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Bilevel Programming Approach to Optimizing a Time-variant Electricity Tariff for Demand Response

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***Abstract*—This paper proposes a bilevel programming model to day-ahead electricity tariff optimization in smart grids to balance grid-level demand and supply at all times. In this Stackelberg game approach, the leader is the grid operator, who aims to set the tariff to ensure the balance of supply and demand. The followers are groups of consumers, who, in response to the observed tariff, schedule their controllable consumption and determine the charging/discharging policy of their batteries to minimize their cost. The bilevel optimization problem is reformulated into a single-level quadratically constrained quadratic program (QCQP), which is then solved by a successive linear programming (SLP) algorithm. The approach is illustrated on an example with three different consumer groups.**

I. Introduction

A key challenge in the operation of smart grids is achieving optimal demand response management to balance electricity demand and supply at all times. The primary motivation for consumers to contribute to the stability of the grid is considered to be an appropriate, time-varying electricity tariff; hence, the grid operator can exploit the consumers' cost minimizing behavior to pursue the grid-level, social objective. In this paper, we investigate an electricity tariff optimization problem in a smart grid setting where the grid operator must announce time-varying electricity prices one day ahead, with hourly resolution. Consumers respond to the announced energy tariff by optimizing the schedule of their controllable loads and by determining their battery charging/discharging policy. It is assumed that the objective of the grid operator is to maintain the balance between demand and supply, whereas the consumers' optimization criterion is a combination of their cost of electricity and their achieved utility.

This problem is analyzed in a game theoretical perspective. A bilevel optimization model is proposed, where the grid operator is the leader and the consumer groups (CGs), formed of individual consumers with similar characteristics, act as multiple independent followers. For solving this bilevel problem, we reformulate it to a single level optimization problem by exploiting duality for the followers' problem, which is represented as a linear program. Reformulation results in a quadratically constrained quadratic program (QCQP). Since this QCQP does not exhibit properties that would render it tractable by exact optimization methods, we use *successive linear programming* (SLP) for solving it [1].

This paper is organized as follows. First, a brief review of the related literature is given. Then, the tariff optimization problem is defined formally, and a solution approach is proposed. The approach is illustrated on an example with three CGs, and the results are discussed in detail. Finally, conclusions are drawn and directions for future research are proposed.

II. Literature review

Game theoretic approaches to tariff optimization in smart grids have received considerable attention recently [2]. Most of the contributions focus on a real-time pricing scenario: a Stackelberg approach is investigated for demand response scheduling under load uncertainty in [3]. In [4], a multi-leader, multi-follower Stackelberg game is defined for demand response for a large population, and closed-form expressions are derived for the unique equilibrium solution. Electric vehicle charging is modeled as a generalized Stackelberg game with the grid as the leader and multiple plug-in electric vehicle groups as followers in [5]. The followers play a generalized Nash game to establish their equilibrium strategies, which determines their response to the prices set by the grid. A similar game model is applied in [6] to the management of consumer-to-grid systems to encourage consumers to feed just the required amount of energy into the grid while maintaining social optimality. Regarding day-ahead tariff models, [7] formulates a Stackelberg game and solves it using an iterative heuristic approach. Two different games related to demand side management are studied in [8]; a Nash game between consumers equipped with batteries and a Stackelberg game between the utility provider and the consumers.

For modeling the consumers' energy management problem, linear programming (LP) models capturing active-power-only power flow equations are extensively used in the literature, see, e.g., [9]. More sophisticated models that cover reactive power and voltage magnitudes, as well, are discussed in [10, 11].

An introduction to bilevel programming, including basic modeling and solution techniques is given in [12, 13]. Approaches to encoding continuous bilevel optimization problems into equivalent single level problems using the optimal value and the KKT reformulations are investigated in [14]. Bilevel programming approaches, similar to the models presented in this paper, have been investigated in various fields of application, such as inventory management [15] or tariff setting for the airline industry [16]. A recent survey on bilevel programming for price setting problems is given in [17].

III. Problem Definition

Consider a smart power grid that serves various consumers in a given area. Each consumer can be ordered into one of the finitely many consumer groups (CGs), each comprising consumers with similar profiles regarding energy consumption (controllable and uncontrollable), production, and storage requirements and capabilities. The system architecture is depicted in Figure 1.

The ensemble of consumers in CG$_i$ is characterized by its fixed, uncontrollable production $C^+_{i,t}$ and consumption $C^-_{i,t}$ in each time period $t = 1, ..., T$. In addition, CG$_i$ requires a (potentially zero) controllable consumption of M_i over the finite time horizon, where in each time period, a maximum of $\bar{L}_{i,t}$ can be consumed. Setting $\bar{L}_{i,t} = 0$ for some t can be used to define time windows for the controllable consumption. To capture the CGs' preferences on the timing of the controllable consumption, (the monetary equivalent of) the utility of scheduling one unit of controllable consumption in time period t is denoted by $U_{i,t}$.

Moreover, CG$_i$ may be equipped with a battery to store energy. The battery is characterized by its capacity $\overline{B}_i$, the maximum charge and discharge rates R^+_i and R^-_i, the cycle efficiency of the battery η_i, and the initial battery state of charge $b_{i,0}$. The consumer may wish to maintain a given, time-varying minimum state-of-charge $\underline{B}_{i,t}$ in the battery in order to safeguard from unexpected power outages.

It is assumed that every CG chooses its battery charge and discharge rates $r^+_{i,t}$ and $r^-_{i,t}$, as well as its controllable consumption $L_{i,t}$ in such a way that it optimizes the objective function composed of maximizing the utility and minimizing the cost of energy w.r.t. the energy tariff set by the grid operator. It is emphasized that this CG model is generic enough to capture a wide variety of consumers, including households or offices with uncontrollable consumption only (and therefore, unresponsive to the energy tariff), consumers equipped with renewable energy generation and/or storage devices, owners of plug-in electric vehicles, or complex sub-grid systems.

In this paper, we focus on the problem faced by the grid operator, who wishes to apply demand response management by setting the electricity tariff to match future electricity demand and supply in the best possible way. The grid operator applies the same time-of-use, variable electricity tariff to all consumers. The tariff is specified in the form of the day-ahead electricity purchase price Q^+_t and feed-in price Q^-_t for periods $t = 1, ..., T$. The objective of the grid operator is to minimize the difference between the target grid-level consumption D_t and the actual consumption of the consumers, $\sum_{i=1}^{N}(x^+_{i,t} - x^-_{i,t})$, in a quadratic norm.

The applied notation is summarized in Table I. In order to ensure the feasibility of the followers' problem, the following straightforward assumptions are made:

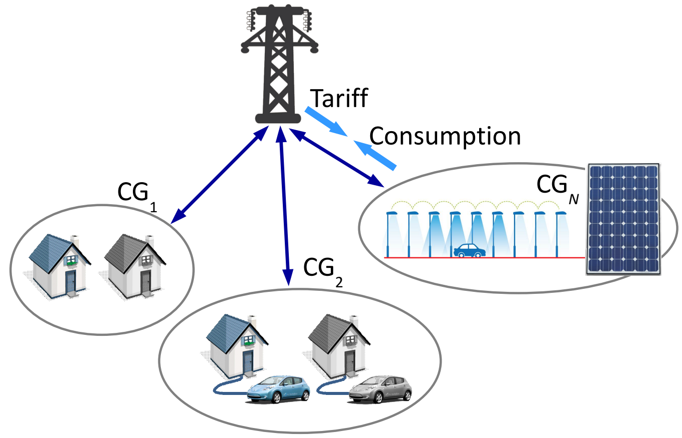

Fig. 1. System architecture with a grid operator and multiple CGs.

$$\sum_{t=1}^{T} \bar{L}_{i,t} \geq M_i \qquad \forall i \tag{1}$$

$$\underline{B}_{i,t} \leq \overline{B}_i \qquad \forall i, t \tag{2}$$

$$b_{i,0} \leq \overline{B}_i \qquad \forall i \tag{3}$$

$$b_{i,0} + tR^+_i \leq \underline{B}_{i,t} \qquad \forall i, t \tag{4}$$

With the assumptions stated, the followers' problem has a feasible solution under any decision of the leader on Q^+ and Q^-, since ignoring the battery ($r^+_{i,t} = r^-_{i,t} = 0$, $b_{i,t} = b_{i,0}$, $\forall i, t$), and distributing the controllable consumption arbitrarily among time periods in such a way that the bounds $\bar{L}_{i,t}$ are respected always leads to a feasible solution.

It is assumed that the grid operator can compensate for an arbitrary deviation of the grid-level consumption from D_t using backup generators. The grid operator is aware of the decision model and the parameters of the CGs, which leads to a bilevel optimization problem with the grid operator as the leader and the CGs as multiple followers. We adopt the *optimistic* bilevel assumption, i.e., in case a followers has multiple optimal solutions, then it selects one among them that is the *most favorable* for the leader.

IV. Solution Approach

A. Overview

Before the detailed presentation of the proposed models and algorithms, we give an overview of the solution approach to the above energy tariff optimization problem, to be solved by the grid operator. The approach exploits that the optimal response of an individual follower depends solely on the energy tariff set by the leader, while it is independent of the responses of fellow followers. Hence, the problem can be modeled as a bilevel optimization problem with a single leader (the grid operator) and multiple independent followers (the CGs).

First, the models of an individual follower and the leader are formally defined. Then, these are combined into a single

TABLE I
THE NOTATION USED IN THE PAPER.

Dimensions	
T	Number of time periods
N	Number of consumer groups
Parameters	
$C^+_{i,t}$	Uncontrollable production of CG i in period t
$C^-_{i,t}$	Uncontrollable consumption of CG i in period t
M_i	Total controllable consumption of CG i during the horizon
$\bar{L}_{i,t}$	Maximum controllable consumption of CG i in period t
$U_{i,t}$	Utility of controllable consumption of CG i in period t
D_t	Target grid-level consumption in period t
$\overline{Q}$	Maximum electricity price
$\underline{Q}$	Minimum electricity price
R^+_i	Maximum battery charge rate at CG i
R^-_i	Maximum battery discharge rate at CG i
η_i	Cycle efficiency of the battery at CG i
$b_{i,0}$	Initial battery state of charge at CG i
$\overline{B}_i$	Battery capacity at CG i
$\underline{B}_{i,t}$	Minimum battery state of charge at CG i in period t
Leader's variables	
Q^+_t	Electricity purchase price in period t
Q^-_t	Electricity feed-in price in period t
Δ_t	Absolute difference between target and actual demand in period t
Followers' variables	
$x^+_{i,t}$	Electricity purchase rate of CG i in period t
$x^-_{i,t}$	Electricity feed-in rate of CG i in period t
$L_{i,t}$	Controllable consumption of CG i in period t
$r^+_{i,t}$	Battery charge rate at CG i in period t
$r^-_{i,t}$	Battery discharge rate at CG i in period t
$b_{i,t}$	Battery state of charge at CG i at the end of period t
Objective values	
f	Objective value of the leader (grid operator)
g_i	Objective value of follower (CG) i

level QCQP reformulation of the bilevel problem. Finally, an SLP algorithm is proposed for solving the resulting QCQP formulation.

B. Consumer Groups' (Followers') Problem

An individual follower's optimization problem looks at scheduling the controllable load, determining the battery charge/discharge policy, as well as the corresponding grid purchase/feed-in policy in order to maximize the individual follower's utility and minimize its energy cost w.r.t. the energy tariff set by the leader. It can be encoded into a linear program (LP) as follows, with the leader's variables appearing in the objective function only.

Minimize

$$g_i(Q^+, Q^-) = \sum_{t=1}^{T} \left(Q^+_t x^+_{i,t} - Q^-_t x^-_{i,t} - U_{i,t} L_{i,t} \right) - \frac{Q^+_T + Q^-_T}{2} b_{i,T} \quad (5)$$

subject to

$$C^+_{i,t} - C^-_{i,t} + x^+_{i,t} - x^-_{i,t} - L_{i,t} = r^+_{i,t} - r^-_{i,t} \quad \forall t \quad (6)$$

$$\eta_i\, r^+_{i,t} - r^-_{i,t} = b_{i,t} - b_{i,t-1} \quad \forall t \quad (7)$$

$$\sum_{t=1}^{T} L_{i,t} = M_i \quad (8)$$

$$0 \le L_{i,t} \le \bar{L}_{i,t} \quad \forall t \quad (9)$$

$$\underline{B}_{i,t} \le b_{i,t} \le \overline{B}_i \quad \forall t \quad (10)$$

$$0 \le r^+_{i,t} \le R^+_i \quad \forall t \quad (11)$$

$$0 \le r^-_{i,t} \le R^-_i \quad \forall t \quad (12)$$

$$0 \le x^+_{i,t}, x^-_{i,t} \quad \forall t \quad (13)$$

The follower's objective (5) is comprised of the total cost of energy, i.e., the cost of energy purchased minus the income from feeding energy into the grid; the CG's utility achieved by the timing of the controllable consumption; plus a valuation of the energy stored in the battery at the end of the planning horizon. The latter component is necessary in order to avoid end-of-horizon effects, e.g., followers selling all the energy stored in the batteries. Constraint (6) encodes that the energy balance at the consumer group is maintained. Equation (7) computes the battery state of charge based on the charge and discharge rates. Constraints (8) and (9) ensure that the amount and the timing of the controllable consumption satisfies the requirements. Finally, inequalities (10-13) define the range of the battery state of charge, the charge and discharge rates, as well as the electricity purchase and feed-in rates at the consumer group.

It is noted that all constraints in the followers' model are linear in the followers' variables, whereas the objective contains the leader's variables as multipliers, making it a bilinear (quadratic) expression. The models of different followers are linked only via the leader's decision problem.

C. Grid Operator's (Leader's) Problem

Minimize

$$f = \sum_{t=1}^{T} \Delta_t^2 \quad (14)$$

subject to

$$\Delta_t \ge D_t - \sum_{i=1}^{N} (x^+_{i,t} - x^-_{i,t}) \quad \forall t \quad (15)$$

$$\Delta_t \ge \sum_{i=1}^{N} (x^+_{i,t} - x^-_{i,t}) - D_t \quad \forall t \quad (16)$$

$$\underline{Q} \le Q^-_t \le Q^+_t \le \overline{Q} \quad \forall t \quad (17)$$

$$\begin{pmatrix} x^+_{i,t} \\ x^-_{i,t} \end{pmatrix} \in \arg\min \left\{ \; g_i(Q^+, Q^-) \mid (6) - (13) \right\} \quad \forall i \quad (18)$$

The leader's objective is to minimize the squared deviation between the target and the actual grid-level consumption. Constraints (15) and (16) calculate this (absolute) deviation from the planned energy purchase and feed-in of the followers.[1] Inequalities (17) define the range of the energy tariff variables, where a strictly positive lower bound $\underline{Q}$ is required to exclude degenerate solutions with $Q_t^- = Q_t^+ = 0$, which would render all solutions of the followers optimal. Finally, constraint (18) states that the grid purchase and feed-in values are determined by the followers using the above optimization model.

D. Single Level QCQP Reformulation

The key to reformulating the above bilevel problem into a single level optimization problem is modeling the followers' optimality condition (18). Exploiting LP duality for the followers' model, the optimality condition (18) can be replaced by the conjunction of the followers' primal formulation, (6)-(13) and its dual formulation, with the primal and the dual objectives connected by an equality constraint. In this representation, the leader's variables, Q_t^+ and Q_t^-, appear only in the objective function in the followers' primal, and hence, in the right hand side of the followers' constraints in the dual formulation. Non-linear terms in this single level equivalent model include the followers' primal objective (5), which is a bilinear expression containing the multiplication of the followers' and the leader's variables; and the leaders quadratic objective (14). Since displaying the complete QCQP model requires extensive additional formalism for the followers' dual model, it is presented in the on-line appendix [18].

E. Solution Method

For solving the above single level QCQP reformulation of the original bilevel optimization problem, we propose using a *successive linear programming* (SLP) approach. SLP solves non-linear problems by iteratively building local LP approximations of the original problem, and solving each approximation using standard LP techniques [1]. The algorithm departs from an initial solution x_0, and in each iteration k, it builds a local linearization of the original problem around x_k, denoted as LP$_k$. The optimal solution of LP$_k$ is looked for with a certain step bound, $-s \leq x - x_k \leq s$. If the optimal LP solution is feasible with a given tolerance, then it is accepted as the next solution x_{k+1} (and possibly s is increased); otherwise $x_{k+1} = x_k$ and s is decreased.

SLP is known to be an efficient heuristic with good convergence properties for problems where most of the constraints are linear, which is the case for the above problem. On the other hand, SLP is not an exact solution approach, i.e., it may converge to a local optimum. In the actual implementation, the algorithms of the SLP package of Fico Xpress were used, with initial solution values set to $Q_t^- = Q_t^+ = \underline{Q}$.

V. Illustrative Example

In the following sample problem, we address the problem of electricity tariff setting for demand response management in a smart grid on a one-day horizon, with the time horizon staring at 8:00, using hourly time units. The grid serves the following three CGs:

- An energy-positive solar street lighting system equipped with PV generation and battery storage, called E+grid [19, 20]. The data used is gathered from a physical prototype system with 191 intelligent LED luminaries and 151.2 m^2 of active PV surface area. On a sunny day in October, the system is a net producer (up to 15 kW) during the day, and a net consumer (up to 3.5 kW) during the night. The lighting system realizes demand response by optimizing the charging/discharging schedule of its 20 kWh battery storage.
- An electric vehicles CG that appears as controllable load. Individual vehicles are connected to the grid between 17:00–20:00 and disconnected between 6:00-8:00 in the morning. Three EVs, with a 24kWh battery pack in each vehicle, have to be recharged from a 50% state to 100%. This is modeled as a controllable load of 36kWh, limited to the above period. The vehicle-to-grid (V2G) option is ignored. The CG has a slight preference for scheduling the charging process as early as possible, which is reflected in utility values $U_{2,t}$ decreasing linearly over time.
- A households and offices CG with uncontrollable consumption only. In the experiment, the data of 15 average Hungarian households with hourly resolution was used, corresponding to consumption profile with high consumption (5–6 kW) during the day, a peak around 19:00–20:00, and a valley period (3.8–5 kW) during the night. There are no decision variables related to this CG.

The grid operator must set time-varying purchase and feed-in prices between $\underline{Q} = 1$ and $\overline{Q} = 100$ c/kWh. We assume that the objective of the grid operator is to ensure a constant target consumption of 2.49 kW throughout the horizon.

The results of tariff optimization are displayed in Figures 2-5, for the overall grid and for each individual CG. The diagrams compare the optimized consumption profile to the baseline consumption, where the latter is received by schedul-

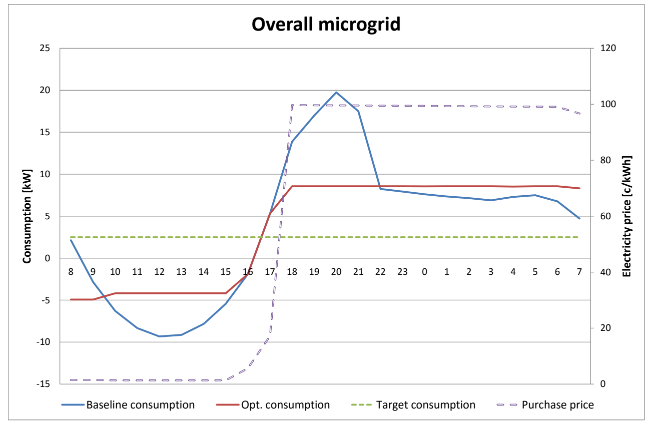

Fig. 2. Solution with optimized tariff: grid-level consumption over time.

[1] Formally equivalent simplifications of this model exist, e.g., by lifting up $\Delta_t = \sum_{i=1}^{N}(x_{i,t}^+ - x_{i,t}^-) - D_t$ into the objective. However, the SLP solution approach is sensitive to the representation, and the proposed model achieved reliably better solutions in experiments than trivial simplifications.

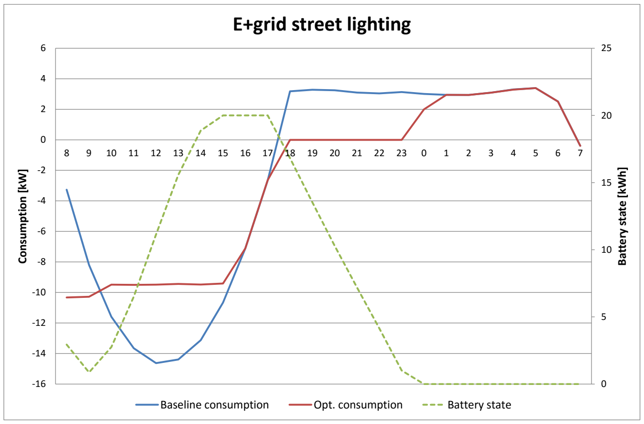

Fig. 3. Solution with optimized tariff: consumption and battery state of the E+grid lighting system CG over time.

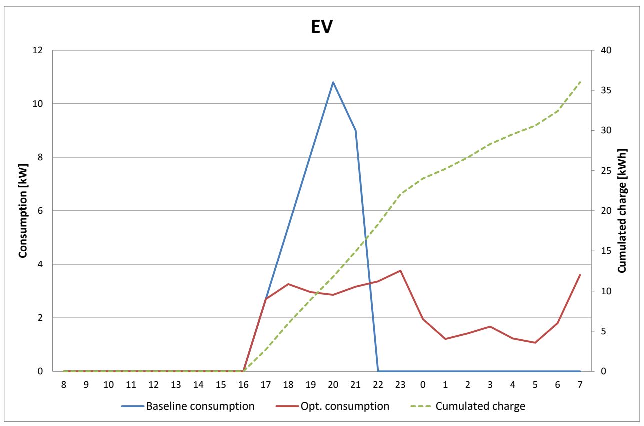

Fig. 4. Solution with optimized tariff: consumption and cumulated load of the electronic vehicle CG over time.

ing the controllable loads as early as possible and not using the batteries. The optimized purchase tariff is also shown in Figure 2: low prices are applied in the valley period (<1.5 c/kWh until 16:00, <18 c/kWh until 18:00), whereas high, slightly decreasing prices are used afterwards (99.68 c/kWh at 18:00, decreasing by 0.05 c/kWh per hour). This corresponds to a peak-to-average ratio of 1.68.

As a result of the optimized tariff, the grid-level consumption was considerably smoothed compared to the baseline consumption, but constant consumption throughout the horizon could not be achieved. The optimized consumption is characterized by two separate, even periods. The time period until 16:00 is dominated by the PV production of the E+grid system, with a nearly constant consumption of -4 kW (production of 4 kW). After a short transition, the consumption is stabilized at 8.56 kW after 18:00.

The applied tariff motivated the E+grid CG to charge its battery in the valley period, to reach a fully charged state at 17:00, and to gradually discharge the battery afterwards. The time period relevant for the EV CG is after 17:00, when the vehicles are connected to the grid. In this period, the slight decrease of the purchase prices over time compensates the CG for its

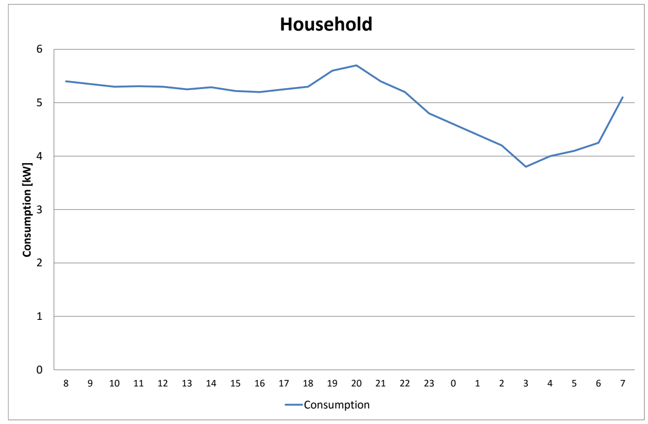

Fig. 5. Solution with optimized tariff: consumption of households CG over time.

linearly decreasing utility function, and therefore, an arbitrary scheduling of the controllable load became optimal for the CG. Together with the optimistic bilevel assumption applied, this tariff facilitated a perfectly balanced consumption profile within the relevant time period. There are no controllable variables for the households CG. This tariff and consumption profile is globally optimal for the grid operator, since the two periods before and after 17:00 cannot be balanced better: controllable loads cannot be anticipated and all batteries are fully charged at 17:00.

An aspect of the results that requires further discussion is the application of the optimistic bilevel assumption. As seen above, the assumption may lead to favorable results in the computational model that are hard to reproduce in practice, unless some sophisticated communication and control method is implemented between the grid operator and the CGs. Nevertheless, by the structure of the model, the grid operator can abuse this assumption only for at most one CG at a time. Hence, the significance of this unrealistic effect diminishes as the number of CGs increases.

For the actual implementation in Fico Xpress 7.5, solving problem instances similar to the above sample problem (with $N = 3$ and $T = 24$) took less than 1 second on a personal computer with an Intel i5 2.40Ghz CPU and 4GB RAM. The SLP solution procedure converged to a locally optimal solution, which was often also the global optimum, within 5-20 iterations.

VI. Conclusions and Future Research

This paper proposed a game theoretic approach to energy tariff optimization in a smart grid. In the bilevel optimization model, the grid operator is the leader who aims to set the energy tariff in such a way that the grid-level consumption, emerging from the tariff-aware, rational behavior of the consumers as followers, tracks the desired profile. The bilevel optimization problem was first compiled into a single level QCQP, then solved by a successive linear programming algorithm using a black box solver. The novelty of the proposed method lies in the application of formal mathematical programming techniques to a game theoretical approach to demand side management. The efficiency of the method was illustrated in an example with three CGs.

Obviously, there are numerous straightforward extensions of the above model, e.g., with multiple controllable loads and batteries for each CG. Constraints can be added to protect the consumers from extreme energy tariffs set by the grid operator, e.g., by fixing the average tariff over time. It is also possible to combine the goal of load balancing and the economic objective of the grid operator into a single objective.

Future research will address the detailed analysis of algorithm performance. The computational complexity of the problem must be proven (NP-hardness is conjectured), and relevant polynomially solvable cases should be identified. Extensive computational experiments, including instances with more consumer groups, must be performed.

Another important pre-condition of the applicability of bilevel models is the ability of the leader to identify the models and parameters applied by its followers, which is a non-trivial problem in practical application scenarios characterized by information asymmetry. For this purpose, we plan to investigate an inverse optimization approach; given historical pairs of a follower's input and response, the inverse optimization model seeks for parameters that ensure that every response is optimal for the corresponding input. We plan to implement a model similar to an analogous case in inventory control [21].

VII. Acknowledgments

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