The eigenvalue-eigenvector equation

$$\begin{pmatrix} 1 & p & p & p & \dots & p & p \\ 1/p & 1 & q & 1 & \dots & 1 & 1/q \\ 1/p & 1/q & 1 & q & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\ 1/p & 1 & 1 & 1 & \dots & 1 & q \\ 1/p & q & 1 & 1 & \dots & 1/q & 1 \end{pmatrix} \begin{pmatrix} w_1^{EM} \\ w_2^{EM} \\ \\ \vdots \\ \vdots \\ \\ w_n^{EM} \end{pmatrix} = \lambda_{\max} \begin{pmatrix} w_1^{EM} \\ w_2^{EM} \\ \\ \vdots \\ \vdots \\ \\ w_n^{EM} \end{pmatrix},$$

where

$$\lambda_{\max} = \frac{\sqrt{q^4 + (2n-8)(q^3+q) + (n^2-4n+14)q^2 + 1} + q^2 + (n-2)q + 1}{2q},$$
$$w_1^{EM} = p\frac{\sqrt{q^4 + (2n-8)(q^3+q) + (n^2-4n+14)q^2 + 1} - [q^2 + (n-4)q + 1]}{2q},$$
$$w_i^{EM} = 1, \qquad i = 2, 3, \dots, n$$

can be rewritten as follows:

$$\begin{pmatrix} 1 & p & p & p & \dots & p & p \\ 1/p & 1 & q & 1 & \dots & 1 & 1/q \\ 1/p & 1/q & 1 & q & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\ 1/p & 1 & 1 & 1 & \dots & 1 & q \\ 1/p & q & 1 & 1 & \dots & 1/q & 1 \end{pmatrix} \begin{pmatrix} w_1^{EM} \\ 1 \\ 1 \\ \vdots \\ \vdots \\ \\ 1 \end{pmatrix} = \lambda_{\max} \begin{pmatrix} w_1^{EM} \\ 1 \\ 1 \\ \vdots \\ \vdots \\ \\ 1 \end{pmatrix}.$$

First, $w_1^{EM} + (n-1)p = \lambda_{\max} w_1^{EM}$, or, equivalently,
$(n-1)p = (\lambda_{\max} - 1)w_1^{EM}$ follows from

$$(\lambda_{\max} - 1)w_1^{EM} =$$
$$= p\frac{\left(\sqrt{q^4 + (2n-8)(q^3+q) + (n^2-4n+14)q^2 + 1}\right)^2 - [q^2 + (n-4)q + 1]^2}{4q^2} =$$
$$= p\frac{4q^2 n - 4q^2}{4q^2} = (n-1)p.$$

Second,

$$\frac{1}{p}w_1^{EM} + q + \frac{1}{q} + n - 3 = \lambda_{\max}$$

follows from

$$\lambda_{\max} - \frac{1}{p} w_1^{EM} = \frac{2 + 2nq - 6q + 2q^2}{2q} = q + \frac{1}{q} + n - 3$$

We also need to confirm that the maximal eigenvalue and the associated eigenvector are found. It follows from the assumptions $n \geq 4$ and $p, q > 0$ that $w_1^{EM} > 0$ and certainly $w_i^{EM} > 0\,(i = 2, 3, \ldots, n)$, meaning that the eigenvector is positive. Sekitani and Yamaki (15, Lemma 5) proved that any positive eigenvector belongs to $\lambda_{\max}$, which completes the proof.