Kovács, A.: On the Computational Complexity of Tariff Optimization for Demand Response Management. IEEE Transactions on Power Systems, 33(3):3204-3206, 2018.
https://doi.org/10.1109/TPWRS.2018.2802198

On the Computational Complexity of Tariff Optimization for Demand Response Management

András Kovács

***Abstract*—Different modeling and solution approaches to electricity tariff optimization for demand response management have received considerable attention recently. Yet, there are hardly any results available on the computational complexity of these problems. The clarification of the complexity status is crucial to understand for which models an efficient, polynomial algorithm or a closed-form analytical solution can be expected, and when the application of heuristics delivering sub-optimal solutions or time consuming search procedures is justifiable. In this note, we define the *Simple Multi-period Energy Tariff Optimization Problem* (SMETOP) and prove its NP-hardness. The result naturally applies to many models in the literature that generalize SMETOP, and whose complexity status has been unknown to date.**

***Index Terms*—Demand-side management, optimization, computational complexity.**

I. Introduction

A wide variety of models and algorithms for electricity tariff optimization have been advised and investigated recently for demand response management, where Stackelberg game approaches are dominant for capturing the interplay of the self-interested energy retailers and consumers. For some simpler problem models, e.g., most of those focusing on a single time instant, the Stackelberg equilibrium (i.e., optimal solution) can be computed easily in closed form or by polynomial algorithms [1]. On the other hand, richer models, e.g., those that investigate a discrete time horizon consisting of multiple time periods, are typically addressed either by heuristics that may produce sub-optimal solutions [2] or by computationally demanding exact approaches [3, 4]. However, there are hardly any results available on the computational complexity of these models, which implies that it is unclear to date when an efficient algorithm can be expected, and when the application of sub-optimal heuristics or computationally expensive search approaches is justifiable.

In this note, we prove for the first time in the literature that computing the optimal solution of multi-period electricity tariff optimization problems is NP-hard. For this purpose, we define the *Simple Multi-period Energy Tariff Optimization Problem* (SMETOP), which captures the core features of Stackelberg game models for tariff optimization, including a profit-maximizing electricity retailer and multiple consumers who schedule their controllable loads to maximize their utility and minimize their cost of energy. We formally prove that SMETOP is NP-hard. It is emphasized that the simplicity of SMETOP is not a shortcoming, but a benefit: any richer model that generalizes SMETOP is also NP-hard. This result also clarifies the complexity status of various earlier problem formulations from the literature.

This research was supported by the GINOP Grant No. 2.3.2-15-2016-00002 and the János Bolyai Research Fellowship.

A. Kovács is with the Institute for Computer Science and Control, Hungarian Academy of Sciences. E-mail: andras.kovacs@sztaki.mta.hu.

II. Background on Stackelberg Games, Bilevel Programming, and Complexity Theory

Demand response management in smart grids is a paragon of decision situations whose outcome is decided by multiple players with conflicting interests. Operational-level models typically focus on the interaction of a profit-maximizing electricity retailer and its multiple consumers interested in minimizing their cost of energy and maximizing their utility. Game theoretic approaches therefore provide a natural means for modelling these decision situations. Specifically, *Stackelberg games* can be applied to modelling tariff optimization for demand response. In these sequential games, the decision maker called the *leader* makes its choice first (the retailer determines the electricity tariff). Then, a single or multiple *followers* optimize their response (consumers schedule their consumption) according to their own objective in view of the leader's choice. The outcome of the game, as well as the payoffs of both the leader and the followers, are mutually affected by the decisions of the other players.

The particularly interesting optimization problem is that of the leader, who must make its choice by taking into consideration the rational response of the followers. Obviously, this assumes that the leader has a perfect knowledge of the followers' decision model. In mathematical programming, this problem can be formulated as a so-called *bilevel program*, i.e., an optimization problem that includes a nested, parametric optimization problem (the followers' problem embedded into the leader's one) [5, 6]. Bilevel optimization problems have been applied to solving problems in various domains, such as toll setting in networks [7], regulatory problems in economic or social systems [8], or more recently, diverse problems in energy systems [9, 10].

While bilevel programs are powerful tools for modeling Stackelberg games, solving them is inherently challenging. Even linear bilevel programs are NP-complete in general [5], whereas problems with discrete variables in the followers' sub-problems are representatives of the still more complicated PSPACE complexity class. Yet, this does not mean that *all* Stackelberg game models would be NP-hard, as demonstrated by, e.g., the efficiently solvable tariff optimization problem with a single time instant in [1]. A methodological approach to developing efficient algorithms for a novel class of optimization problems requires first understanding which problems and special cases can be solved in polynomial time, and which ones are NP-hard. The most frequently applied method to prove the NP-hardness of a decision problem is *Karp reduction* [11, 12] from a source problem that is already know to be NP-hard. This involves the polynomial conversion of *any* instance of the source problem into an instance of the target problem that gives the same output as the source instance. It is noted that the result of the conversion is often a target problem with some special structure, but irrespective of this, the complexity result is valid for the target problem without any restrictions. This paper applies Karp reduction from the *set cover problem* (SCP) [11, 13] into SMETOP in order to prove the NP-completeness of the latter.

III. Problem Definition

SMETOP captures the optimization problem faced by an electricity retailer (leader in the Stackelberg game) who buys electric energy from the wholesale market and supplies it to multiple independent consumers ($i = 1, ..., N$) in a smart grid setting. The problems is solved on a finite time horizon ($t = 1, ..., T$), where all parameters are assumed to be deterministic and fully known. The retailer addresses the maximization of its profit, $\sum_{t=1}^{T}\sum_{i=1}^{N}(Q_t - P_t)\, x_{i,t}$, where Q_t is the electricity price offered to the consumers (decided by the retailer), P_t is the price on the wholesale market (input parameter), and $x_{i,t}$ denotes the consumption of consumer i (decided by the consumer).

As a response to the tariff announced by the retailer, each consumer i (followers in the Stackelberg game) determines its consumption by scheduling a total consumption of M_i over the time horizon to maximize its linear utility, $\sum_{t=1}^{T} U_{i,t}x_{i,t}$, and to minimize its cost of energy, $\sum_{t=1}^{T} Q_t x_{i,t}$. Here, $U_{i,t}$ denotes (the monetary equivalent of) the utility of scheduling one unit of controllable consumption in time period t. Consumption in period t is bounded from above by $\bar{L}_{i,t}$.

The relation of the parties is regulated by an a priori contract that defines minimum, maximum, and max. average electricity prices $\underline{Q}$, $\overline{Q}$, and $\tilde{Q}$, respectively. Such an agreement is necessary to prevent the profit maximizing retailer from increasing purchase prices without any limit. The system architecture is depicted in Figure 1. SMETOP can be cast as a bilevel program as follows.

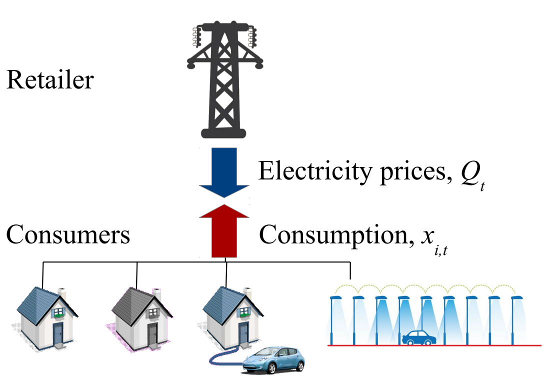

Fig. 1. System architecture with a retailer and multiple consumers.

Maximize

$$\sum_{t=1}^{T}\sum_{i=1}^{N}(Q_t - P_t)\, x_{i,t} \tag{1}$$

subject to

$$\underline{Q} \leq Q_t \leq \overline{Q} \quad \forall\, t \tag{2}$$

$$\frac{1}{T}\sum_{t=1}^{T} Q_t \leq \tilde{Q} \tag{3}$$

$$(x_{i,t}) \in \arg\min \left\{ \sum_{t=1}^{T}(Q_t x_{i,t} - U_{i,t}x_{i,t}) \;\middle|\; \begin{array}{ll} x_{i,t} \leq \bar{L}_{i,t} \quad \forall\, t & \text{(4b)} \\ \sum_{t=1}^{T} x_{i,t} = M_i & \text{(4c)} \end{array} \right\} \forall\, i \quad \text{(4a)}$$

In this formulation, the upper-level objective (1) is maximizing the retailer's profit, while constraints (2) and (3) enforce the a priori contract on electricity prices. Lines (4a)-(4c) capture the followers' optimization problem embedded as a constraint into the leader's problem, which states that consumption $x_{i,t}$ is chosen by follower i to minimize its objective, computed as its energy cost minus the (monetary equivalent) utility. Inequality (4b) sets an upper bound on the consumption in each time period, while constraint (4c) ensures that consumption over the horizon sums up to M_i. It is noted that this formulation implicitly includes the so-called *optmistic assumption*, which states that if a follower has multiple optimal solutions according to its own objective, then it chooses the optimal response that is the most favorable for the leader.

IV. Computational Complexity

Lemma 1: The decision version of SMETOP is NP-complete.

Proof: In the decision version of SMETOP, the questions asked is whether there exist a solution in which the retailer's profit is at least κ. NP-hardness is shown by Karp reduction from the set cover problem (SCP) to the decision version of SMETOP. In an instance of SCP, there is given a universe $V = \{1, 2, .., N\}$, a collection of sets over this universe $\mathcal{S} = \{S_1, S_2, ..., S_T\}$ with $S_t \subseteq V$ and $\bigcup_{t=1}^{T} S_t = V$, and an integer k. The question is whether the universe can be covered by at most k members of $\mathcal{S}$, i.e., if there exist sets $\{S_{[1]}, S_{[2]}, ..., S_{[k]}\}$ such that $\bigcup_{j=1}^{k} S_{[j]} = V$.

This SCP instance can be reduced to an instance of SMETOP with $N + 1$ consumers and $T + 1$ time periods, where consumers $i = 1, ..., N$ represent the elements of the universe V, and each time period $t = 1, ..., T$ corresponds to set S_t in the SCP. Covering element i by set S_t in the SCP corresponds to offering an attractive electricity price to consumer i in time period t in SMETOP. The additional time period $t = 0$ is a peak period from which as much load as possible should be deferred, whereas the additional consumer $i = 0$ is required for encoding the SCP cover size into the optimal SMETOP solution value.

Formally, such a SMETOP instance is constructed as follows. For all consumers $i = 1, ..., N$, the total consumption is $M_i = 1$ and it can be scheduled to time period t (i.e., $\bar{L}_{i,t} = 1$) if and only if $i \in S_t$. Otherwise, $\bar{L}_{i,t} = 0$. There is an initial time period $t = 0$ with $\bar{L}_{i,0} = 1\ \forall i$. All consumers prefer scheduling their load to the initial time period ($U_{i,0} = 2\ \forall i$) to scheduling it later ($U_{i,t} = 1\ \forall i, t \geq 1$). Finally, the consumption profile of the additional consumer 0 is perfectly determined by input parameters $\bar{L}_{N+0,0} = 0$, $\bar{L}_{0,t} = 1\ \forall t \geq 1$, and $M_0 = T$, resulting in $x_{0,0} = 0$ and $x_{0,t} = 1$ for $t \geq 1$.

The agreement between the retailer and the consumers defines the price limits as $\underline{Q} = 1$ and $\overline{Q} = \tilde{Q} = 2$. The retailer can buy electricity from the market at a price extremely high in the initial period ($P_0 > 2(N + T)$) and constantly low afterwards ($P_t = 1\ \forall t \geq 1$).

In the optimal solution of the SMETOP instance, the retailer must motivate each consumer to postpone their controllable load from the preferred, but extremely costly, initial time period to some later period. This can be achieved only by setting $Q_0 = 2$, and $Q_t = 1\ \forall t \in \Theta$ for an appropriately selected set of *cheap* time periods Θ. Then, Θ must be selected in such a way that it *covers the universe of all consumers* in the sense that $\forall i \exists t \in \Theta :\ i \in S_t$. Obviously, there exists such a Θ with $|\Theta| \leq k$ in the SMETOP instance if and only if the SCP is feasible with constant k. Finally, the retailer must set $Q_t = 2$ for $t \notin \Theta$ to maximize its income from consumer 0.

The retailer's profit in this optimal solution can be calculated as follows. The cost paid by the retailer on the market is $N+T$ (one unit for the consumption of each consumer $i = 1, ..., N$ and T units for consumer 0), whereas the income from the consumers is $N + 2T - |\Theta|$ (one unit from each consumer $i = 1, ..., N$, $|\Theta|$ units from consumer 0 in the cheap periods, and $2(T-|\Theta|)$ units in the expensive periods). Hence, the SCP decision problem is solvable with the given constant k if and only if the SMETOP instance admits a solution with $|\Theta| \leq k$, i.e., where the retailer's profit is at least $\kappa = N + 2T - k$. With this, SCP is successfully reduced to SMETOP, and the NP-hardness of SMETOP is proven.

Finally, the membership of the decision version of SMETOP in NP is obvious, since with a given price vector Q_t, calculating the consumers' response and solving the decision problem reduces to solving a linear program, which can be carried out in polynomial time, see, e.g., [3]. □

V. Discussion and Conclusions

This note introduced SMETOP, a simple Stackelberg game model that captures the decision situation of a profit-maximizing electricity retailer and its consumers with controllable consumption on a finite time horizon. It has been formally proven that SMETOP is NP-hard. The significance of this result is that it naturally applies to all richer multi-period tariff optimization models in the literature that generalize SMETOP, and whose complexity status has not been proven to date. Such earlier models include an extension of SMETOP with more sophisticated cost and utility models [2], another extension with battery storage and energy production at consumers [14], and a two-stage robust optimization model with battery storage at the retailer [4]. The proof of complexity can be adjusted trivially to other optimization criteria of the retailer that assign different costs to consumption in different time periods. An example is minimizing the squared deviation from a target consumption profile, investigated in [3].

The same result also suggests that polynomial-time solution approaches can be expected only for simpler models that lack the complicated interdependency between sub-problem related to different time periods in SMETOP, such as real-time pricing models focusing on a single time instant.

References

[1] S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing, and T. Basar. Demand response management in the smart grid in a large population regime. *IEEE Trans. Smart Grid*, 7(1):189–199, 2016.

[2] M. Yu and S. H. Hong. Supply-demand balancing for power management in smart grid: A Stackelberg game approach. *Applied Energy*, 164:702–710, 2016.

[3] A. Kovács. Bilevel programming approach to optimizing a time-variant electricity tariff for demand response. In *IEEE SmartGridComm'2016, IEEE International Conference on Smart Grid Communications*, pages 674–679, 2016.

[4] W. Wei, F. Liu, and S. Mei. Energy pricing and dispatch for smart grid retailers under demand response and market price uncertainty. *IEEE Trans. Smart Grid*, 6(3):1364–1374, 2015.

[5] S. Dempe. *Foundations of Bilevel Programming*. Kluwer, 2002.

[6] B. Colson, P. Marcotte, and G. Savard. Bilevel programming: A survey. *4OR: A Quarterly Journal of Operations Research*, 3:87–107, 2005.

[7] M. Labbé and A. Violin. Bilevel programming and price setting problems. *4OR*, 11(1):1–30, 2013.

[8] J. F. Bard, J. Plummer, and J. C. Sourie. A bilevel programming approach to determining tax credits for biofuel production. *European Journal of Operational Research*, 120(1):30–46, 2000.

[9] S. Dempe, V. Kalashnikov, G. A. Pérez-Valdés, and N. Kalashnykova. *Bilevel Programming Problems: Theory, Algorithms and Applications to Energy Networks*. Springer, 2015.

[10] B. P. Esther and K. S. Kumar. A survey on residential Demand Side Management architecture, approaches, optimization models and methods. *Renewable and Sustainable Energy Reviews*, 59:342–351, 2016.

[11] R. M. Karp. Reducibility among combinatorial problems. In R. E. Miller and J. W. Thatcher, editors, *Complexity of Computer Computations*, pages 85–103. Springer, 1972.

[12] O. Goldreich. *Computational Complexity: A Conceptual Perspective*. Cambridge University Press, 2008.

[13] B. Korte and J. Vygen. *Combinatorial Optimization: Theory and Algorithms (5 ed.)*. Springer, 2012.

[14] A. Kovács. Bilevel programming approach to demand response management with day-ahead tariff. *Energy*, submitted paper, 2017.