Manuscript of / please cite as
Bozóki, S., [2017]: Two short proofs regarding the logarithmic least squares optimality in Chen, K., Kou, G., Tarn, J.M., Song, J. (2015): Bridging the gap between missing and inconsistent values in eliciting preference from pairwise comparison matrices, Annals of Operations Research 235(1):155-175,
Annals of Operations Research 253(1) pp.707-708
http://dx.doi.org/10.1007/s10479-016-2396-9

Addendum:
Two short proofs regarding the logarithmic least squares optimality in Chen, K., Kou, G., Tarn, J.M., Song, J. (2015): Bridging the gap between missing and inconsistent values in eliciting preference from pairwise comparison matrices, Annals of Operations Research 235(1):155-175

Sándor Bozóki[1,2,3]

The incomplete logarithmic least squares (LLS) problem has been solved in [1, Section 4]. Theorems 1 and 2 in [2] are special cases, and short proofs can be given with the help of the Laplacian matrix.

Proof of Theorem 2 in [2]: We can assume without loss of generality that $i = 1, j = 2$ and elements a_{1k}, a_{2k} and their reciprocals are known for $k = 3, 4, \ldots, n-m$, and the remaining elements a_{12}, a_{21} as well as a_{1k}, a_{2k} and their reciprocals are unknown for $k = n-m+1, \ldots, n$. Let us write the conditions of LLS optimality, a system of linear equations (30) in [1], it is sufficient to detail the first two rows of the matrix of coefficients.

$$\left(\begin{array}{cc|cccc|ccc} n-m-2 & 0 & -1 & -1 & \ldots & -1 & 0 & \ldots & 0 \\ 0 & n-m-2 & -1 & -1 & \ldots & -1 & 0 & \ldots & 0 \\ & \vdots & & & \vdots & & & \vdots & \\ & \vdots & & & \vdots & & & \vdots & \\ & \vdots & & & \vdots & & & \vdots & \end{array}\right) \left(\begin{array}{c} y_1 \\ y_2 \\ \hline y_3 \\ \vdots \\ y_{n-m} \\ \hline y_{n-m+1} \\ \vdots \end{array}\right) = \left(\begin{array}{c} \log \prod\limits_{k=3}^{n-m} a_{1k} \\ \log \prod\limits_{k=3}^{n-m} a_{2k} \\ \vdots \\ \vdots \end{array}\right),$$

where $y_i = \log w_i$. The first two equations are

$$(n-m-2)y_1 - (y_3 + \ldots + y_{n-m}) = \log \prod_{k=3}^{n-m} a_{1k},$$
$$(n-m-2)y_2 - (y_3 + \ldots + y_{n-m}) = \log \prod_{k=3}^{n-m} a_{2k},$$

and their difference results in

$$y_1 - y_2 = \frac{\log \prod\limits_{k=3}^{n-m} a_{1k} - \log \prod\limits_{k=3}^{n-m} a_{2k}}{n-m-2},$$

[1]Laboratory on Engineering and Management Intelligence, Research Group of Operations Research and Decision Systems, Institute for Computer Science and Control, Hungarian Academy of Sciences (MTA SZTAKI); Mail: 1518 Budapest, P.O. Box 63, Hungary. E-mail: bozoki.sandor@sztaki.mta.hu
[2]Department of Operations Research and Actuarial Sciences, Corvinus University of Budapest, Hungary
[3]Research was supported by the János Bolyai Research Fellowship no. BO/00154/16/3 of the Hungarian Academy of Sciences and by the Hungarian Scientific Research Fund, grant OTKA K111797.

or equivalently,

$$\frac{w_1}{w_2} = \left(\prod_{k=3}^{n-m} \frac{a_{1k}}{a_{2k}} \right)^{\frac{1}{n-m-2}}. \qquad \square$$

Proof of **Theorem 1** in [2]: Apply the previous proof with $m = 0$. $\square$

References

[1] Bozóki, S., Fülöp, J., Rónyai, L. (2010): On optimal completion of incomplete pairwise comparison matrices, Mathematical and Computer Modelling, 52(1-2):318–333

[2] Chen, K., Kou, G., Tarn, J.M., Song, J. (2015): Bridging the gap between missing and inconsistent values in eliciting preference from pairwise comparison matrices, Annals of Operations Research 235(1):155–175