Global sensitivity analysis in PROMETHEE

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A global sensitivity analysis is proposed within the framework of the PROMETHEE methodology.

Global sensitivity analysis: all the weights can change simultaneously

Preliminaries 1

Partial sensitivity analysis: a single criterion weight is allowed to change at a time, as in Visual Promethee, Decision Lab 2000 and PROMCALC & GAIA (walking weights & **stability interval**s)

The simultaneous change of two criterion weights are analyzed by calculating **stability polygon**s in PROMCALC & GAIA.

Preliminaries 2

Mareschal (1988) showed that PROMETHEE is an **additive** MCDM method: the net outranking flow values of the alternatives can be written in the form of a weighted sum of ‘criterion-wise net outranking flows’, where the weights are the criterion weights themselves.

Buying a car, Visual Promethee's default example

	Criterion C_1 (Price)	Criterion C_2 (Power)	Criterion C_3 (Consumption)	Criterion C_4 (Habitability)	Criterion C_5 (Comfort)
unit	€	kW	liter/100km	5-point	5-point
min/max	min	max	min	max	max
type	V-shape	linear	V-shape	level	level
Indifference threshold q	-	5	-	1	0.5
Preference threshold p	15000	30	2	2.5	2.5
Weight	$v_1 = 1/5$	$v_2 = 1/5$	$v_3 = 1/5$	$v_4 = 1/5$	$v_5 = 1/5$
Alternative A_1 (Tourism B)	25500	85	7.0	4	3
Alternative A_2 (Luxury 1)	38000	90	8.5	4	5
Alternative A_3 (Tourism A)	26000	75	8.0	3	3
Alternative A_4 (Luxury 2)	35000	85	9.0	5	4
Alternative A_5 (Economic)	15000	50	7.5	2	1
Alternative A_6 (Sport)	29000	110	9.0	1	2

Positive, negative and net flows

P	A_1	A_2	A_3	A_4	A_5	A_6	Φ^+	Φ^-	Φ
A_1	0	0.32	0.15	0.33	0.45	0.55	0.36	0.10	0.26
A_2	0.10	0	0.18	0.15	0.50	0.45	0.28	0.22	0.05
A_3	0.00	0.21	0	0.22	0.26	0.34	0.21	0.19	0.01
A_4	0.10	0.04	0.24	0	0.60	0.30	0.26	0.26	0.00
A_5	0.14	0.30	0.20	0.35	0	0.34	0.26	0.42	–0.16
A_6	0.16	0.24	0.20	0.24	0.30	0	0.23	0.39	–0.17

Criterion-wise positive, negative and net flows

$\mathbf{P}_1$	A_1	A_2	A_3	A_4	A_5	A_6	Φ^+_1	Φ^-_1	Φ_1
A_1	0	0.83	0.03	0.63	0	0.23	0.35	0.14	0.21
A_2	0	0	0	0	0	0	0.00	0.69	–0.69
A_3	0	0.8	0	0.6	0	0.2	0.32	0.15	0.17
A_4	0	0.2	0	0	0	0	0.04	0.53	–0.49
A_5	0.7	1	0.73	1	0	0.93	0.87	0.00	0.87
A_6	0	0.6	0	0.4	0	0	0.20	0.27	–0.07

Net flow written as the weighted sum of criterion-wise net flows ($\Phi = \Sigma_{k=1..5}\, v_k \Phi_k$)

	$v_1 = 1/5$	$v_2 = 1/5$	$v_3 = 1/5$	$v_4 = 1/5$	$v_5 = 1/5$	
	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ
A_1	0.21	0.08	0.70	0.30	0.00	0.26
A_2	–0.69	0.16	–0.20	0.30	0.70	0.05
A_3	0.17	–0.20	0.10	0.00	0.00	0.01
A_4	–0.49	0.08	–0.50	0.50	0.40	0.00
A_5	0.87	–0.96	0.40	–0.40	–0.70	–0.16
A_6	–0.07	0.84	–0.50	–0.70	–0.40	–0.17

Preliminaries 2

Mareschal (1988) showed that PROMETHEE is an **additive** MCDM method: the net outranking flow values of the alternatives can be written in the form of a weighted sum of ‘criterion-wise net outranking flows’, where the weights are the criterion weights themselves.

Preliminaries 3

A global sensitivity analysis is proposed for additive methods by Mészáros and Rapcsák (1996)

	weights of criteria				
	v_1	v_2	…	v_m	total score
A_1	s_{11}	s_{12}	…	s_{1m}	$\Sigma_{k=1..m} v_k s_{1k}$
A_2	s_{21}	s_{22}	…	s_{2m}	$\Sigma_{k=1..m} v_k s_{2k}$
⋮	⋮	⋮		⋮	⋮
A_n	s_{n1}	s_{n2}	…	s_{nm}	$\Sigma_{k=1..m} v_k s_{nk}$

Preliminaries 3

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What is the largest **simultaneous** change in the weights and in the criterion-wise scores such that no rank reversal occurs within a certain set of pairs of alternatives?

Global sensitivity analysis in PROMETHEE

Assume that only weights of criteria change such that w_k, the modified weight of criterion k, remains in the interval

$[v_k(1–\lambda);v_k(1+\lambda)]$ (relative) or

$[v_k–\lambda;v_k+\lambda]$ (absolute)

for all $1 \leq k \leq m$.

Example: if $v_k = 0.2$ and $\lambda = 0.1$, then

$w_k \in [0.18; 0.22]$ (relative)

$w_k \in [0.1; 0.2]$ (absolute)

Global sensitivity analysis in PROMETHEE

Let the whole ranking be $A_1, A_2, \ldots, A_{n-1}, A_n$ from $\Phi(A_1) \geq \Phi(A_2) \geq \ldots \geq \Phi(A_{n-1}) \geq \Phi(A_n)$ calculated with the original weights $v_1, v_2, \ldots, v_m$

Select a set S of pairs of alternatives. Set S includes those pairs of alternatives, the relations of which should be kept.

For example, if only the winner is of interest, then $S = \{(A_1,A_2), (A_1,A_3), \ldots, (A_1,A_n)\}$.

Global sensitivity analysis in PROMETHEE

If the stability of the whole ranking is investigated, then

$S = \{(\mathrm{A}_i, \mathrm{A}_j)\}$ for all $1 \leq i < j \leq n$.

If the set of the first three alternatives is required to be fixed, independently of their inner relations, then

$$S = \{(\mathrm{A}_1,\mathrm{A}_4),(\mathrm{A}_1,\mathrm{A}_5),\ldots,(\mathrm{A}_1,\mathrm{A}_n),(\mathrm{A}_2,\mathrm{A}_4),(\mathrm{A}_2,\mathrm{A}_5),\ldots,(\mathrm{A}_2,\mathrm{A}_n),(\mathrm{A}_3,\mathrm{A}_4),(\mathrm{A}_3,\mathrm{A}_5),\ldots,(\mathrm{A}_3,\mathrm{A}_n)\}.$$

Global sensitivity analysis in PROMETHEE

The optimization problems in the

relative case:

$$\max\{\ \lambda \mid \Phi(A_i) > \Phi(A_j) \text{ for all } (A_i,A_j) \in S \text{ and}$$
$$v_k\,(1-\lambda) \le w_k \le v_k\,(1+\lambda) \text{ for all } k\,\}$$

absolute case:

$$\max\{\ \lambda \mid \Phi(A_i) > \Phi(A_j) \text{ for all } (A_i,A_j) \in S \text{ and}$$
$$v_k - \lambda \le w_k \le v_k + \lambda \text{ for all } k\,\}$$

where $\Phi = \Sigma_{k=1..m}\, w_k\, \Phi_k$

Absolute and relative changes of weights coincide if $v_k = 1/5$ ($k = 1,\ldots,5$).

Test 1. Global sensitivity analysis provides

$\lambda = 0.0022$ if the whole ranking is set.

Modified weights

$w_1 = 1/5 - \lambda$	$\Phi_1(A_5) > \Phi_1(A_6)$
$w_2 = 1/5 + \lambda$	$\Phi_2(A_5) < \Phi_2(A_6)$
$w_3 = 1/5 - \lambda$	$\Phi_3(A_5) > \Phi_3(A_6)$
$w_4 = 1/5 - \lambda$	$\Phi_4(A_5) > \Phi_4(A_6)$
$w_5 = 1/5 + \lambda$	$\Phi_5(A_5) < \Phi_5(A_6)$

result in a tie between alternatives A_5 and A_6.

Test 2. If we focus on the first position only, then global sensitivity analysis provides

$\lambda = 0.07875$

Modified weights

$w_1 = 1/5 - \lambda$ $\Phi_1(A_1) > \Phi_1(A_2)$

$w_2 = 1/5 + \lambda$ $\Phi_2(A_1) < \Phi_2(A_2)$

$w_3 = 1/5 - \lambda$ $\Phi_3(A_1) > \Phi_3(A_2)$

$w_4 = 1/5$ $\Phi_4(A_1) = \Phi_4(A_2)$

$w_5 = 1/5 + \lambda$ $\Phi_5(A_1) < \Phi_5(A_2)$

results in a tie between alternatives A_1 and A_2

Test 3. If we require that A_1 and A_2 should be in the first two positions, but not necessarily in this order, then global sensitivity analysis provides $\lambda = 0.01644$ and

$w_1 = 1/5+\lambda$	$\Phi_1(A_2) < \Phi_1(A_3)$
$w_2 = 1/5-\lambda$	$\Phi_2(A_2) > \Phi_2(A_3)$
$w_3 = 1/5+\lambda$	$\Phi_3(A_2) < \Phi_3(A_3)$
$w_4 = 1/5-\lambda$	$\Phi_4(A_2) > \Phi_4(A_3)$
$w_5 = 1/5-\lambda$	$\Phi_5(A_2) > \Phi_5(A_3)$

result in a tie between alternatives A_2 and A_3 in the second place, while A_1 remains the winner (according to Test 2).

Now let us depart from non-equal weights of criteria in the example in order to demonstrate the global sensitivity analysis with *relative* changes:

$$v_1 = 0.1$$
$$v_2 = 0.2$$
$$v_3 = 0.2$$
$$v_4 = 0.1$$
$$v_5 = 0.4$$

Test 4. Sensitivity calculation with the whole ranking gives $\lambda = 0.0472$

$w_1 = v_1(1+\lambda) = 0.1(1+\lambda)$	$\Phi_1(A_1) < \Phi_1(A_2)$
$w_2 = v_2(1-\lambda) = 0.2(1-\lambda)$	$\Phi_2(A_1) > \Phi_2(A_2)$
$w_3 = v_3(1+\lambda) = 0.2(1+\lambda)$	$\Phi_3(A_1) < \Phi_3(A_2)$
$w_4 = 0.1$	$\Phi_4(A_1) = \Phi_4(A_2)$
$w_5 = v_5(1-\lambda) = 0.4(1-\lambda)$	$\Phi_5(A_1) > \Phi_5(A_2)$

Test 5. The level of uncertainty may vary from criteria to criteria. Let the vector

$$(10, 5, 1, 10, 2)$$

express that the weights' changes are bounded by the following inequalities:

$$v_1(1-10\lambda) \le w_1 \le v_1(1+10\lambda)$$
$$v_2(1-5\lambda) \le w_2 \le v_2(1+5\lambda)$$
$$v_3(1-\lambda) \le w_3 \le v_3(1+\lambda)$$
$$v_4(1-10\lambda) \le w_4 \le v_4(1+10\lambda)$$
$$v_5(1-2\lambda) \le w_5 \le v_5(1+2\lambda)$$

With the whole ranking we get $\lambda = 0.01556$

Open questions

The degree of weight changes can be significantly different before and after the re-normalization of the modified weights, a methodology to track and compare the two settings is to be developed.

Can an arbitrary order of the alternatives be realized by an appropriate modification of the weights?

If it is possible, what is the smallest level of modification to achieve it?

Open questions

How to include the uncertainties of the evaluations of the alternatives with respect to the criteria?

If we depart from the criterion-wise net flows, the global sensitivity analysis can be extended accordingly.

Open questions

However, if the starting point is the decision table as in Table 1, the use of discontinuous preference functions, such as the U-shape or the step (level) function, makes the calculations more difficult and all possible jumps within the region analyzed have to be considered.

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	Criterion C_1 (Price)	Criterion C_2 (Power)	Criterion C_3 (Consumption)	Criterion C_4 (Habitability)	Criterion C_5 (Comfort)
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Thank you for your attention

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