems working in t mode.

Theorem 4.6. $\mathcal{L}_t(CD\text{-}mLIN) = \mathcal{L}(ET0L_{mLIN})$

Proof. In [2] theorem 3.10 it is shown that $\mathcal{L}(ET0L) \subseteq \mathcal{L}_t(CDCF)$. When the construction is applied to a metalinear CD grammar system the structure of the productions is not changed and we obtain $\mathcal{L}(ET0L_{mLIN}) \subseteq \mathcal{L}_t(CD\text{-}mLIN)$.
Now we consider an m-linear CD grammar system $G = (N, T, P_1, \ldots, P_n, S)$. We construct an m-linear $ET0L$ system G' that generates $\mathcal{L}_t(G)$. $G' = (\{A^i \mid 1 \leq i \leq n \mid A \in N\} \cup \{X\} \cup T, T, \{Q_1, \ldots, Q_n\} \cup \{Q_{j,k} \mid 1 \leq k \neq j \leq n\}, S)$. The tables are defined as follows

$$\begin{aligned}
Q_i &= \{A^i \to \alpha B^i \beta \mid A \to \alpha B \beta \in P_i\} \cup \\
&\quad\ \{S \to A_1^i \ldots A_{m_0}^i \mid S \to A_1 \ldots A_{m_0} \in P_i\}, \\
&\quad\ 1 \leq i \leq n \\
Q_{j,k} &= \{A^j \to A^k \mid A \notin Dom(P_j)\} \cup \\
&\quad\ \{A^j \to X \mid A \in Dom(P_j)\}, \\
&\quad\ 1 \leq k \neq j \leq n
\end{aligned}$$

Note that G' is metalinear and of the same width as G. A derivation in G with component P_i is simulated in G' by applying Q_i one ore more times. Note that a change of superscript i of the nonterminals in G' is only possible if there is no nonterminal from $Dom(P_i)$ in the derivation string. In this way the stop conditions of the t-mode derivations in G are correctly simulated in G'. In summary we have $\mathcal{L}_t(CD\text{-}mLIN) \subseteq \mathcal{L}(ET0L_{mLIN})$ □

5 Closure Properties

Since metalinear CD grammar systems that are working in t-mode are equivalent to metalinear $ET0L$ systems, the closure properties for this mode can be found in [6]. The following proofs hold for all the derivation modes considered in this paper.

Theorem 5.1. *Let $f \in \{t, = k, \geq k \mid k \geq 2\}$, then $\mathcal{L}_f$(CD-mLIN) is closed with respect to*

1. *union*
2. *intersection with regular languages*
3. *homomorphism*
4. *inverse homomorphism.*

Proof. 1. Let L_1 and L_2 be in $\mathcal{L}_f$*(CD-mLIN)* and $G_1 = (N_1, T_1, P_{1,1}, \dots, P_{1,n_1}, S_1)$ and $G_2 = (N_2, T_2, P_{2,1}, \dots, P_{2,n_2}, S_2)$ two m-linear CD grammar systems where $\mathcal{L}_f(G_1) = L_1$ and $\mathcal{L}_f(G_2) = L_2$. W.l.o.g. let N_1 and N_2 be disjoint.

Then let S be a new symbol where $S \notin (N_1 \cup N_2)$. Construct $P'_{i,j}$ from $P_{i,j}$ in the following way: For each production $S_i \to \alpha$ in $P_{i,j}$ add a new production $S \to \alpha$ to $P'_{i,j}$. The m-linear CD grammar system $G = (N_1) \cup (N_2) \cup \{S\}, T_1 \cup T_2, P'_{1,1}, \dots, P'_{1,n_1}, P'_{2,1}, \dots, P'_{2,n_2}, S)$ generates $L_1 \cup L_2$.
2. Let L be a language in $\mathcal{L}_f$*(CD-mLIN)* and $G = (N, T, P_1, \dots, P_n, S)$ be a grammar system with $L(G) = L$. Furthermore, let R be a regular set which is accepted by the deterministic finite automaton $M = (\Sigma, Q, \delta, q_0, F)$. Let S' be a new symbol with $S' \notin N$ and define $N' = \{[q, A, p] \mid q, p \in Q, A \in N\} \cup \{S'\}$. For all i with $1 \leq i \leq n$ and each production $p : A \to A_1 \dots A_l, A \in N, A_1, \dots, A_n \in N \cup T$ in P_i we define the productions $p' = \{[q, A, p] \to [q, A_1, q_1][q_1, A_2, q_2] \dots [q_{l-1}, A_l, p] \mid q, q_1, \dots, q_{l-1}, p \in Q\}$. We construct p'' from p' by replacing all symbols $[q, a, p]$, $q, p \in Q, a \in T$ and $\delta(q, a) = p$ with a. Furthermore, the productions containing symbols $[q, a, p]$ with $q, p \in Q, a \in T$ and $\delta(q, a) \neq p$ are deleted from p''. Let $P'_i = \bigcup_{p \in P} p''$ for $1 \leq i \leq n$. If P_i contains a production with S on its left hand side, then replace in P'_i the nonterminals $[q_0, S, f]$, $f \in F$ with S'. The m-linear CD grammar system $G' = (N', T, P'_1, \dots, P'_n, S')$ generates $L \cap R$.
3. Let L be a language in $\mathcal{L}_f$*(CD-mLIN)* and $G = (N, T, P_1, \dots, P_n, S)$ be a grammar system with $L(G) = L$. Furthermore, let $h : T \to T'^*$ be a homomorphism that is continued on T^* as usual. We construct a production set P'_i for $1 \leq i \leq n$ by replacing each production $p : A \to \alpha B \beta$ with $A, B \in N$ and $\alpha, \beta \in T^*$ in P_i by $p' : A \to h(\alpha) B h(\beta)$. The m-linear CD grammar system $G' = (N, T', P'_1, \dots, P'_n, S)$ generates $h(L)$.
4. Let L be a language in $\mathcal{L}_f$*(CD-mLIN)* and $G = (N, T, P_1, \dots, P_n, S)$ be a grammar system with $L(G) = L$. Furthermore, let $h : T' \to T^*$ be a homomorphism. W.l.o.g. $T \cap T' = \emptyset$. The language

$$\begin{aligned} K &= \{y \in (T \cup T')^* \mid y = z_0 x_1 z_1 \dots z_{l-1} x_l z_l,\ x_1 \dots x_l \in L(G), \\ &\quad z_0, \dots, z_l \in T'^2 \cup T' \cup \{\varepsilon\}\} \end{aligned}$$

is generated by the m-linear CD grammar system G' which is constructed from G as follows. For each $a \in T$ we define the set $L_a = \{xay \mid x, y \in T' \cup \{\varepsilon\}\}$. The sets L_a are finite. Now, if a production $p : A \to a_1 \ldots a_{r_1} B b_1 \ldots b_{r_2}$ $r_1 \geq 0, r_2 \geq 0$ $B \in N \cup \{\varepsilon\}$, $A \in N$, $a_1, \ldots, a_{r_1}, b_1, \ldots, b_{r_2} \in T$ occurs in a component, it is replaced by the set of productions $L_p = \{A \to a'_1 \ldots a'_{r_1} B b'_1 \ldots b'_{r_2} \mid a'_i \in L_{a_i}, b'_j \in L_{b_j},\ 1 \leq i \leq r_1,\ 1 \leq j \leq r_2\}$.

We consider the set

$$M = \{h(y_1)y_1h(y_2)y_2 \ldots h(y_n)y_n \mid n \geq 1, y_i \in T'\}.$$

It can be seen that M is regular. The intersection between K and M can be described as follows:

$$K \cap M = \{h(y_1)y_1h(y_2)y_2 \ldots h(y_n)y_n \mid h(y_1) \ldots h(y_n) \in L(G)\}.$$

Now we define the homomorphism $j : (T \cup T') \to T'$, $j(a) = a$ if $a \in T'$ and $j(a) = \varepsilon$ if $a \in T$. It is easy to see that $j(K \cap M) = h^{-1}(L(G))$. Since $\mathcal{L}_f$*(CD-mLIN)* is closed under intersection with regular sets and homomorphism, it follows that $\mathcal{L}_f$*(CD-mLIN)* is closed under inverse homomorphism. □

Theorem 5.2. *Let $f \in \{t, = k, \geq k \mid k \geq 2\}$, then $\mathcal{L}_f$(CD-mLIN) is not closed with respect to*

1. *concatenation,*
2. *Kleene-closure,*
3. *intersection,*
4. *complement.*

Proof. 1. With [7] Theorem 4.1 we can show that $L_m = \{a^n b^n \mid n \geq 0\}^{2m-1}$ is in $\mathcal{L}_f$*(CD-mLIN)* for $f \in \{t, = k, \geq k \mid k \geq 2\}$. But $L_m L_m$ contains words $w = (a^h b^h)^{4m-2}$. With the technique from Lemma 4.1 it can be shown that $L_m L_m$ is not in $\mathcal{L}_f$*(CD-mLIN)*.
2. follows from 1.
3. Consider the languages $L_1 = \{a^* b^* (a^n b^n)^m \mid n \geq 1\}$ and $L_2 = \{(a^n b^n)^m a^* b^* \mid n \geq 1\}$. L_1 can be generated by the grammar system

$$\begin{aligned}
G_1 &= \{\{S, A, B, C\}, \{a, b\}, P_1, P_2, P_3, P_4, P_5, S\} \\
P_1 &= \{S \to CA^k\} \\
P_2 &= \{C \to aC, C \to aB, B \to bB, B \to bA\} \\
P_3 &= \{A \to aA'b\} \\
P_4 &= \{A' \to A\} \\
P_5 &= \{A \to ab\}
\end{aligned}$$

using the t-mode. With similar methods as in [7] Theorem 4.1 an m-linear CD grammar system that accepts L_1 in the $\{= k, \geq k \mid k \geq 2\}$-modes can be constructed. Now, we change $S \to CA^k$ to $S \to A^k C$ and obtain an m-linear CD grammar system

G_2 that generates L_2. With similar methods as in [7] Theorem 4.1 an m-linear CD grammar system that accepts L_2 in the $\{= k, \geq k \mid k \geq 2\}$-modes can be constructed. The intersection between the two languages is $L_1 \cap L_2 = \{(a^n b^n)^{m+1}\}$. We know from Theorem 4.2 that this language is not in $\mathcal{L}_f$*(CD-mLIN)*.
4. With De Morgan's Theorem and the fact that $\mathcal{L}_f$*(CD-mLIN)* is closed under union but not closed under intersection we obtain that $\mathcal{L}_f$*(CD-mLIN)* is not closed with respect to complement. □

6 Conclusion

We have defined classes of metalinear CD grammar systems and intensely investigated their generative capacity. To obtain the results a pumping lemma was shown. The following holds

$$\begin{aligned} LIN &= \mathcal{L}_f(CD\text{-}LIN) \subset \mathcal{L}_f(CD\text{-}2LIN) \subset \ldots \\ &\subset \mathcal{L}_f(CD\text{-}mLIN) \subset \mathcal{L}_f(CD\text{-}(m+1)LIN) \subset \ldots . \end{aligned}$$

Furthermore it is

$$\begin{aligned} \mathcal{L}_t(CD_1\text{-}mLIN) = \mathcal{L}_t(CD_2\text{-}mLIN) &\subset \\ \mathcal{L}_t(CD_3\text{-}mLIN) &= \mathcal{L}_t(CD\text{-}mLIN) = \mathcal{L}(ET0L_{mLIN}). \end{aligned}$$

and

$$\mathcal{L}_f(CD\text{-}mLIN) \subseteq \mathcal{L}(MAT_{mLIN}), \; f \in \{= k, \geq k \mid k \geq 2\}.$$

We have shown that there is a context-free language that can not be generated with metalinear CD grammar systems although many context-sensitive languages can be. At last we have investigated the closure properties of metalinear CD grammar systems. The researched language classes are closed under union, intersection with regular languages, homomorphism and inverse homomorphism. They are not closed under concatenation, Kleene closure intersection and complement.

Metalinear CD grammar systems were defined to obtain grammars that are more powerful than context free grammars but have the simplest possible derivation structure. We got a deep insight into how the width of a metalinear CD grammar system influences its generative capacity. We have also obtained a pumping lemma which allows us to decide for certain languages if they are not generated by metalinear CD grammar systems. To complete the research of the generative power it would be interesting to know how the number of components influence the generative capacity of the metalinear CD grammar systems in $(= k)$-mode and $(\geq k)$-mode.

References

[1] H. Bordihn, B. Sunckel, On the Number of Active Symbols in CD grammar systems. Unpublished manuscript.

[2] E. Csuhaj-Varjú, J. Dassow, J. Kelemen, G. Păun, *Grammar Systems: A Grammatical Approach to Distribution and Cooperation.* Gordon and Breach, London, 1994.

[3] J. Dassow, G. Păun, *Regulated Rewriting in Formal Language Theory.* Springer, Berlin, 1989.

[4] J. Dassow, G. Păun , G. Rozenberg, Grammar Systems. In: G. Rozenberg, A. Salomaa (ed.), *Handbook of Formal Languages Vol. 2.* Springer-Verlag, Berlin, 1997, 155–213.

[5] J. E. Hopcroft, J. D. Ullman, *Introduction to Automata Theory, Languages and Computation.* Addison-Wesley, London, 1979.

[6] G. Rozenberg, D. Vermeir, On metalinear ET0L systems. *Fundamenta Informaticae* **3(1)** (1980), 15–36.

[7] B. Sunckel, On the descriptional complexity of metalinear CD grammar systems, In: L. Ilie, D. Wotschke (ed.), *Pre-proceedings of the Sixth International Workshop on Descriptional Complexity of Formal Systems (DCFS 2004)*, Report No. 619, Department of Computer Science, The University of Western Ontario, London, Ontario, Canada, 2002.