Sensitivity analysis in PROMETHEE

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Outline

- PROMETHEE
- Partial sensitivity analysis of the weights
- Global sensitivity analysis of the weights

	C_1	C_2	C_3	C_4
Type	V-shape	V-shape	Linear	U-shape
Weight	0.2	0.4	0.3	0.1
A_1	5	3	8	1
A_2	7	5	4	5
A_3	8	2	3	2

$\mathbf{P_1}$ =

C_1	A_1	A_2	A_3
A_1	0	0	0
A_2	1	0	0
A_3	1	0.5	0

$\mathbf{P_2}$ =

C_2	A_1	A_2	A_3
A_1	0	0	0.25
A_2	0.5	0	0.75
A_3	0	0	0

$\mathbf{P_3}$ =

C_3	A_1	A_2	A_3
A_1	0	0	1
A_2	0	0	0
A_3	0	0	0

$\mathbf{P_4}$ =

C_4	A_1	A_2	A_3
A_1	0	0	0
A_2	1	0	0
A_3	0	0	0

$\mathbf{P} = w_1\mathbf{P_1} + w_2\mathbf{P_2} + w_3\mathbf{P_3} + w_4\mathbf{P_4} =$

	A_1	A_2	A_3
A_1	0	0.3	0.4
A_2	0.5	0	0.3
A_3	0.2	0.1	0

Outgoing and incoming flows:

$\mathbf{P} =$

	A_1	A_2	A_3	Φ^+
A_1	0	0.3	0.4	0.35
A_2	0.5	0	0.3	0.4
A_3	0.2	0.1	0	0.15
Φ^-	0.35	0.2	0.35	

Net flows:

$$\Phi(A_1) = \Phi^+(A_1) - \Phi^-(A_1) = 0.35 - 0.35 = 0$$
$$\Phi(A_2) = \Phi^+(A_2) - \Phi^-(A_2) = 0.4 - 0.2 = 0.2$$
$$\Phi(A_3) = \Phi^+(A_3) - \Phi^-(A_3) = 0.15 - 0.35 = -0.2$$

Criterionwise outgoing and incoming flows:

$\mathbf{P_1}$ =

C_1	A_1	A_2	A_3	$\Phi^+_{C_1}$
A_1	0	0	0	0
A_2	1	0	0	0.5
A_3	1	0.5	0	0.75
$\Phi^-_{C_1}$	1	0.25	0	

Criterionwise net flows:

$$\Phi_{C_1}(A_1) = \Phi^+_{C_1}(A_1) - \Phi^-_{C_1}(A_1) = 0 - 1 = -1$$

$$\Phi_{C_1}(A_2) = \Phi^+_{C_1}(A_2) - \Phi^-_{C_1}(A_2) = 0.5 - 0.25 = 0.25$$

$$\Phi_{C_1}(A_3) = \Phi^+_{C_1}(A_3) - \Phi^-_{C_1}(A_3) = 0.75 - 0 = 0.75$$

Criterionwise outgoing and incoming flows:

$\mathbf{P_2}$ =

C_2	A_1	A_2	A_3	$\Phi^+_{C_2}$
A_1	0	0	0.25	0.125
A_2	0.5	0	0.75	0.625
A_3	0	0	0	0
$\Phi^-_{C_2}$	0.25	0	0.5	

Criterionwise net flows:

$$\Phi_{C_2}(A_1) = \Phi^+_{C_2}(A_1) - \Phi^-_{C_2}(A_1) = 0.125 - 0.25 = -0.125$$

$$\Phi_{C_2}(A_2) = \Phi^+_{C_2}(A_2) - \Phi^-_{C_2}(A_2) = 0.625 - 0 = 0.625$$

$$\Phi_{C_2}(A_3) = \Phi^+_{C_2}(A_3) - \Phi^-_{C_2}(A_3) = 0 - 0.5 = -0.5$$

Criterionwise outgoing and incoming flows:

$\mathbf{P_3}$ =

C_3	A_1	A_2	A_3	$\Phi^+_{C_3}$
A_1	0	0	1	1
A_2	0	0	0	0
A_3	0	0	0	0
$\Phi^-_{C_3}$	0	0.5	0.5	

Criterionwise net flows:

$$\Phi_{C_3}(A_1) = \Phi^+_{C_3}(A_1) - \Phi^-_{C_3}(A_1) = 1 - 0 = 1$$

$$\Phi_{C_3}(A_2) = \Phi^+_{C_3}(A_2) - \Phi^-_{C_3}(A_2) = 0 - 0.5 = -0.5$$

$$\Phi_{C_3}(A_3) = \Phi^+_{C_3}(A_3) - \Phi^-_{C_3}(A_3) = 0 - 0.5 = -0.5$$

Criterionwise outgoing and incoming flows:

$\mathbf{P_4}$ =

C_4	A_1	A_2	A_3	$\Phi^+_{C_4}$
A_1	0	0	0	0
A_2	1	0	0	0.5
A_3	0	0	0	0
$\Phi^-_{C_4}$	0.5	0	0	

Criterionwise net flows:

$$\Phi_{C_4}(A_1) = \Phi^+_{C_4}(A_1) - \Phi^-_{C_4}(A_1) = 0 - 0.5 = -0.5$$

$$\Phi_{C_4}(A_2) = \Phi^+_{C_4}(A_2) - \Phi^-_{C_4}(A_2) = 0.5 - 0 = 0.5$$

$$\Phi_{C_4}(A_3) = \Phi^+_{C_4}(A_3) - \Phi^-_{C_4}(A_3) = 0 - 0 = 0$$

$0 = \Phi(A_1) = w_1\Phi_{C_1}(A_1) + w_2\Phi_{C_2}(A_1) + w_3\Phi_{C_3}(A_1) + w_4\Phi_{C_4}(A_1) = 0.2 \times -1 + 0.4 \times -0.125 + 0.3 \times 1 + 0.1 \times -0.5 = 0.$
and the same holds for $\Phi(A_2), \Phi(A_3), \Phi(A_4)$.

Mareschal (1998) showed that PROMETHEE is an **additive** MCDM method.

Sensitivity analysis for additive MCDM models (Mészáros, Rapcsák, 1996)

Criteria	C_1	C_2	$\dots$	C_n	
Weight	w_1	w_2	$\dots$	w_n	Total
A_1	e_{11}	e_{12}	$\dots$	e_{1n}	$\sum_{i=1}^{n} w_i e_{1i}$
A_2	e_{21}	e_{22}	$\dots$	e_{2n}	$\sum_{i=1}^{n} w_i e_{2i}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\dots$
A_m	e_{m1}	e_{m2}	$\dots$	e_{mn}	$\sum_{i=1}^{n} w_i e_{mi}$

Does the final rank change as input data (weights and evaluations) change?

Sensitivity analysis for additive MCDM models (Mészáros, Rapcsák, 1996)

Criteria	C_1	C_2	$\dots$	C_n	
Weight	w_1	w_2	$\dots$	w_n	Total
A_1	e_{11}	e_{12}	$\dots$	e_{1n}	$\sum_{i=1}^{n} w_i e_{1i}$
A_2	e_{21}	e_{22}	$\dots$	e_{2n}	$\sum_{i=1}^{n} w_i e_{2i}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\dots$
A_m	e_{m1}	e_{m2}	$\dots$	e_{mn}	$\sum_{i=1}^{n} w_i e_{mi}$

$$w_i \in [w_i - \lambda D_i^- \,,\, w_i + \lambda D_i^+]$$

$$e_{ji} \in [e_{ji} - \lambda d_{ji}^- \,,\, e_{ji} + \lambda d_{ji}^+]$$

For example, $D_i^- = D_i^+ = w_i$, $d_{ji}^- = d_{ji}^+ = e_{ji}$ and $\lambda = 0.1$ means that each datum can change +/- 10% (relative), independently from each other.

Sensitivity analysis for additive MCDM models (Mészáros, Rapcsák, 1996)

$$w_i \in [w_i - \lambda D_i^- \, , \, w_i + \lambda D_i^+]$$

$$e_j i \in [e_{ji} - \lambda d_{ji}^- , \, e_{ji} + \lambda d_{ji}^+]$$

Let an arbitrary subset of ordered pairs of alternatives is fixed (even the whole rank). Then the maximal value of λ such that the rank of the alternatives does not change can be computed efficiently and fast.

For example, if $\lambda^* = 20\%$ then all the data can change +/- 20% (relative) such that the considered orders remain. However, one can find an appropriate +/- 21% (relative) change in the data such that at least one of the considered pairs becomes reversed.

Partial sensitivity analysis in Decision Lab 2000, PROMCALC:

- *walking weights*: a single weight can be changed
- *stability intervals*: lower and upper bounds are computed as a single weight can be changed

In PROMCALC (which is older than Decision Lab 2000) the changes of two criterion weights can be also analysed, stability polygons are plotted.

In our proposed method, **any subset** (including all) of the weights can be changed independently from each other.

Special case: only one criteria is considered, then we get the same results as in partial sensitivity analysis.

Questions:

- How about sensitivity of the evaluations?
- Interdependence of the weights

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Thank you for attention.

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