An Inverse Economic Lot-sizing Approach to Eliciting Supplier Cost Parameters

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Abstract

This paper proposes an inverse lot-sizing model for eliciting the cost parameters (the setup, holding, and backlog costs) of a supplier from earlier demand vs. optimal lot-size pairs. It is assumed that the supplier solves a single-item, multi-period, uncapacitated lot-sizing problem with backlogs to optimality to calculate its lot-sizes, and the buyer is aware of this fact. The inverse optimization problem is reformulated to a linear program. The approach is evaluated on a set of randomly generated problem instances.

Keywords: Economic lot-sizing, inverse combinatorial optimization, eliciting cost parameters.

1. Introduction

Planning inventories in a supply chain necessarily calls for the interaction of autonomous partners operating with distinct, potentially conflicting objectives, different decision mechanism and asymmetric information, even in the simplest single-buyer, single-supplier models. The literature offers a wide spectrum of coordination mechanisms (Albrecht 2010, Váncza et al. 2011) based on game theoretic and optimization approaches, which make different assumptions on the information available to the different partners. Nevertheless, there is a considerable gap between the *incomplete information* (or asymmetric information) models, which usually assume a single encounter of the buyer and the supplier with some well-defined information situation, and the *complete information* models, which consider that the companies are mutually aware of their partners decision situation. Namely, in case of *repeated encounters*, a significant amount of information is hidden in historic records of earlier interactions. Exploiting this information enables a company to use well-informed, e.g., *Stackelberg* or *bilevel optimization* approaches for planning its production and logistics, providing a considerable competitive advantage compared to models with restricted information.

In this paper, we tackle the issue of how the historic records of earlier encounters between a buyer and a supplier can be utilized in decision making. We take the stance of the buyer and aim at eliciting the cost parameters of the rational supplier's assumed decision problem. Specifically, we introduce an *inverse combinatorial* approach to eliciting the cost parameters of a supplier who determines its delivery periods and quantities by solving a single-item, multi-period, uncapacitated lot-sizing problem with backlogs (ULSB). The input of the approach is a historic record of demand vs. delivery lot-size pairs. To the best of our knowledge, this is the first *inverse lot-sizing* model investigated in the literature. The output of this model can be useful in various scenarios involving a buyer-supplier relationship. Beyond the general benefit of knowing the partner's cost parameters, e.g., in price negotiations, a specific application is the utilization of the elicited cost parameters as inputs to one of the recent Stackelberg or bilevel approaches to lot-sizing in supply chains. Note that such models require the knowledge of the supplier's cost parameters, but the coordination mechanisms themselves do not present any incentives for the supplier to reveal their true values. Hence, our method can be a precious complement of those coordination approaches.

In what follows, after surveying related works (Section 2) and defining formally the problem (Section 3) the inverse lot-sizing problem will be encoded as a linear program using a shortest path representation (Section 4). Next, we present results of computational experiments (Section 5) and conclude the paper with discussing application opportunities and further extensions of the model (Section 6).

2. Literature review

2.1. Lot-sizing

Fundamental results on dynamic lot-sizing models have been published in (Wagner and Whitin, 1958) and (Zangwill 1969). These papers consider uncapacitated lot-sizing models where the deterministic, time varying demand is known in advance over a finite planning horizon. Over the past decades the basic models have been extended by production capacities and various side constraints, for an overview see, e.g., (Axsäter 2006, Pochet 2001, Pochet and Wolsey 2006). The modeling of various features in lot-sizing by mixed-integer programs (MIP) are investigated in e.g., (Belvaux and Wolsey 2001, Cordier et al. 1999).

The need for studying the interacting lot-sizing decisions of multiple autonomous parties in a supply chain is widely recognized. One of the possible approaches is *integration*, when the different parties jointly solve the interrelated planning problems, see, e.g., (Li and Wang 2007). A drawback of integration is the mutual sharing of all the planning relevant information, which is sometimes unrealistic. A game theoretic approach alleviates this burden by using coordination mechanisms between the parties to drive the supply chain towards a system-wide optimal performance (Albrecht 2010, Cachon 2003). Four different computational approaches (decentralized planning, integration, coordination, and bilevel optimization) to the same lot-sizing problem in a two-player supply chain are compared in (Kovács et al. 2010).

2.2. Game theory

Most papers in supply chain research assume *complete information*, i.e., that the game structure is common knowledge for the players (Wu and Parlar 2011). In realistic situations however, there is an information gap between them–typically concerning either the cost structure or the demand forecast–which justifies the application of *incomplete information* (also called asymmetric information) models. Such approaches usually necessitate Bayesian setting, where the players have some common belief about the private information of the others. The inverse lot-sizing model presented in this paper provides exactly such kind of information to the underinformed player by characterizing the feasible cost parameters. For further details on game theoretical models and their applications in inventory management problems, we refer to (Wu and Parlar 2011).

In two-player games with incomplete information, the process of learning the private information of the other player is called *screening*. Such models are widespread in the supply chain management literature, for a recent overview see (Voigt 2011). For instance, Corbett (2001) considers the case where either the setup cost or the backorder cost of the supplier is a private information. Xu et al. (2010) present a model where the supplier's cost–which is inversely proportional to the required delivery time–is only known by the supplier, which is an obstacle for the buyer in optimizing its purchasing. Wang et al. (2009) investigate situations where it is beneficial for the supplier to share its production cost information with the buyer truthfully. The two-player setting is more exhaustively investigated in (Esmaeili and Zeephongsekul 2010), which also assumes that the price- and marketing-dependent demand rate is a private information of the buyer. We have studied an extended newsvendor type model in (Egri and Váncza 2012), where the buyer has private information about the uncertain demand forecast, while the supplier knows the various cost factors. The suggested coordination protocol and payment scheme provide both partners the right incentive for minimizing the total cost: the buyer is interested in sharing its unbiased information on the demand forecast and its uncertainty, while the supplier's rational decision concurs with the overall optimum.

A different approach for decreasing the effects of information asymmetry and ensuring win-win situation for the players is called *collaborative planning*. In this case the goal is not eliciting the missing information, but the cooperative iterative improvement of the supply chain plan by non-hierarchical players. For uncapacitated dynamic lot-sizing in assembly networks Chu and Leon (2009) present an iterative planning procedure. For the finite capacity case Dudek and Stadtler (2005) developed a solution. A general overview of collaborative planning problem can be found in (Stadtler 2009).

2.3. Inverse combinatorial optimization

Inverse combinatorial optimization is a relatively new field of operations research. A survey of this field, including the studied problem models, algorithms, and the main results achieved has been given in (Heuberger 2004). Most of the previous work in the field focused on graph theoretical problems, such as the inverse shortest path problem (Burton and Toint 1992) or the inverse center location problem (Cai et al. 1999). A generic optimization model for a class of inverse problems has been introduced in (Zhang and Liu 2002), together with a Newton-type algorithm that runs in strongly polynomial time under mild conditions.

A closely related field of operations research is *bilevel programming*. It addresses decision and optimization problems whose outcome is determined by the interplay of two self-interested decision makers who decide sequentially, and whose pay-off functions mutually depend on the decision of the other party. The basic modeling and solution techniques in bilevel programming are presented in (Dempe 2002), while results in bilevel inventory control include (de Kok and Muratore 2010, Ryu et al. 2004, Yang 2007).

The inverse lot-sizing problem model introduced in this paper is partly motivated by the bilevel lot-sizing model presented in (Kis and Kovács 2012). The current inverse model and the follower's optimality condition in the bilevel model are based on a common idea, and an upfront application of the inverse model is to compute the supplier's cost parameters for the bilevel problem.

3. Problem definition

A problem of cost elicitation in a dyadic supply chain, consisting of a single buyer and a single supplier is investigated. The buyer aims at eliciting the cost parameters of its supplier, who is known to solve a single-item uncapacitated lot-sizing problem with backlogs (ULSB) to compute its delivery quantities based on the demand received from its buyer.

Formally, the input contains a set of M samples, each of a length of T time periods. Each sample consists of a vector of demand values over time, d_i^m, and a vector of corresponding delivery lot-sizes, x_i^m, $m = 1...M, i = 1...T$. In every sample, the lot-sizes x_i^m are computed from the demand d_i^m by solving an ULSB problem with common parameters, namely the setup cost, f, a per period and per unit holding cost, h, and a per period and per unit backlog cost, g. It is assumed that all cost parameters are non-negative numbers. The values of f, h, and g are unknown, and the objective of the cost elicitation problem is to characterize the possible values of these parameters. We note that, in general, the samples do not determine the values of (f,h,g) unambiguously.

Finally, we note that the solution of the ULSB problem is invariant to the multiplication of the cost parameters, i.e., for a given demand vector d_i^m and cost parameters (f,h,g), the optimal lot-sizes x_i^m are the same as with d_i^m and (cf,ch,cg), where c is an arbitrary positive constant. Therefore, the approach is unsuitable for finding the absolute values of (f,h,g); only their ratios can be determined. Therefore, in the sequel we assume that f is fixed, and address finding the feasible range of the other parameters, namely, $h_{\min}$, $h_{\max}$, $g_{\min}$, and $g_{\max}$.

4. The solution approach

Before presenting the details of the solution approach, we characterize the range of feasible values of the cost parameters (h,g) by the following two lemmas.

Lemma 1. The range of feasible values of (h,g) is a convex 2-dimensional polygon.

Proof. An equivalent form of the lemma states that if (h_1,g_1) and (h_2,g_2) are two feasible solutions of the above inverse optimization problem, then for any constant c such that $0 \le c \le 1$, $\left(ch_1+(1-c)h_2, cg_1+(1-c)g_2\right)$ is also a feasible solution.

Assume that the statement is false, i.e., for some sample m, the lot-sizes x_i^m (and the implied stock quantities s_i^m, backlogs r_i^m, and production events y_i^m) are not an optimal solution for demands d_i^m of the corresponding ULSB. Then, there exists a lot-size vector $x_i'^m$ (and implied values $s_i'^m$, $r_i'^m$, and $y_i'^m$) that incurs lower cost than x_i^m. Since x_i^m is optimal for (h_1,g_1) and (h_2,g_2), but not for $\left(ch_1+(1-c)h_2, cg_1+(1-c)g_2\right)$, we have

$$\sum_{i=1}^{T}(fy_i^m + h_1 s_i^m + g_1 r_i^m) \le \sum_{i=1}^{T}(fy_i'^m + h_1 s_i'^m + g_1 r_i'^m)$$

$$\sum_{i=1}^{T}(fy_i^m + h_2 s_i^m + g_2 r_i^m) \le \sum_{i=1}^{T}(fy_i'^m + h_2 s_i'^m + g_2 r_i'^m)$$

$$\sum_{i=1}^{T}\left(fy_i^m + \left(ch_1+(1-c)h_2\right)s_i^m + \left(cg_1+(1-c)g_2\right)r_i^m\right) >$$
$$\sum_{i=1}^{T}\left(fy_i'^m + \left(ch_1+(1-c)h_2\right)s_i'^m + \left(cg_1+(1-c)g_2\right)r_i'^m\right)$$

This is a contradiction, since the last inequality can be received as a linear combination of the above two inequalities, with the inequality symbol pointing to the other direction. □

Lemma 2. The convex 2-dimensional polygon describing the range of feasible values of (h,g) does not necessary contain any of the four corners of its bounding box.

This lemma has been proven by constructing suitable problem instances. These instances are omitted in this paper.

In the sequel we present an inverse combinatorial optimization approach for computing the *bounding box* of the feasible range of (h, g) This involves computing four values, namely, $h_{\min}$, $h_{\max}$, $g_{\min}$, and $g_{\max}$. The four values will be computed by solving a series of four inverse problems that differ in their objective functions only. Each problem instance refers to the complete set of all the M samples.

4.1. A linear programming formulation

The reformulation of the inverse optimization problem to a linear program (LP) is based on the idea of *regeneration intervals* (Zangwill 1969) and a shortest path representation of the ULSB problem, see, e.g., (Kis and Kovács 2012). Namely, any instance of the ULSB problem admits an optimal solution in which the time horizon can be subdivided into a series of regeneration intervals $[i,k]$, such that all demand in the interval $[i,k]$ is met by production in a single period j with $i \le j \le k$.

Then, the graph representation of our inverse problem consists of M disjoint components for the M samples. In each component, there are $T+1$ nodes, corresponding to time periods $0,1,...,T$. Within a component, between any pair of nodes i',k with $i' < k$, there are $k-i'$ parallel directed edges pointing from node i' to node k, which correspond to a possible regeneration interval $[i,k]$ with $i = i'+1$ and the $k-i' = k-i+1$ possible positions of the single production period in the interval. The length of edge (m,i,j,k) is denoted by c_{ijk}^m, and it corresponds to the cost incurred if, in sample m, all demand in the regeneration interval $[i,k]$ is covered by production in period j. Hence,

$$c_{ijk}^m = f + \sum_{u=i}^{j-1}(j-u)g * d_u^m + \sum_{u=j+1}^{k}(u-j)h * d_u^m .$$

In order to maintain the distance of a node from the first node of the corresponding component, potential values π_i^m are assigned to each node. The potential values are consistent with the edge lengths if and only if $c_{ijk}^m \ge \pi_k^m - \pi_{i-1}^m$ holds for each edge, and $c_{ijk}^m = \pi_k^m - \pi_{i-1}^m$ for the edges of the shortest path.

Then, the optimal solution of the ULSB corresponding to sample m is encoded in the shortest path between the first and last nodes of component m of this graph. Notice that the optimal ULSB solutions, and therefore the set of all edges along the corresponding shortest paths are now given in the input of our inverse problem. Let us denote this set of edges by OPT.

Now, we are ready to define our formulation of the problem, called LP1, whose variables are the cost parameters h and g, as well as the edge lengths c_{ijk}^m and potential values π_i^m:

Minimize or maximize

$$h \text{ or } g \tag{1}$$

subject to

$$c_{ijk}^m - \pi_k^m + \pi_{i-1}^m = 0 \qquad \forall (m,i,j,k) \in OPT \qquad (2)$$

$$c_{ijk}^m - \pi_k^m + \pi_{i-1}^m \geq 0 \qquad \forall (m,i,j,k) \notin OPT \qquad (3)$$

$$c_{ijk}^m = f + \sum_{u=i}^{j-1} (j-u) g * d_u^m + \sum_{u=j+1}^{k} (u-j) h * d_u^m \qquad \forall m,i,j,k \qquad (4)$$

$$\pi_0^m = 0 \qquad \forall m \qquad (5)$$

$$c_{ijk}^m, \pi_k^m, h, g \geq 0 \qquad \forall m,i,j,k \qquad (6)$$

In this LP formulation, constraints (2) and (3) ensure the consistency of the edge lengths and node potentials with the shortest paths encoded in OPT. Constraint (4) defines the edge lengths, while lines (5) and (6) set the ranges of the variables.

4.2. Extension to noisy samples

Model LP1 finds cost parameter values in clear theoretic situations when the supplier uses fixed parameters throughout the time of collecting the historical samples. However, in a more realistic setting, the supplier may slightly modify its cost parameters over time or deviate from the optimal ULSB solution for some reason. In such a case, the samples may become noisy, and LP1 may not observe any feasible solution.

A natural extension of the model for such a case is to look for a good approximation of the cost parameters by minimizing the error according to some measure. For this purpose, we introduce individual cost variables (f_m, h_m, g_m) for each sample, and look for an approximation of the common cost parameters (h, g) that minimize the error E. The error is measured as the maximum difference between the individual and the common cost parameters, scaled by constants q_f, q_h, and q_g. The extended LP model, called LP2, is presented below.

Minimize

$$E \qquad (7)$$

subject to

$$f - q_f E \leq f_m \leq f + q_f E \qquad \forall m \qquad (8)$$

$$h - q_h E \leq h_m \leq h + q_h E \qquad \forall m \qquad (9)$$

$$g - q_g E \leq g_m \leq g + q_g E \qquad \forall m \qquad (10)$$

$$c_{ijk}^m = f_m + \sum_{u=i}^{j-1} (j-u) g_m * d_u^m + \sum_{u=j+1}^{k} (u-j) h_m * d_u^m \qquad \forall m,i,j,k \qquad (11)$$

(2), (3), (5), (6)

5. Experiments

Computational experiments investigated the accuracy of the elicited cost parameters, including the dependence of the accuracy on various factors, such as the number of samples or the characteristics of the instance. Two types of problem instances have been generated: in the *random samples* instances, the samples were generated using independent random demand, $d_i^m \leftarrow U[1,10]$, where $U[a,b]$ denotes the integer uniform random distribution over the interval $[a,b]$. In the *rolling horizon* instances, the demand in the first sample, x_i^1 was generated in a similar fashion, whereas subsequent demand vectors were generated by shifting the previous demand earlier by one period, and perturbing the demand value by at most 10%, i.e., $d_i^{m+1} = d_{i+1}^m (U[90,110]/100)$. The setup cost was fixed to $f = 1000$, while the holding and backlogging costs were randomized, $h \leftarrow U[10,100]$ and $g \leftarrow U[20,200]$. Obviously, these cost parameters were only used for generating the samples, and were not included in the input parameters of the inverse problem. The lot-sizes in the samples were generated by solving a standard MIP formulation of the ULSB problem. 100 instances were generated of either type, with a horizon of 10 time periods and 50 historic samples in each instance.

Two measures were used for characterizing the instances: the *stocking ratio*, *S*, denotes the ratio of periods over all samples where in the optimal solution, the supplier satisfied demand from stock. Analogously, the *backlogging ratio*, *R*, denotes the ratio of periods where the supplier backlogs.

The experiments were performed on an implementation of LP1 in FICO Xpress 7.2 using the Mosel programming language. All instances could be solved to optimality in less than one second, i.e., the exact values of $h_{\min}$, $h_{\max}$, $g_{\min}$, and $g_{\max}$ could be computed. For the same reason, the solution times are not displayed below.

The *accuracy* of the parameter elicitation was measured by calculating the gap between the computed lower and upper bounds of the given cost parameter, i.e., $\frac{h_{\max} - h_{\min}}{h_{\min}} \cdot 100\%$ for the holding cost and $\frac{g_{\max} - g_{\min}}{g_{\min}} \cdot 100\%$ for the backlog cost.

Figure 1 shows the results achieved for a random samples instance with $S = 59.2\%$ and $R = 3.2\%$, i.e., very rare backlogs. The diagram on the left displays the computed bounds of the holding cost parameter, while the diagram on the right refers to the backlog cost parameter. The horizontal axis shows the number of samples applied for the parameter elicitation. The maximum and minimum cost parameter curves indicate that the accuracy of the elicitation improved gradually until up to 25 samples, but no improvement was experienced afterwards. The final gap was 4.5% for parameter h and 32.7% for parameter g. Figure 2 displays the results for a different, frequent-backlog instance with $S = 24.2\%$ and $R = 41.4\%$ using a similar diagram design. The proposed method achieved a much better accuracy for this instance, 1.51% for h and 0.81% for g, and the accuracy improved continuously until the 50th sample.

The summary of the experimental results on the 200 sample instances is presented in Table 1 for the holding cost parameter h, and in Table 2 for the backlog cost parameter g. The tables display the results by different values of the stocking and backlogging ratios *S* and *R* (in different rows), and by different number of samples used for the elicitation (10, 25, and 50, in different columns). Each cell contains the average gap measured on the 25 corresponding instances. The numbers in parentheses indicate the number of instances where this gap was infinitely large. This occurred when the samples used for the elicitation did not contain any backlogging action, and therefore, no finite $g_{\max}$ could be computed.

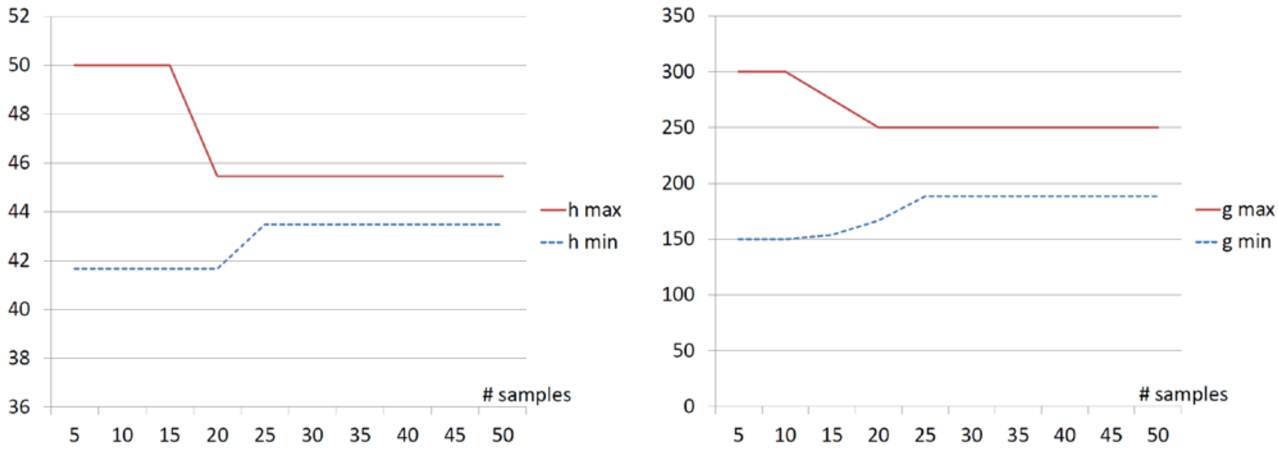

Figure 1: Accuracy of the elicited holding cost h (left) and backlog cost g (right) for a rare-backlog sample instance.

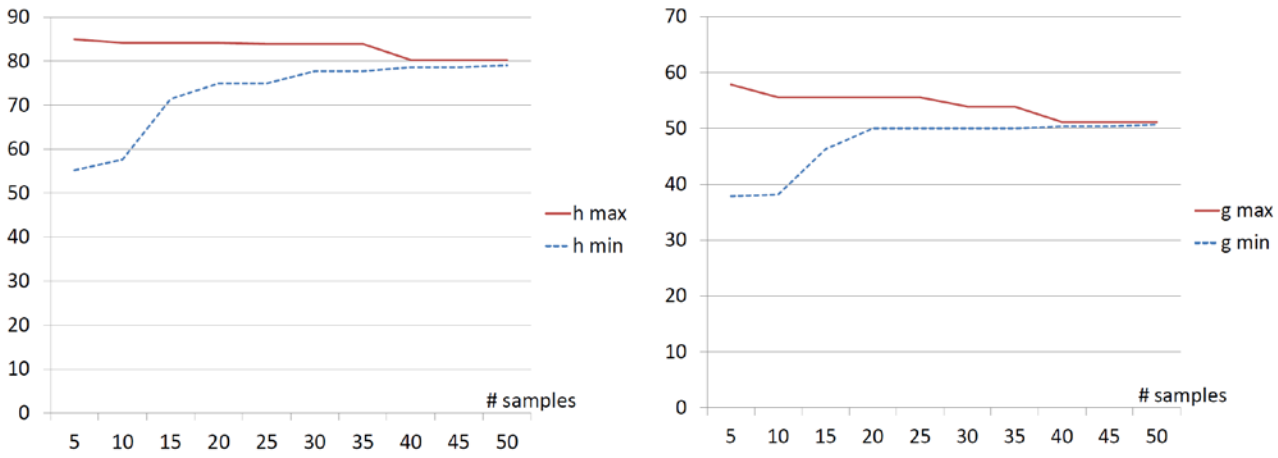

Figure 2: Accuracy of the elicited holding cost h (left) and backlog cost g (right) for a frequent-backlog sample instance.

The results show that the average gap decreased continuously with the increasing number of samples considered, and in the end it reached 5-10% for parameter h and 5-20% for parameter g. We note that h and g have symmetric roles in the model, but the different random distributions used for generating the instances caused a difference in the accuracy for h and g. For very low and high values of S, the accuracy of the elicitation for h is worse than for medium values. This occurs because for low (high) S, too few data is available to predict $h_{\max}$ ($h_{\min}$). The same holds for R and g. Furthermore, the elicitation is somewhat more accurate on random samples than on the rolling horizon instances, since the samples in the latter case are interrelated, and therefore contain fewer information.

Random samples			
S	No. of samples		
	10	25	50
11.8-33.6%	20.00	11.58	6.82
33.8-42.2%	19.14	9.05	5.93
42.6-58.0%	14.55	9.10	5.46
58.4-77.8%	26.07	10.36	5.71

Rolling horizon			
S	No. of samples		
	10	25	50
7.4-35.0%	35.13	15.66	8.20
35.2-45.2%	29.83	11.05	7.79
46.0-61.0%	21.98	11.19	6.94
61.4-77.0%	29.34	11.63	6.49

Table 1. Average accuracy of the elicitation for h, for different stocking ratios S and numbers of samples.

Random samples			
R	No. of samples		
	10	25	50
0.8-6.0%	109.20 (3)	49.40	15.42
6.2-14.8%	26.69	14.65	7.83
15.0-29.2%	22.38	10.51	5.06
29.8-55.0%	22.07	10.38	5.71

Rolling horizon			
R	No. of samples		
	10	25	50
0.6-4.2%	135.88 (1)	47.59	18.65
4.8-12.4%	37.00	17.07	9.13
13.2-27.2%	32.64	13.41	7.56
27.6-64.4%	26.38	11.30	6.41

Table 2. Average accuracy of the elicitation for g, for different backlogging ratios R and numbers of samples. The numbers in parentheses indicate the number of instances for which $g_{\max}$ was infinite.

A different aspect for the evaluation of the approach would be measuring the efficiency of the prediction of the supplier's future actions, i.e., after eliciting the supplier's cost parameters from m samples, computing predicted lot-sizes for the $(m+1)$ th sample. The question is how often the predicted lot-sizes match the real actions of the supplier. The preliminary experiments suggest that this measure approaches 100% even after learning from as few as 10 samples. This experiment, and in general, the thorough evaluation of the approach on a larger set of data will consist the subject of future work.

6. Conclusions and future work

This paper proposed a novel technique for eliciting a supplier's cost parameters from earlier records of demand vs. delivery lot-size pairs by using an inverse optimization approach. An inverse ULSB lot-sizing model, and its formulation as a linear program was introduced. The accuracy of the elicited cost parameters was measured in initial computational experiments. Based on these experiments, we conclude that the approach is efficient enough to predict the future actions of the supplier. On the other hand, the approach is not sufficient on its own for learning cost parameters, e.g., for price discussions, since it elicits only the ratios of the different cost parameters, not their absolute values. An interesting direction for future work is the extension of the model towards more realistic assumptions, e.g., rolling horizon models and costs varying over time.

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