Central European Journal of Operations Research manuscript No.
(will be inserted by the editor)

Inverse Optimization Approach to the Identification of Electricity Consumer Models

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Received: date / Accepted: date

Abstract Stackelberg game models for demand response management in smart electricity grids have been studied extensively in the scientific literature. Still, a barrier to their practical applicability is the assumption that the retailer (leader in the game) has perfect knowledge about the consumers' (followers') decision model. This paper investigates the possibilities of reconstructing the consumers' decision model from historic tariff and consumption data. For this purpose, it introduces an inverse optimization approach to eliciting the parameters of electricity consumer models formulated as linear programs from the historic samples. The inverse problem is first transformed into a quadratically constrained quadratic program, and then solved using successive linear programming techniques. The approach is demonstrated on a common consumer model with multiple types of deferrable loads behind a single smart meter. Experimental results are presented, and directions for future research are proposed.

Keywords Smart grids · demand response management · parameter elicitation · inverse optimization

1 Introduction

An utmost challenge in the operation of future smart electricity grids is developing effective practices for demand response management (DRM): with the increasing share of renewables in the electricity mix, power generation is becoming less and less flexible. This implies that the traditional *supply follows demand* approach, i.e., power plants at any point in time generating exactly

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as much electricity as required by consumers, seems less and less feasible. Fortunately, in parallel, with the appearance of new, flexible types of load (e.g., electric vehicle charging) and new ICT devices to control traditional loads, the alternative *demand follows supply* paradigm came into the limelight. The collective name of the practices for adjusting consumer behavior to better match available supply is *demand response management* (DRM, with a focus on short-term, e.g., intra-day behavior) or *demand-side management* (DSM, allowing a longer time horizon). The primary means to achieving DRM is an appropriate financial incentive, such as a time-of-use electricity tariff. However, designing a suitable electricity tariff requires a deep understanding of consumer behavior.

The natural and commonly applied mathematical model for DRM is a multi-follower Stackelberg game [14, 16, 29]. The leader in the game is an electricity retailer, who aims to motivate its consumers to adjust their electricity consumption to the system-level objectives. Consumers are multiple independent followers, who respond to the electricity tariff by scheduling their load to minimize their cost of electricity and to maximize their utility.

A common critical assumption of the above Stackelberg approaches is that the retailer has perfect information about the decision model and the parameters of the consumers. Obviously, this assumption cannot be met in practice: consumption is determined partly by hard-to-predict human behavior (which probably dominates for residential consumers), and partly by economic rationale (which prevails in case of industrial consumers). It is particularly challenging to characterize consumer responsiveness to electricity tariff, i.e., to find out how consumers will react to different candidate tariffs. At the same time, due to the ever wider availability of smart metering technologies [4, 11], the retailer holds a huge amount of historic data about the behavior of its individual consumers, including corresponding electricity tariff and consumption time series.

This paper investigates how these historic records can be exploited to reconstruct the decision model of the followers. For this purpose, it assumes that some of the consumer models frequently applied in the literature, with stationary parameters over time, capture consumer behavior with a suitable precision. Then, it applies an *inverse optimization* approach to elicit parameter values for that decision model from historic data. The proposed approach is applicable to arbitrary consumer models formulated as *linear programs* (LP), which is the most common representation in literature, see, e.g., [22]. The generic approach is illustrated on a typical consumer model, with multiple deferrable loads behind a single smart meter at the consumer. The effectiveness of the approach is investigated in computational experiments. The need for extending inverse optimization models typically addressed by the operations research community is also highlighted.

This paper is a substantially extended version of the earlier conference paper [15]. Extensions include a proper positioning of the novel results in the state-of-the-art of inverse optimization, involving a formal demonstration that the problem at hand is non-convex, as well as a thorough experimental investigation of the proposed approach. An additional outlook to alternative modeling approaches is also given, including a mixed-integer linear program (MILP) that exploits a structural property of the consumer's optimal solutions.

The paper is structured as follows. A review on DRM and the relevant mathematical methodologies is presented in Section 2. The parameter elicitation problem is defined formally in Section 3. Then, the proposed solution approach is introduced (Section 4). The approach is validated in computational experiments in Section 5. A discussion on potential alternative solution methods is presented in Section 6. Finally, conclusions are drawn and directions for future research are discussed.

2 Literature review

2.1 Demand response management in smart grids

Stackelberg game approaches are widely used to address optimization problems in DRM [8]. In most such models, the leader is an electricity retailer, while the followers are its consumers, who aim to schedule deferrable [27] or curtailable [16] loads, charging their batteries [14] or electric vehicles [23] taking into account the electricity tariff set by the retailer. The Stackelberg equilibrium can be computed analytically for some simpler models, typically those considering a single time period [16]. At the same time, the solution of more sophisticated multi-period models requires search. The commonly applied solution method is transforming the corresponding bilevel program into a single-level MILP using the Karush-Kuhn-Tucker (KKT) conditions [29,23]. Recently, *successive linear programming* (SLP) has also shown favorable performance on some models [14].

The practical applicability of the above Stackelberg approaches has been criticized for two main reasons: (1) the assumption that the retailer has perfect information about its consumers; and (2) for the simplistic models applied to characterize consumer behavior.

Indeed, the modeling of consumer behavior from the viewpoint of DRM has received significant attention recently. Approaches can be roughly classified as *technological engineering models* and *econometric empirical studies* [24]. The former group of methods build detailed models of the main load components, and aggregate these components to calculate the grid-level electricity consumption. This allows the investigation of power systems and their operation practices in simulated environments. However, the accuracy of the models is often disputed, and matching these formal models with observed consumption is a challenge. A particular difficulty is a mismatch in their granularity: apart from experimental scenarios, smart electricity meters measure the total consumer-level (e.g., household-level) consumption, whereas the load profile of individual appliances is not readily accessible. *Load disaggregation* addresses the decomposition of the total consumption to device-level load using machine learning and background knowledge to achieve *non-intrusive appliance load monitoring* (NIALM) [17,28].

Econometric empirical studies, in contrast, rely on statistical data obtained from measurements on the physical system, without a formal model of the individual load components. They look for correlation between the measured consumption, the electricity tariff, and other external variables using statistical methods. A probabilistic characterisation of the load flexibility of residential consumers is given in [24]. Price and volume signals are considered as incentives from the demand response provider to consumers, which offer monetary benefits for consumers in exchange for the modifications of their consumption in given time intervals. The DRM potential in electric vehicle charging has received special attention: while classical contributions depart from the fraction of the charging time and the connection time [12], some recent studies aim at composing a more sophisticated statistical characterization [19]. A methodology for analyzing the reflectivity of electricity tariffs in simulation models is proposed in [10] for residential consumers with photovoltaic (PV) generation and battery storage.

2.2 Inverse optimization

In mathematical programming terms, the problem of finding parameter values for an optimization problem that lead to a given optimal solution is called *inverse optimization*. Models and algorithms for inverse optimization are reviewed in [9]. The classical work of Ahuja and Orlin [2] addresses inverse optimization problems where a cost vector is looked for that makes a given solution optimal, while it causes the smallest perturbation compared to an initial cost vector under the L_1 or the L_∞ norm. The constraint coefficients are assumed to be fixed. With this restriction, polynomial algorithms are given for various inverse problems, including the inverse variants of linear programming and different graph problems. An outlook to the L_2 norm, with the same restriction, is given in [3]. The paper [20] investigates inverse integer programming, again, with unknown parameters in the objective only.

A substantial generalization where both the cost vector and the constraint coefficients can be varied within a closed polyhedron for inverse linear problems is studied in [6]. Given a desired solution x_0, parameter values and a modified solution x are looked for that minimize the distance $|x, x_0|$ in the Euclidian norm. It is shown that in the general case, this problem is non-convex with multiple locally optimal solutions. Necessary and sufficient conditions of optimality are proven.

Many of the above papers highlight the relation of inverse optimization to (or its potential applications in) other fields of computer science, including parameter identification and machine learning. The recent survey [26] highlights the similarities between the challenges faced by the optimization and the machine learning communities in solving inverse problems, and investigates the possibilities of cross-fertilization. From among optimization approaches, this survey stresses the common application of iterative, gradient-based methods for solving non-linear problems.

A key novelty of the problem addressed in this paper compared to the above state-of-the-art is that suitable parameter values should be found for a large *set* of historical solutions, rather than a *single* desired solution. We are aware of a single earlier contribution from the literature where inverse optimization is used for parameter elicitation of a set of historic solutions to a lot sizing problem [7]. Still, in that problem, all unknown parameters were located in the objective, and it was assumed that there exists a combination of parameters that renders all historical solution a feasible solution of the lot sizing problem (i.e., there is no noise on the historical data). The current problem lifts both of these assumptions.

3 Problem definition

3.1 Direct problem

This paper illustrates the proposed parameter elicitation technique on a common electricity consumer model with multiple controllable loads behind a single smart meter. This problem, solved by the consumer to schedule its loads subject to the time-of-use electricity tariff, constitutes the *direct problem* in our inverse optimization approach. A similar consumer model is used, e.g., in [13, 27].

In this model, the consumer schedules N different types of controllable loads over a finite time horizon divided into T time periods of equal length. For each type of load $i = 1, ..., N$, the total demand M_i over the horizon is given. The load scheduled into period t is bounded from above by $L_{i,t}$. Scheduling a unit of load of type i into period t incurs a utility of $U_{i,t}$ for the consumer.[1] The unit price of electricity Q_t also varies over time. Then, the consumer aims to maximize its total utility incurred and minimize the total cost of electricity. This problem can be formulated as an LP as follows. Symbols in brackets on the r.h.s. of the constraints represent the dual variables assigned to the constraint. The applied notation is displayed in Table 1.

[1] Utility aims to quantify in monetary terms the tangible or intangible gain of the consumer by scheduling some energy-intensive activity into a given time unit. For example, an industrial consumer that operates electric furnaces, and needs to decide on scheduling the heat treatment processes into the night or during the day, may set the utilities of different time intervals according to the extra costs of the night shift, stemming from night shift differentials paid to workers. For a residential consumer, the utility of washing clothes in the convenient evening hours, rather than early in the morning, is equivalent to the difference in the electricity cost for which it is willing to wake up earlier to switch on the mashing machine in the early morning valley period. Similar utility models are widely applied in game theoretic models of DRM: it is typical to adopt simple linear or piecewise linear utility functions [1, 14, 25], but more sophisticated, e.g., quadratic, piecewise quadratic, or logarithmic functions are also used [16, 27].

Maximize

$$\sum_{t=1}^{T}\sum_{i=1}^{N}\left(U_{i,t}x_{i,t}^{k} \;-\; Q_{t}^{k}x_{i,t}^{k}\right) \tag{1}$$

subject to

$$\sum_{t=1}^{T} x_{i,t}^{k} = M_i \quad \forall\, i \qquad [\alpha_i] \tag{2}$$

$$x_{i,t}^{k} \leq L_{i,t} \quad \forall\, i,t \qquad [\beta_{i,t}] \tag{3}$$

$$x_{i,t}^{k} \geq 0 \quad \forall\, i,t \tag{4}$$

In this LP formulation, the objective (1) states that the consumer maximizes its total utility minus the cost of electricity. Equality (2) declares that the total load of type i must equal M_i, whereas constraint (3) specifies the upper bound on the per period load for each type. The decision variables in this direct problem are the scheduled load values $x_{i,t}^{k}$ over time for each type of load. Upper indices k in the electricity tariff Q_t^k and the scheduled load $x_{i,t}^{k}$ refer to different instances of the problem solved by the consumer over time, leading to historic samples $k = 1, ..., K$ of corresponding tariff and consumption values.

A core assumption of the approach is that consumer behavior can be characterized *sufficiently well* by the above model with *stationary parameters* over time. Accordingly, M_i, $L_{i,t}$, and $U_{i,t}$ are common over all historic samples, where samples correspond to repetitive time intervals that are characteristic for the given type of load, such as days, workdays, or weekend days. To reflect that *sufficiently good* characterization is assumed instead of *perfect* characterization, we allow the realized total load of the consumer, z_t^k, to deviate from the load predicted by the model, $\sum_{i=1}^{N} x_{i,t}^{k}$, i.e., it is assumed that $z_t^k \approx \sum_{i=1}^{N} x_{i,t}^{k}$. The extent of tolerable deviation will be analyzed in computational experiments.

3.2 Inverse problem

Let us assume that the above direct problem captures the consumer's behavior with a reasonable accuracy. Then, the goal is to elicit the parameter values M_i, $L_{i,t}$, and $U_{i,t}$ applied by the consumer to schedule its loads.

The electricity retailer is aware of the electricity tariff Q_t^k and the per period total consumption z_t^k (with $z_t^k \approx \sum_{i=1}^{N} x_{i,t}^{k}$), i.e., the consumer's past demand responses to the variation of the electricity tariff from smart meter readings. At the same time, it is unable to directly observe the detailed, per device consumption $x_{i,t}^{k}$, and does not dispose of any background information on the parameter values M_i, $L_{i,t}$, and $U_{i,t}$. Then, the inverse problem consist in determining these unknown parameter values from historic data. This inverse problem can be formulated as follows.

Table 1 Notation used in the paper.

Notation		Observable
Dimensions		
T	Number of time periods	Yes
N	Number of load types	Yes
K	Number of historic samples	Yes
Indices		
t	Time period index	Yes
i	Load type index	Yes
k	Historic sample index	Yes
Grid parameters		
Q_t^k	Unit price of electricity [$/kWh]	Yes
Consumer's parameters		
M_i	Total load during the horizon [kWh]	No
$L_{i,t}$	Maximum load scheduled [kWh]	No
$U_{i,t}$	Utility of load scheduled [$/kWh]	No
Decision variables of the consumer		
$x_{i,t}^k$	Load of type i scheduled to period t [kWh]	No
z_t^k	Cumulated load of the consumer [kWh]	Yes
Auxiliary variables		
α_i^k	Dual variable for constraint (2)	No
$\beta_{i,t}^k$	Dual variable for constraint (3)	No
ε_t^k	Model prediction error	No

Minimize

$$\sum_{k=1}^{K}\sum_{t=1}^{T}\varepsilon_t^k \tag{5}$$

subject to

$$\varepsilon_t^k \geq \left| z_t^k \; - \; \sum_{i=1}^{N} x_{i,t}^k \right| \qquad \forall\, k,t \tag{6}$$

$$\left\{x_{i,t}^k\right\} \in \arg\min \left\{ \sum_{t=1}^{T}\sum_{i=1}^{N} U_{i,t}x_{i,t}^k - Q_t^k x_{i,t}^k \mid (2)-(4) \right\} \quad \forall\, k \tag{7}$$

$$U_{i,t},\ L_{i,t},\ M_i,\ x_{i,t}^k,\ \varepsilon_t^k \geq 0 \qquad \forall\, k,i,t \tag{8}$$

The objective (5) is minimizing the misfit of the model, calculated as the absolute difference between the measured historic consumption and the consumption predicted by the model (6). The equilibrium constraint (7) states that load values $x_{i,t}^k$ are derived from solving the parametric direct problem to optimality.

3.3 Problem characteristics

In the above formulation, the parameters to be elicited appear both in the objective and on the r.h.s. of the constraints of the direct problem (and accordingly, of the equilibrium constraint in the inverse problem), and hence, this problem does not fit into the classes of inverse problems typically investigated in the literature, with unknown parameters appearing only in the objective [2, 20]. Moreover, this problem is non-convex, as it is shown in the lemma below.

Lemma 1 *The above inverse optimization problem is non-convex.*

Proof: Consider an instance of the inverse problem with a single type of load, two time periods, and a single historic sample ($N = 1$, $T = 2$, and $K = 1$), where the sample contains one unit of load in the first time period, and no load in the second period ($z_1^1 = 1, z_2^1 = 0$). Observe that the following solutions are both optimal. In solution S_1, the utility of the second period is very high, but the maximum load is zero ($S_1 : \ U_{1,1} = 1, U_{1,2} = 1000, \ L_{1,1} = 1, L_{1,2} = 0, \ M_1 = 1$); whereas in S_2, the maximum load is very high, but the utility is zero ($S_2 : \ U_{1,1} = 1, U_{1,2} = 0, \ L_{1,1} = 1, L_{1,2} = 1000, \ M_1 = 1$). Both of these parameter combinations induce that all load will be scheduled into the first time period, and therefore, these solutions incur zero error ($x^k_{1,1} = 1, x^k_{1,2} = 0, \ \sum_{k,t} \varepsilon^k_t = 0$).

Now, a convex combination of S_1 and S_2 with identical weights has high utility and high maximum load in the second period ($S' : \ U_{1,1} = 1, U_{1,2} = 500, \ L_{1,1} = 1, L_{1,2} = 500, \ M_1 = 1$). With these parameters, the consumer schedules all load into the second period ($x^k_{1,1} = 0, x^k_{1,2} = 1$), which incurs a positive error ($\sum_{k,t} \varepsilon^k_t = 2$). This contradicts convexity. □

4 Solution approach

The above defined inverse optimization problem is a mathematical program with an equilibrium constraint, which is not directly trackable using classical tools of operations research. Therefore, it is first reformulated to a *quadratically constrained quadratic program* (QCQP). Since the resulting QCQP is non-convex, SLP is applied to solving it.

4.1 Reformulation to QCQP

Reformulation to QCQP is performed by exploiting strong duality for the direct problem, formulated as an LP. Accordingly, in the resulting QCQP, the equilibrium constraint (7) is replaced by the following set of constraints: the primal constraints of the direct problem (constraints (12)-(13) below); the dual constraints of the direct problem (14); and finally, a constraint stating that the primal and the dual objectives are equal (15). For self-containedness, the complete QCQP reformulation is presented below:

Minimize

$$\sum_{k=1}^{K} \sum_{t=1}^{T} \varepsilon_t^k \tag{9}$$

subject to

$$\varepsilon_t^k \geq \sum_{i=1}^{N} x_{i,t}^k \; - \; z_t^k \qquad \forall\, k, t \tag{10}$$

$$\varepsilon_t^k \geq z_t^k \; - \; \sum_{i=1}^{N} x_{i,t}^k \qquad \forall\, k, t \tag{11}$$

$$\sum_{t=1}^{T} x_{i,t}^k = M_i \qquad \forall\, k, i \tag{12}$$

$$x_{i,t}^k \leq L_{i,t} \qquad \forall\, k, i, t \tag{13}$$

$$\alpha_i^k + \beta_{i,t}^k \geq U_{i,t} - Q_t^k \qquad \forall\, k, i, t \tag{14}$$

$$\sum_{i=1}^{N} \sum_{t=1}^{T} \left(U_{i,t} x_{i,t}^k \; - \; Q_t^k x_{i,t}^k \right) \; = \; \sum_{i=1}^{N} M_i \alpha_i^k \; + \; \sum_{i=1}^{N} \sum_{t=1}^{T} L_{i,t} \beta_{i,t}^k \qquad \forall\, k \tag{15}$$

$$U_{i,t},\; L_{i,t},\; M_i,\; x_{i,t}^k,\; \beta_{i,t}^k,\; \varepsilon_t^k \geq 0 \qquad \forall\, k, i, t \tag{16}$$

Quadratic terms in the above formulation appear solely in constraint (15): $U_{i,t} x_{i,t}$ from the primal objective, while $M_i \alpha_i^k$ and $L_{i,t} \beta_{i,t}^k$ from the dual objective.

4.2 Solution by SLP

Since the above inverse optimization problem is non-convex (see Lemma 1), there is little hope for finding efficient exact solution approaches for solving it. For this reason, an SLP solution approach has been implemented, which has been successfully applied to similar problems in demand response management [14]. SLP solves non-linear problems by iteratively building local LP approximations of the original problem, and solving each approximation using standard LP techniques [5,18]. Departing from some initial solution X_0, in each iterative step j, SLP builds a local linearization of the problem around X_j, denoted by LP_j. Then, it solves LP_j within a bounded environment of X_j. If the resulting LP solution is feasible for the original problem with a given tolerance, then it is accepted as X_{j+1}. Otherwise, another solution for LP_j is looked for within a closer environment of X_j. Since SLP is an iterative heuristic by nature, it may get stuck in local optima and return a sub-optimal solution. The quality of the solutions found and the computational efficiency on the problem at hand will be investigated in computational experiments.

5 Experimental results

5.1 Design of experiments

The effectiveness of the proposed approach was investigated in computational experiments on generated data. Experiments addressed whether a hypothetical retailer can predict the future behavior of its consumers based on model parameters elicited from historical samples using the proposed approach.

For this purpose, consumer models were constructed with randomized parameter values $U^0_{i,t}$, M^0_i, and $L^0_{i,t}$. It was assumed that the retailer cannot observe these original parameter values, but it has access to a set of K historic samples, each sample containing a corresponding vector of tariff and overall consumption values $\left\{Q^k_t, z^k_t\right\}^T_{t=1}$. These samples were generated by solving the direct problem with random tariff values Q^k_t. The resulting overall consumption was perturbed by noise to reflect that the model cannot give a perfect characterization of real consumer behavior, using the formula $z^k_t = U(1-\pi, 1+\pi)\sum_i x^k_{i,t}$. Here, $U(a, b)$ denotes the continuous uniform random distribution over the interval $[a, b]$. The level of relative noise was varied between $\pi = 0$ (i.e., no noise) and $\pi = 0.2$ (significant deviation from the model). Applying the proposed approach to these samples resulted in elicited parameter values $U^E_{i,t}$, M^E_i, and $L^E_{i,t}$. Then, the proposed approach was evaluated by comparing the solutions of the direct problem with the original ($U^0_{i,t}$, M^0_i, $L^0_{i,t}$) and the elicited ($U^E_{i,t}$, M^E_i, and $L^E_{i,t}$) parameter values. A test set of randomized tariff values, independent of the historic samples used for elicitation, was used as additional input for the comparison.

Instances with different sizes were generated by selecting the number of load types from $N \in \{1, 3, 5\}$ and the number of samples from $K \in \{25, 50, 100, 200\}$. The length of the time horizon was fixed to $T = 12$. The test set contained 50 randomized tariff vectors. All experiments were performed using an implementation of the QCQP (9)-(16) in Xpress 7.8 in the Mosel programming language, by applying the SLP package for solving the model. The experiments were run on a personal computer with Intel i7 2.70 Ghz CPU and 16 GB RAM.

5.2 Results with a single type of load

For the special case with a single type of load ($N = 1$), the proposed approach enabled a rather successful prediction of consumer behavior. This is shown in Figure 1, which compares the load curves over time with the original (blue) and the elicited (orange) parameter values, with a low number of samples ($K = 25$) and considerable noise ($\pi = 0.2$). Even better predictions could be achieved with a higher number of samples or less noise. The *root mean square error* (RMSE) of the predicted behavior with different values of π and K is depicted in Figures 2-4. Each diagram shows the mean RMSE over the 50 test instances (red dashed line), and the mean RMSE (blue continuous line) with a 80% confidence interval around it (light blue area). Hence, out of the 50 test instances, the result of 40 tests lie in the blue area, whereas 5 above it and 5 under it.

It is emphasized that for instances without noise ($\pi = 0$), the consumer model could be fitted to the historic samples without any error ($\varepsilon_t^k = 0, \ \forall k, t$ in the inverse problem). Yet, the elicited parameter values do not match perfectly the original ones. The reason is that the consumer may have different motivations behind the same behavior pattern. For instance, scheduling no consumption into a time period t may stem from a very low utility $U_{i,t}$ or a maximum load of zero, $L_{i,t} = 0$. These two possible motivations could be distinguished only by extreme tariff values that are not present in the available historic samples.

Accordingly, for $\pi = 0$, the predicted behavior on the independent test set usually matches the model result with the original parameter values, which is illustrated by both a median RMSE of zero and a confidence interval width of zero. However, for a few outliers outside the 80% confidence interval, the predicted behavior was structurally different from the model behavior, resulting in non-zero mean RMSE.

What is even more important, consumer behavior could be reconstructed from noisy samples as well, which is demonstrated by errors converging to zero as K increases, independently from the value of π. The mean RMSE decreased to 1.15% (median: 0.27%) with $\pi = 0.1$ and to 0.91% (median: 0.54%) with $\pi = 0.2$ as the number of historic samples increased to 200.

Computation times are displayed in Table 2, where each row contains average computation time for solving the elicitation problem with a given number of samples K. Each average value is computed over instances with three different levels of perturbation, with five SLP runs for each instance starting from different random initial solutions, i.e., 15 SLP runs altogether. Solution times range from 0.17 s for small instances ($K = 25$) to 2.03 s for the largest problems investigated ($K = 200$). Instances without perturbation ($\pi = 0$) could be solved quickly, in 0.08 s for $K = 25$ and 0.49 s for $K = 200$, since SLP terminated after a few iterations with an objective value of zero. In contrast,

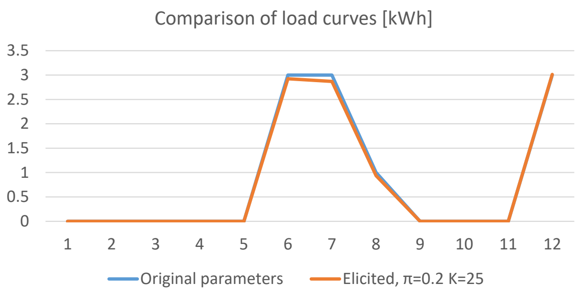

Fig. 1 Comparison of the load curves over time for $N = 1$ with the original parameter values and the elicited parameters ($K = 25$, $\pi = 0.2$).

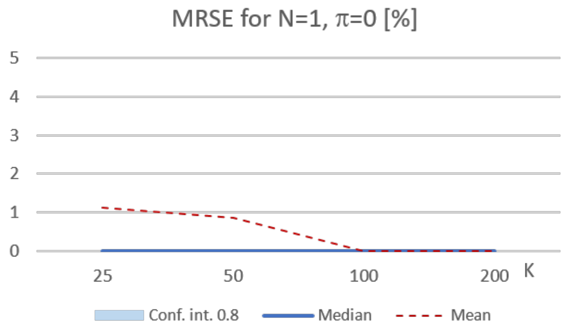

Fig. 2 RMSE of the elicitation for $N = 1$ and $\pi = 0$ over different values of K.

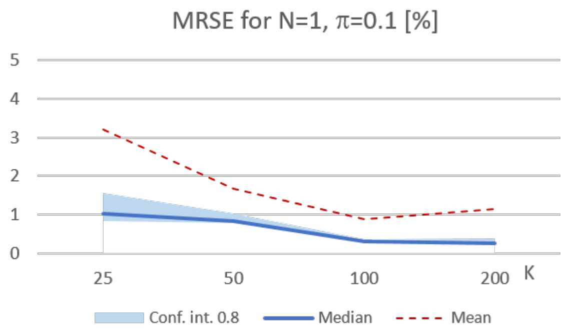

Fig. 3 RMSE of the elicitation for $N = 1$ and $\pi = 0.1$ over different values of K.

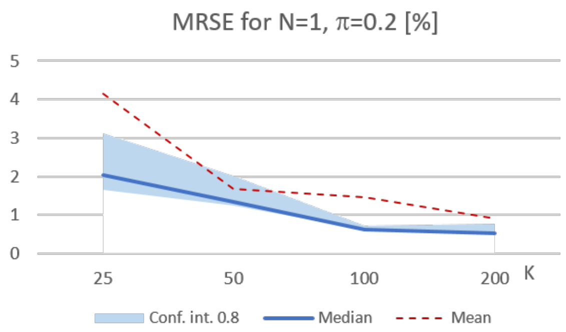

Fig. 4 RMSE of the elicitation for $N = 1$ and $\pi = 0.2$ over different values of K.

there was no clear correlation between the computation times and the level of perturbation for $\pi > 0$.

Table 2 Computation times for $N = 1$.

K	Avg. time (s)
25	0.17
50	0.44
100	0.89
200	2.03

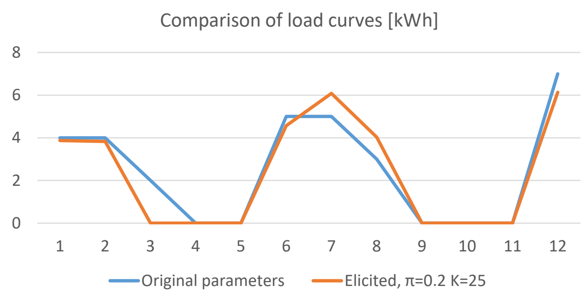

Fig. 5 Comparison of the load curves over time for $N = 3$ with the original parameter values and the elicited parameters ($K = 25$, $\pi = 0.2$).

5.3 Results with multiple types of load

Results for the generic case with multiple types of load ($N \geq 2$) are also promising, but still somewhat more ambiguous. The proposed approach could predict consumer behavior with a reasonable accuracy, as depicted by the power curves for the instance with $K = 25$ and $\pi = 0.2$ in Figure 5, and the MRSE diagrams in Figures 6-11. A typical error of 5-8% in the predicted consumption of an individual consumer, in itself, is acceptable in typical applications. Yet, these errors are an order of magnitude larger than with $N = 1$.

Moreover, the SLP solution approach could not reconstruct the original parameter values ($U^0_{i,t}$, M^0_i, $L^0_{i,t}$) even for instances without noise ($\pi = 0$), where these parameter values would incur an optimal solution with zero error. Here, the MRSE does not converge to zero as the number of samples K increases. These negative results indicate that further research should be invested into developing more efficient algorithms for solving the inverse optimization problem.

Computation times for different problem sizes are shown in Table 2, with average values computed over 15 SLP runs. While small problems could be solved quickly, e.g., in 4.19 s for $N = 3$ and $K = 25$, larger instances required considerable computation times, e.g., 432.20 s for $N = 5$ and $K = 200$. Again, there was no correlation between the computation times and the level of perturbation π.

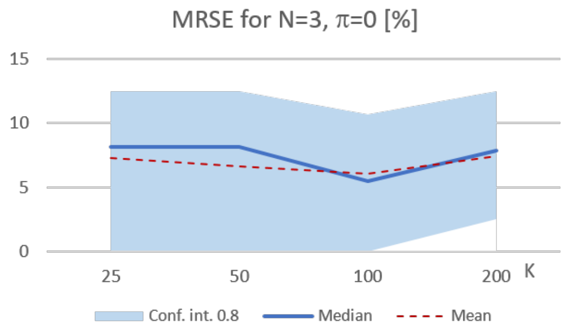

Fig. 6 RMSE of the elicitation for $N = 3$ and $\pi = 0$ over different values of K.

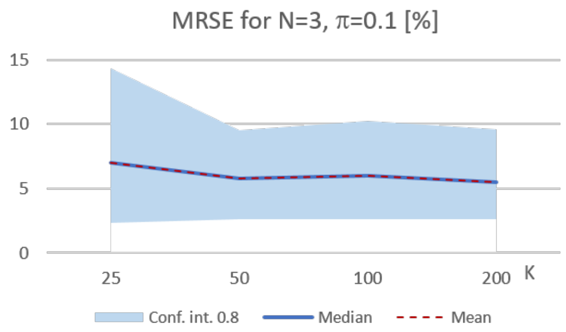

Fig. 7 RMSE of the elicitation for $N = 3$ and $\pi = 0.1$ over different values of K.

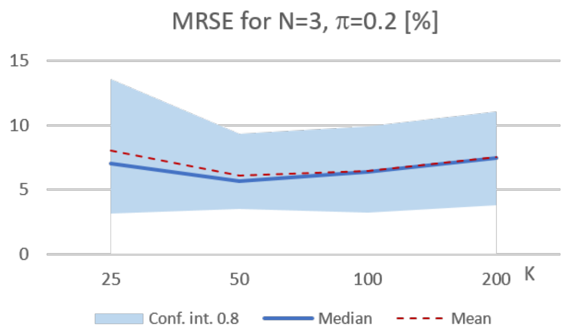

Fig. 8 RMSE of the elicitation for $N = 3$ and $\pi = 0.2$ over different values of K.

6 Discussion on alternative solution approaches

Obviously, the proposed approach is not the only algorithm that might be eligible to solve the above inverse optimization problem. For the specific consumer

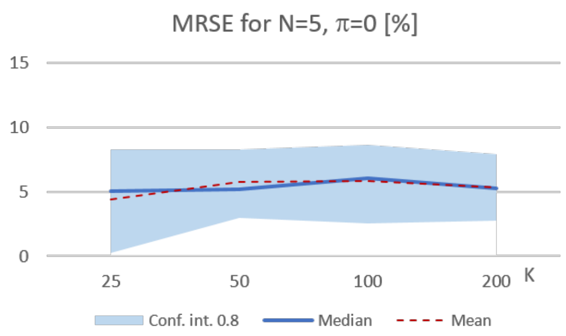

Fig. 9 RMSE of the elicitation for $N = 5$ and $\pi = 0$ over different values of K.

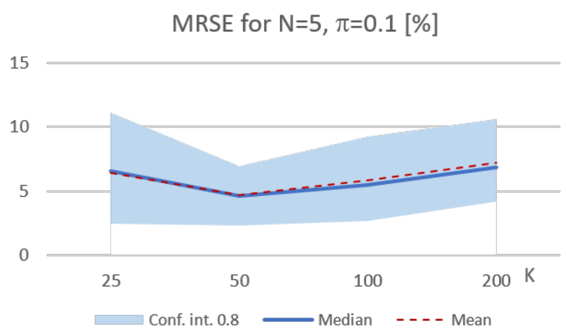

Fig. 10 RMSE of the elicitation for $N = 5$ and $\pi = 0.1$ over different values of K.

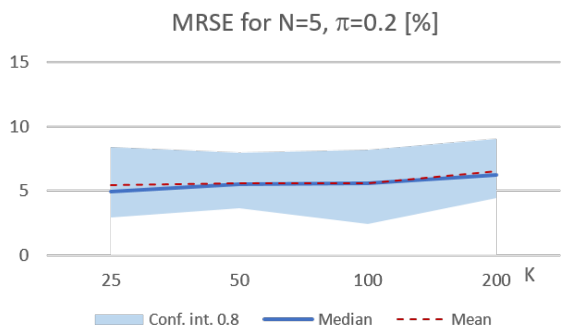

Fig. 11 RMSE of the elicitation for $N = 5$ and $\pi = 0.2$ over different values of K.

model, a straightforward exact approach is exploiting a structural property of the consumer's optimal solutions as follows.

Table 3 Computation times for $N = 3$ and $N = 5$.

K	Avg. time (s)	
K	$N = 3$	$N = 5$
25	4.19	14.96
50	10.62	44.41
100	40.12	121.756
200	147.76	432.20

Lemma 2 *Given an optimal solution to the investigated direct optimization problem, for any load type i, sample k, and for any two time periods t_1 and t_2, at least one of the following statements holds:*

- *No load is scheduled to period t_1, i.e., $x^k_{i,t1} = 0$; or*
- *Period t_2 is saturated, i.e., $x^k_{i,t2} = L_{i,t}$; or*
- *Period t_1 is preferred to period t_2 for the consumer when scheduling load of type i, i.e., $(U_{i,t1} - Q^k_{t1}) \geq (U_{i,t2} - Q^k_{t2})$.*

Proof: Assume that none of the above conditions hold in a given solution. Then, there is a nonzero amount of load, $\min(x^k_{i,t1},\ L_{i,t2} - x^k_{i,t2}) > 0$, that can be rescheduled from period t_1 to period t_2, and this causes an increase of the objective value for the consumer. Hence, the original solution cannot be an optimal one. □

This lemma can be exploited directly by a mixed-integer linear programming (MILP) formulation with two sets of binary variables, one to indicate if a time period is *empty* ($x^k_{i,t} = 0$) and another one to denote that the period is *full* ($x^k_{i,t} = L_{i,t}$). This MILP was implemented in FICO Xpress, and it was used to validate the results of the proposed solution approach on small problem instances. However, this MILP does not scale up to larger instances, due to the large number of binary variables and the weak LP relaxation.

An interesting alternative approach can be the application of stochastic optimization techniques motivated by machine learning, such as the *stochastic gradient descent* (SGD) algorithm or one of its descendants with reduced variance [21,26]. These iterative algorithms have been developed to learn from large data sets. They employ a small batch of data in each individual iteration to increase the speed of learning, while they have shown favorable convergence properties on the whole data set. Finally, the coupling of the inverse optimization approach with load disaggregation techniques is a promising direction for the multiple load types case.

7 Conclusions

This paper introduced a novel approach to elicit the parameters of electricity consumer models from historic time series based on inverse optimization for the purpose of DRM. The approach is applicable to consumer models formulated as a LP, and relies on converting the inverse LP model to a QCQP and solving it using SLP. This requires the generalization of the models commonly applied in inverse optimization in two different ways: by unknown parameters both in the objective and the constraints, and by adopting multiple desired solutions.

The approach was illustrated on a consumer model with multiple types of deferrable load behind a single smart meter, which is a common model in the literature of DRM. In computational experiments on generated data, the approach has shown promising performance. For instances with a single load type, the elicited parameters enable a nearly perfect prediction of future consumer behavior. The relative error of 5-8% is also a reasonable result for instances with multiple load types, but for this generic case, the convergence properties of the SLP algorithm did not satisfy our expectations.

Accordingly, future research has to pursue multiple directions: for improving the performance of the algorithm, the combination of the inverse optimization approach with stochastic optimization methods or load disaggregation using machine learning techniques has to be investigated. Moreover, the evaluation of the proposed approach on more generic consumer models and real measured data is of interest.

Acknowledgements This research has been supported by the NKFIA 129178 grant and the GINOP-2.3.2-15-2016-00002 grant on an "Industry 4.0 research and innovation center of excellence". The author acknowledges the support of the János Bolyai Research Fellowship.

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