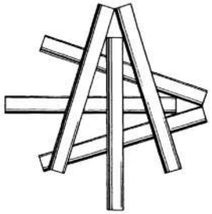 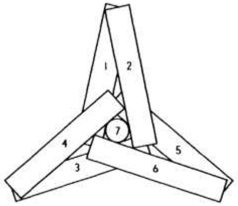 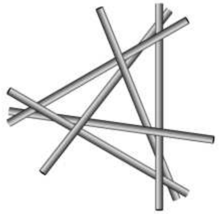 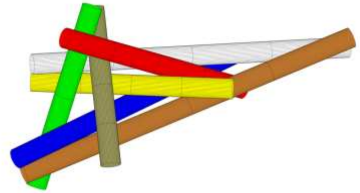 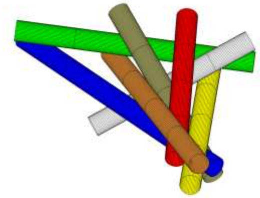  

Seven mutually touching infinite cylinders

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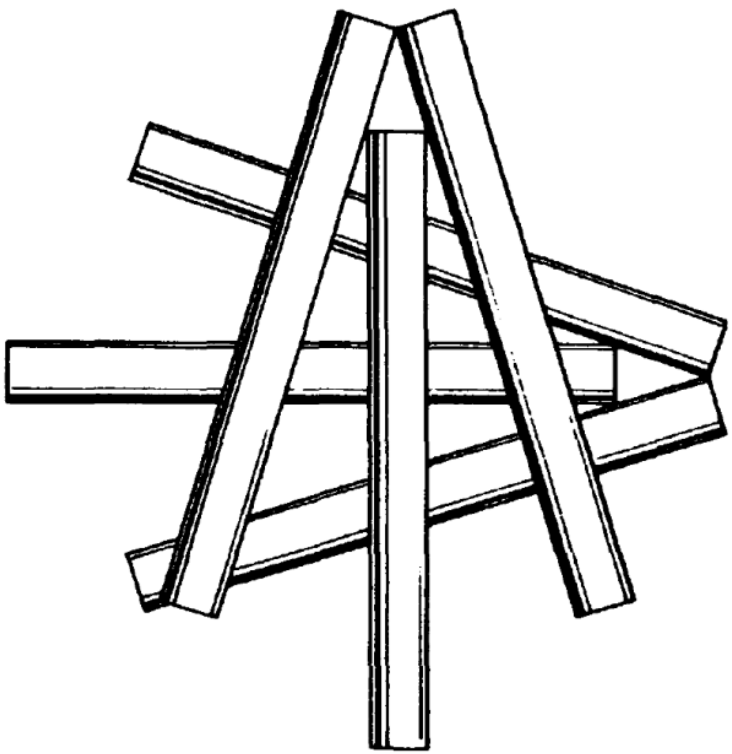
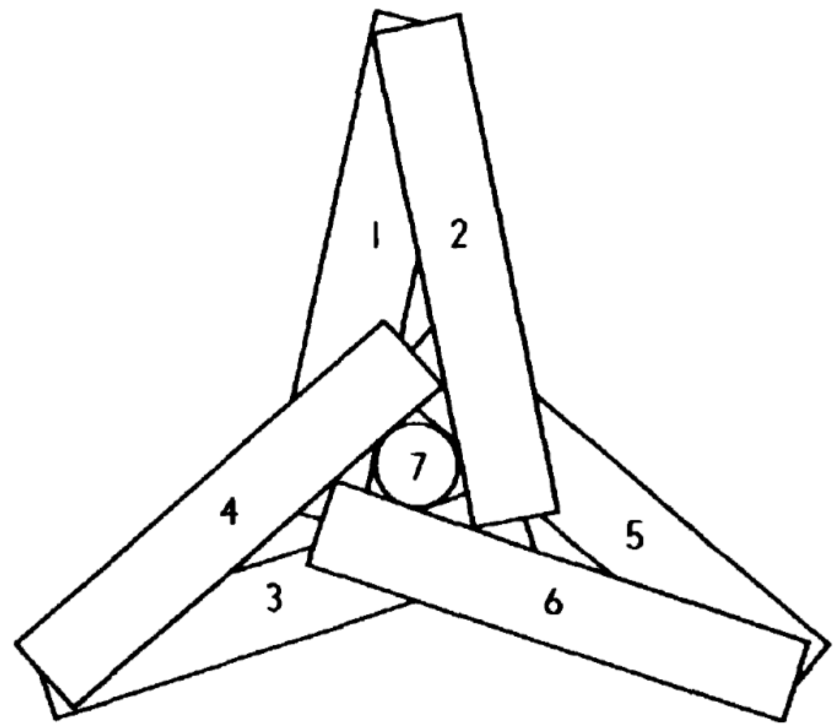
Martin Gardner's touching cigarettes

"Is it possible to place six cigarettes so that each touches the other five? The cigarettes must not be bent or broken." (Gardner, 1959, p. 110)





Littlewood's conjecture

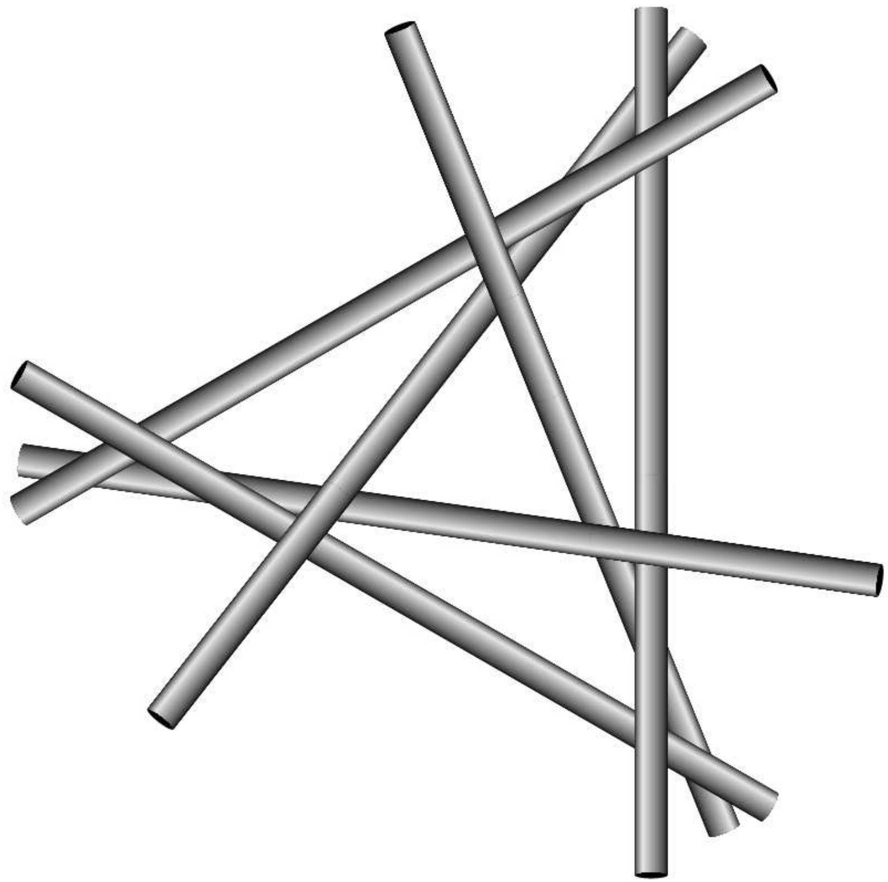
> "*Is it possible in 3-space for seven infinite circular cylinders of unit radius each to touch all the others? Seven is the number suggested by constants.*"
> (Littlewood, 1968, p. 20)

In Ogilvy (1962),

> *„How many lines can be drawn in 3-space, each a unit distant from every one of the others? It is conjectured that seven is the maximum number, but no proof is available. Seven might be too high or too low."* (p. 61)

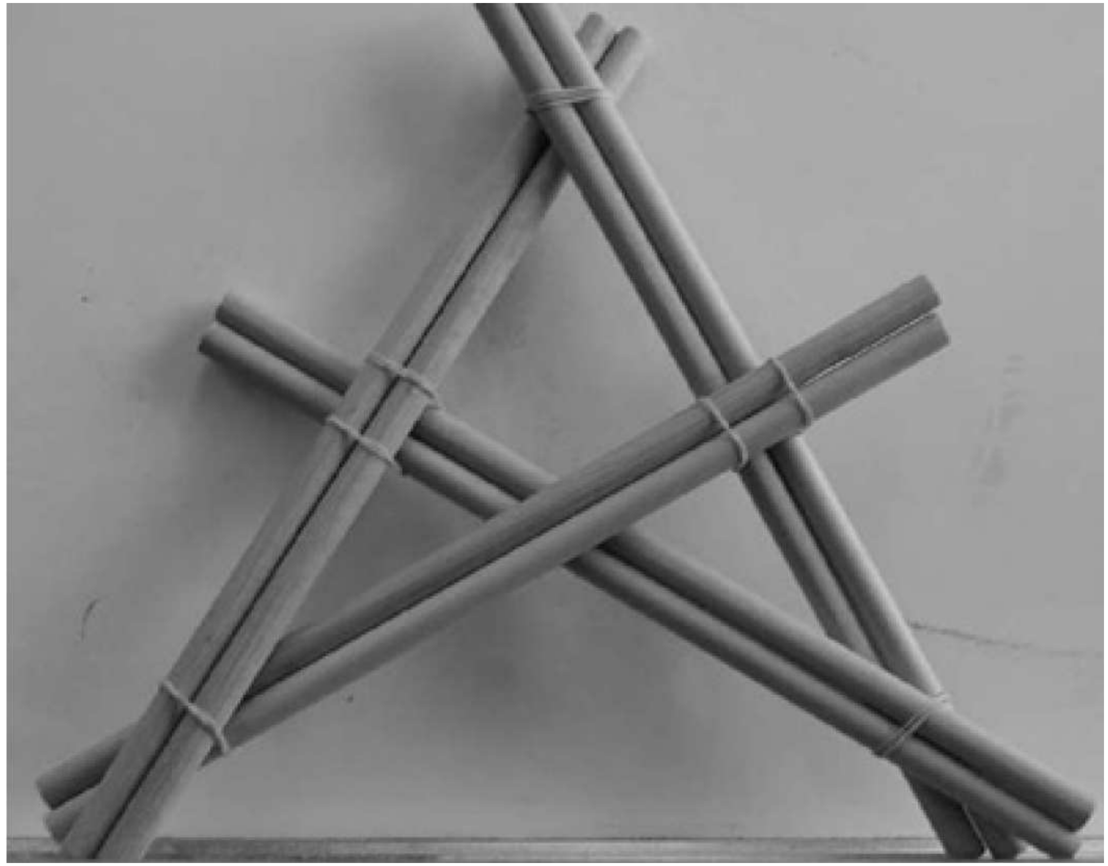
> *„The question on skew lines in 3-space was suggested by Littlewood."* (p. 153)

6 infinite cylinders



source: Brass, Moser, Pach, 2005, p. 98

Kuperberg's arrangement with 8 infinite cylinders



source: Ambrus, Bezdek, 2008, p. 1804

Ambrus and Bezdek disproved Kuperberg's proposal, they showed that at least one pair is not touching in the configuration above.

Lower and upper bounds

Theorem (Bezdek, 2005): *The maximal number of mutually touching identical infinite cylinders is at most 24.*

Best known lower bound: 6.

It is shown in the rest of the talk that the lower bound can be improved to 7.

The system of equations

The distance of lines ℓ_i and ℓ_j $(3 \leq i < j \leq 7)$:

$$-4x_iy_it_iu_it_ju_j +4x_ix_jt_iu_it_ju_j +4x_iy_jt_iu_it_ju_j +4y_ix_jt_iu_it_ju_j +4y_iy_jt_iu_it_ju_j$$
$$-4x_jy_jt_iu_it_ju_j -2x_i^2t_iu_it_ju_j -2y_i^2t_iu_it_ju_j -2x_j^2t_iu_it_ju_j -2y_j^2t_iu_it_ju_j -4x_ix_jt_iu_iu_j$$
$$+4x_ix_jt_iu_j^2 +4x_ix_ju_i^2t_j -4x_ix_ju_it_ju_j +4y_iy_jt_i^2u_j -4y_iy_jt_iu_it_j -4y_iy_jt_it_ju_j$$
$$+4y_iy_ju_it_j^2 +4x_ix_ju_iu_j +4y_iy_jt_it_j +x_i^2t_i^2u_j^2 +x_i^2u_i^2t_j^2 +y_i^2t_i^2u_j^2 +y_i^2u_i^2t_j^2 +x_j^2t_i^2u_j^2$$
$$+x_j^2u_i^2t_j^2 +y_j^2t_i^2u_j^2 +y_j^2u_i^2t_j^2 +2x_iy_it_i^2u_j^2 +2x_iy_iu_i^2t_j^2 -2x_ix_jt_i^2u_j^2 -2x_ix_ju_i^2t_j^2 -2x_iy_jt_i^2u_j^2$$
$$-2x_iy_ju_i^2t_j^2 -2y_ix_jt_i^2u_j^2 -2y_ix_ju_i^2t_j^2 -2y_iy_jt_i^2u_j^2 -2y_iy_ju_i^2t_j^2 +2x_jy_jt_i^2u_j^2 +2x_jy_ju_i^2t_j^2$$
$$-2x_iy_it_i^2u_j -2x_iy_it_iu_j^2 +2x_iy_jt_i^2u_j +2x_iy_jt_iu_j^2 +2x_iy_ju_i^2t_j +2x_iy_ju_it_j^2 -2x_iy_iu_i^2t_j$$
$$-2x_iy_iu_it_j^2 +2y_ix_jt_i^2u_j +2y_ix_jt_iu_j^2 +2y_ix_ju_i^2t_j +2y_ix_ju_it_j^2 -2x_jy_jt_i^2u_j -2x_jy_jt_iu_j^2$$
$$-2x_jy_ju_i^2t_j -2x_jy_ju_it_j^2 -2x_i^2t_iu_j^2 -2x_i^2u_i^2t_j -2y_i^2t_i^2u_j -2y_i^2u_it_j^2 -2x_j^2t_iu_j^2 -2x_j^2u_i^2t_j$$
$$-2y_j^2t_i^2u_j -2y_j^2u_it_j^2 +2x_i^2t_iu_iu_j +2x_i^2u_it_ju_j +2y_i^2t_iu_it_j +2y_i^2t_it_ju_j +2x_j^2t_iu_iu_j$$
$$+2x_j^2u_it_ju_j +2y_j^2t_iu_it_j +2y_j^2t_it_ju_j +2x_iy_it_iu_it_j +2x_iy_it_iu_iu_j +2x_iy_it_it_ju_j$$
$$+2x_iy_iu_it_ju_j -2x_iy_jt_iu_it_j -2x_iy_jt_iu_iu_j -2x_iy_jt_it_ju_j -2x_iy_ju_it_ju_j -2y_ix_jt_iu_it_j$$
$$-2y_ix_jt_iu_iu_j -2y_ix_jt_it_ju_j -2y_ix_ju_it_ju_j +2x_jy_jt_iu_it_j +2x_jy_jt_iu_iu_j +2x_jy_jt_it_ju_j$$
$$+2x_jy_ju_it_ju_j -2x_i^2u_iu_j -2y_i^2t_it_j -2x_j^2u_iu_j -2y_j^2t_it_j -2x_iy_it_iu_i +2x_iy_it_iu_j$$
$$+2x_iy_iu_it_j -2x_iy_it_ju_j +2x_iy_jt_iu_i -2x_iy_jt_iu_j -2x_iy_ju_it_j +2x_iy_jt_ju_j +2y_ix_jt_iu_i$$
$$-2y_ix_jt_iu_j -2y_ix_ju_it_j +2y_ix_jt_ju_j -2x_jy_jt_iu_i +2x_jy_jt_iu_j +2x_jy_ju_it_j -2x_jy_jt_ju_j$$
$$-2x_ix_ju_i^2 -2x_ix_ju_j^2 -2y_iy_jt_j^2 -2y_iy_jt_i^2 +24t_iu_it_ju_j +x_i^2u_i^2 +x_i^2u_j^2 +y_i^2t_i^2 +y_i^2t_j^2 +x_j^2u_i^2$$
$$+x_j^2u_j^2 +y_j^2t_i^2 +y_j^2t_j^2 -12t_i^2u_j^2 -12u_i^2t_j^2 -4t_i^2 -4u_i^2 -4t_j^2 -4u_j^2 -8t_iu_it_j -8t_iu_iu_j$$
$$-8t_it_ju_j +8t_iu_j^2 +8t_i^2u_j +8u_i^2t_j +8u_it_j^2 -8u_it_ju_j +8t_it_j$$
$$+8u_iu_j = 0, \qquad i = 3, \ldots, 6,\ j = i+1, \ldots, 7.$$

Polyhedral homotopy continuation method

The aim is to solve the system of 20 polynomial equations of 20 variables.

There are about 121 billion solutions, but most of them are complex.

We are able to track 20 million solutions per month by the polyhedral homotopy continuation method HOM4PS-2.0.

The first real solution is found after 3 months, and the second one is found after another month.

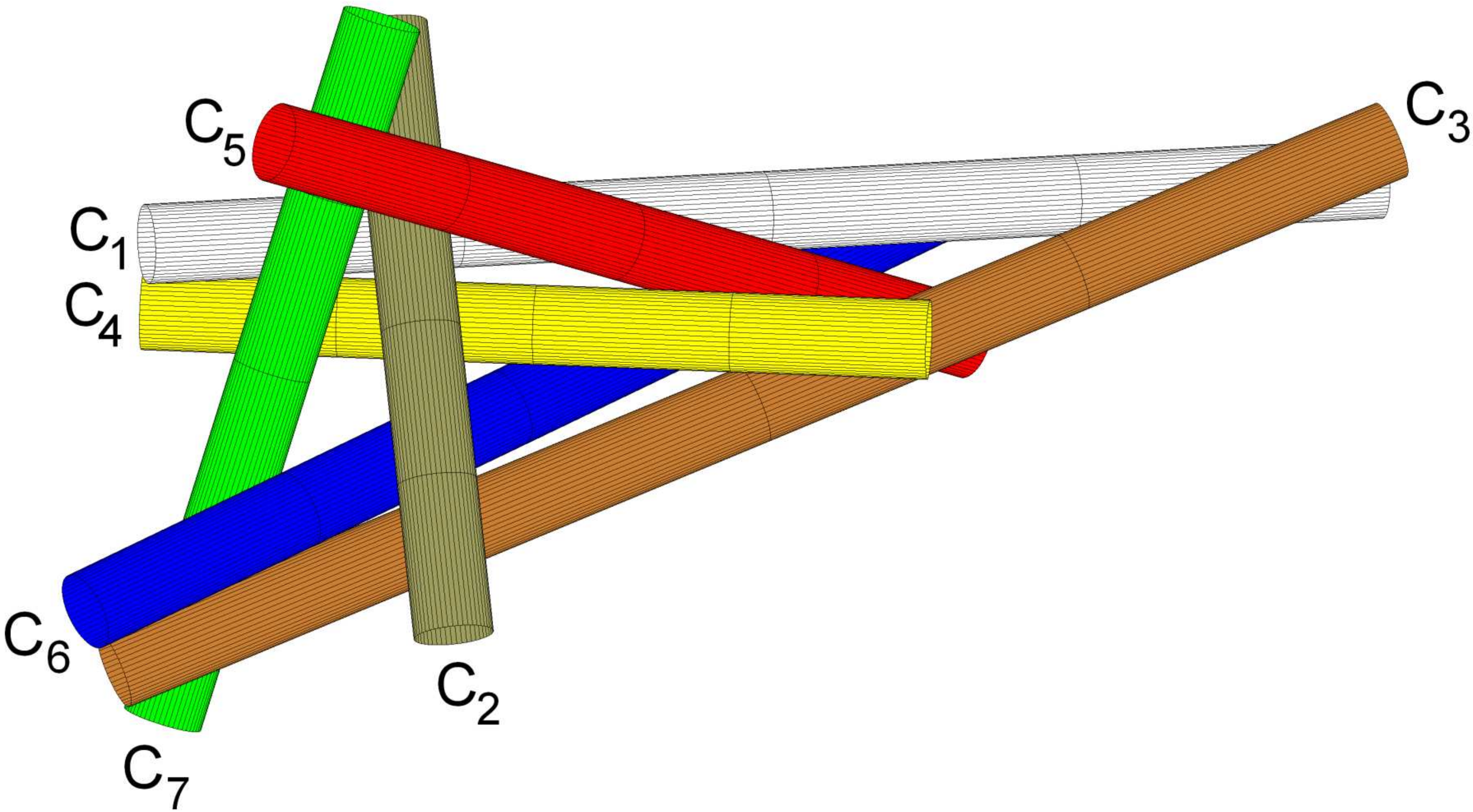
Verification of the roots

alphaCertified by Hauenstein and Sottile (2012) based on Smale's α-theory.

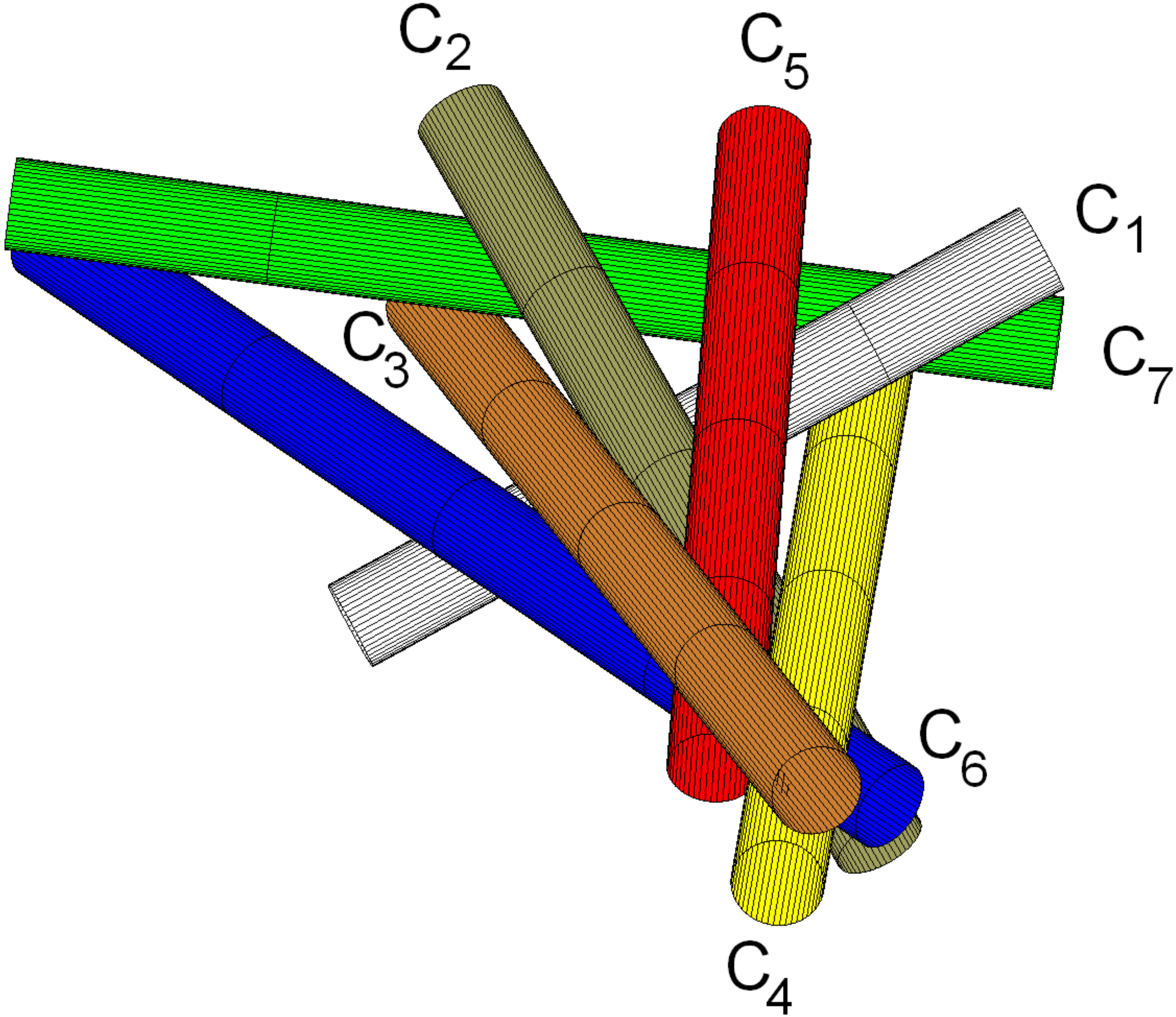
interval Krawczyk method in INTLAB

Both methods have certified that both solutions are real and isolated solutions.

Solution 1



Solution 2

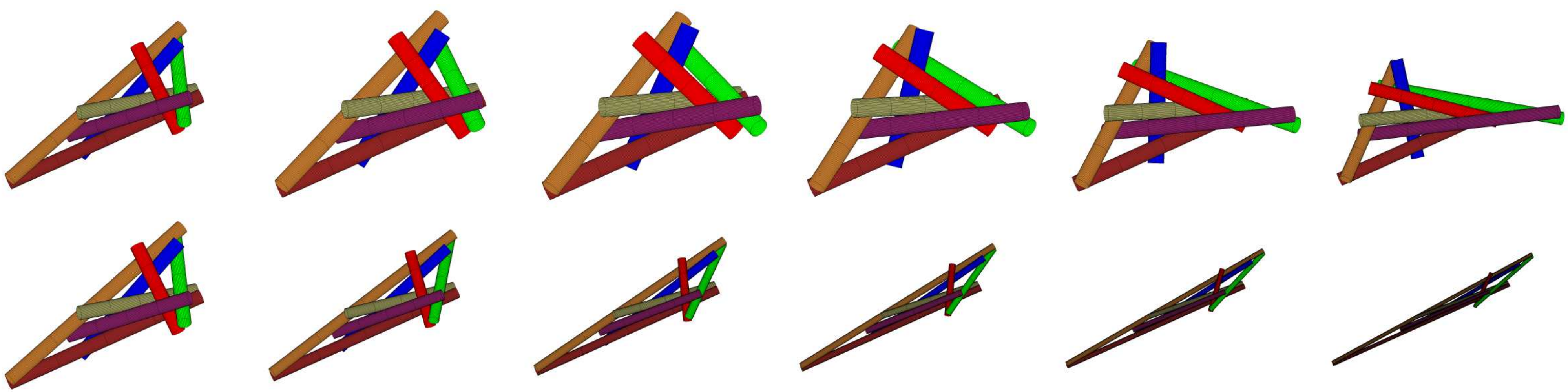


Main results

Theorem: *The maximal number of mutually touching identical infinite cylinders is at least 7.*

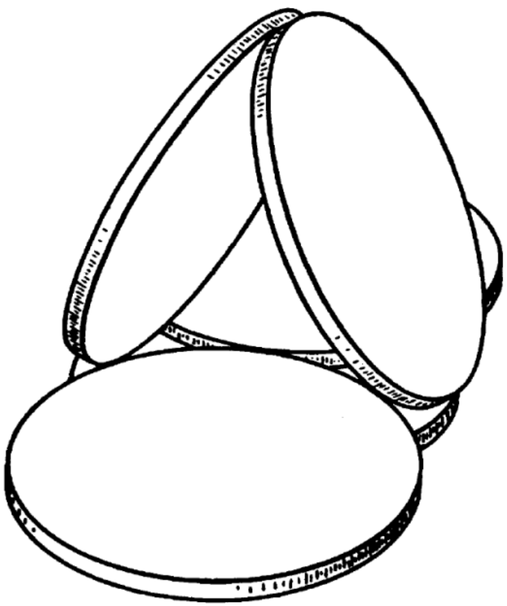
Moreover,

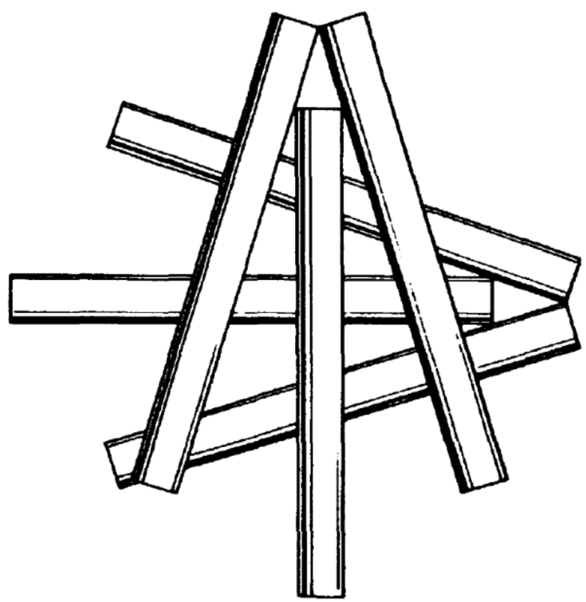
Theorem: *There exists an infinite family of seven mutually touching identical infinite cylinders.*

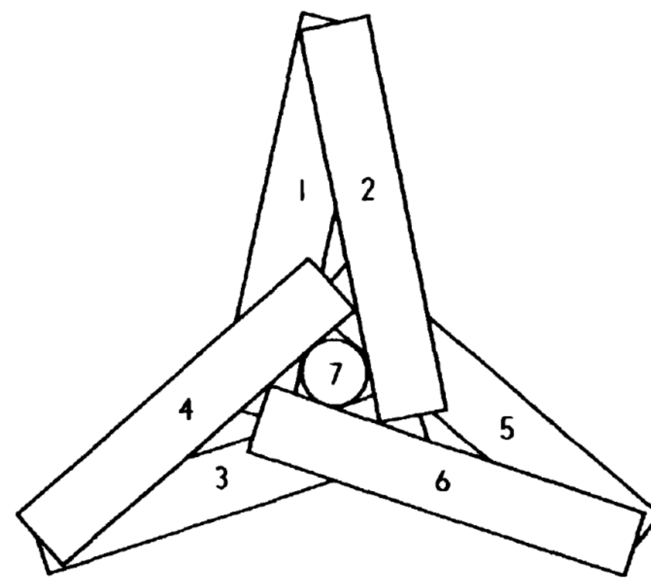


Open problems

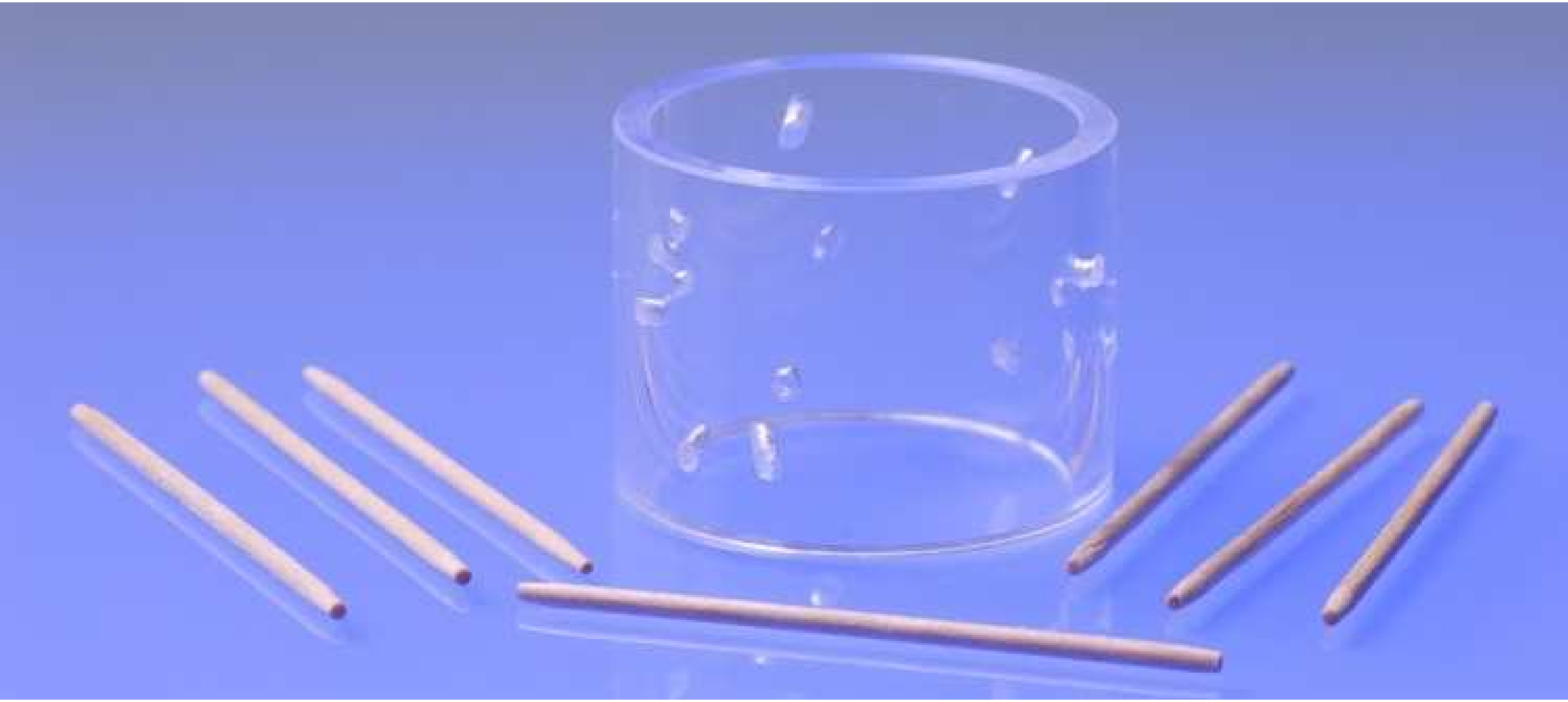
- Can we find 8 or more mutually touching finite cylinders (cigarettes)?
- Can we find 8 or more mutually touching infinite cylinders?
- Intermediate length/diameter ratios







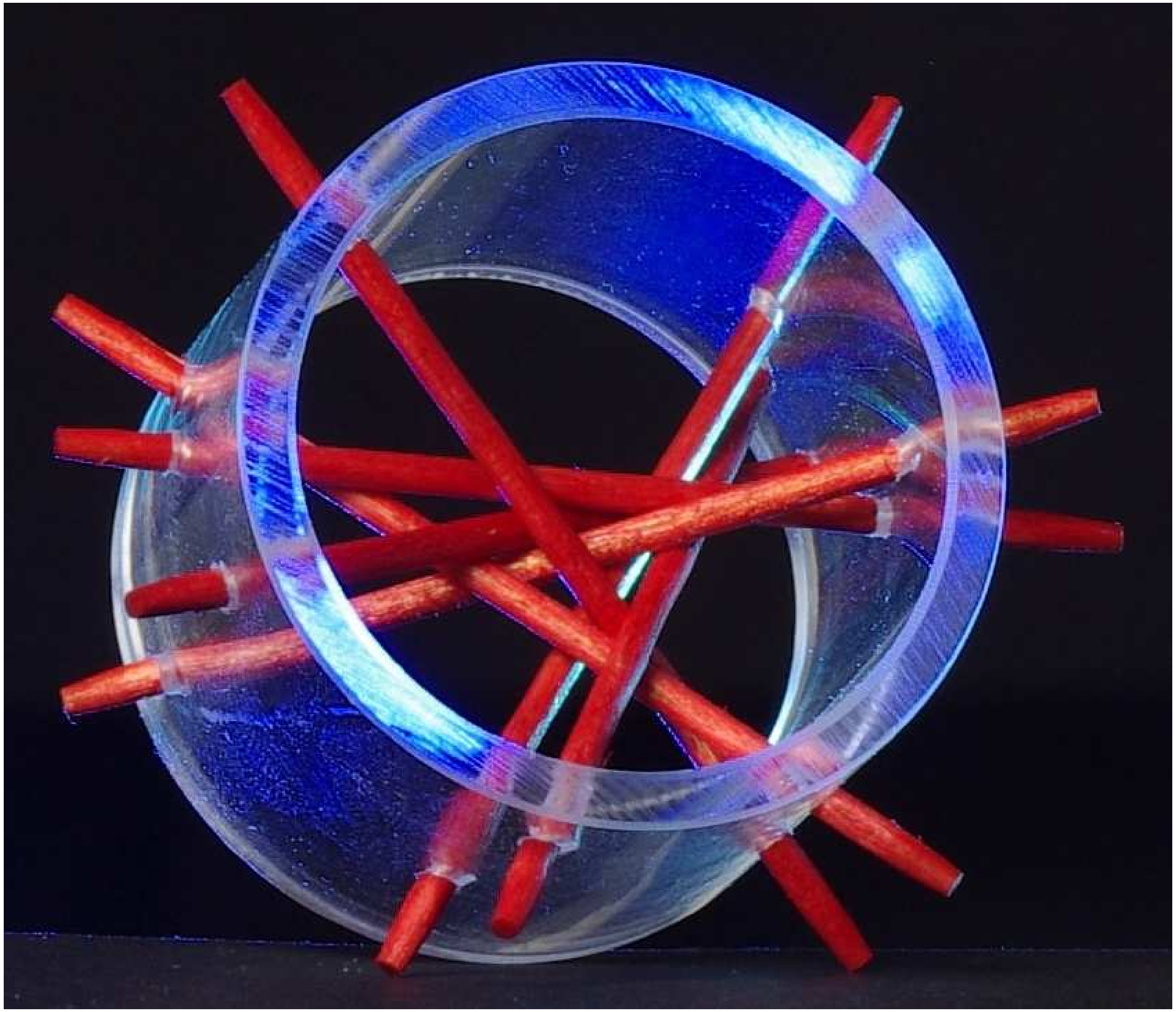
Gift exchange



7+7 sticks are attached. The extra 7 can be colored arbitrary or simply kept as spares.

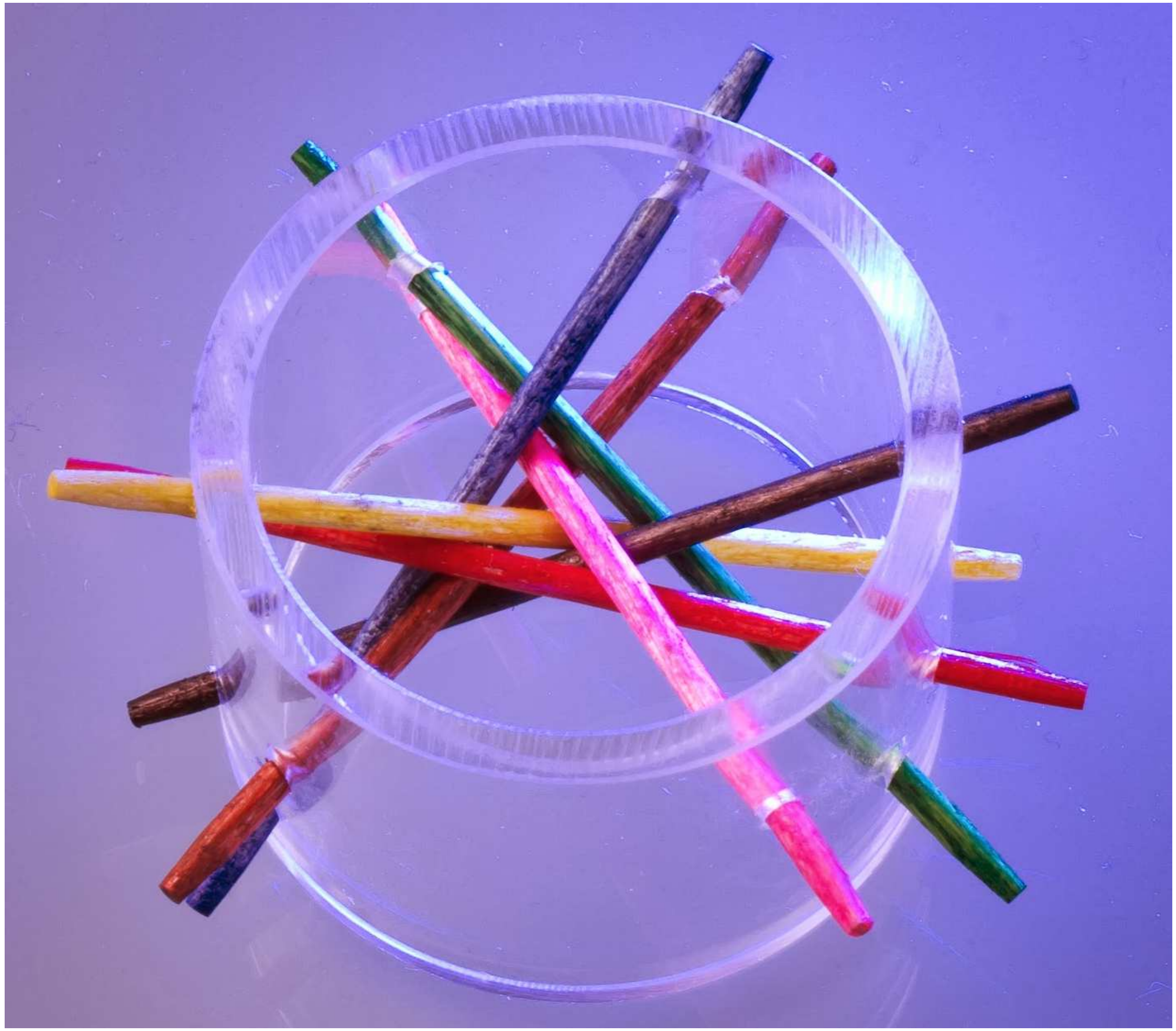
Gift exchange

Gift exchange



(Permanent marker can be used for coloring.)

Gift exchange



(Permanent marker can be used for coloring.)

Details can be found in papers

Bozóki, S., Lee, T.L., Rónyai, L. (2015): Seven mutually touching infinite cylinders, *Computational Geometry: Theory and Applications,* 48(2):87–93.
DOI 10.1016/j.comgeo.2014.08.007
http://arxiv.org/abs/1308.5164

Bozóki, S., Lee, T.L., Rónyai, L. ($\geq$ 2014): Seven mutually touching infinite cylinders: an infinite family of solutions (forthcoming)

Main references to touching cylinders

Ambrus, G., Bezdek, A. (2008): On the number of mutually touching cylinders. Is it 8?, *European Journal of Combinatorics* 29(8):1803–1807.

Bezdek, A. (2005): On the number of mutually touching cylinders, *Combinatorial and Computational Geometry, MSRI Publication*, 52:121–127.

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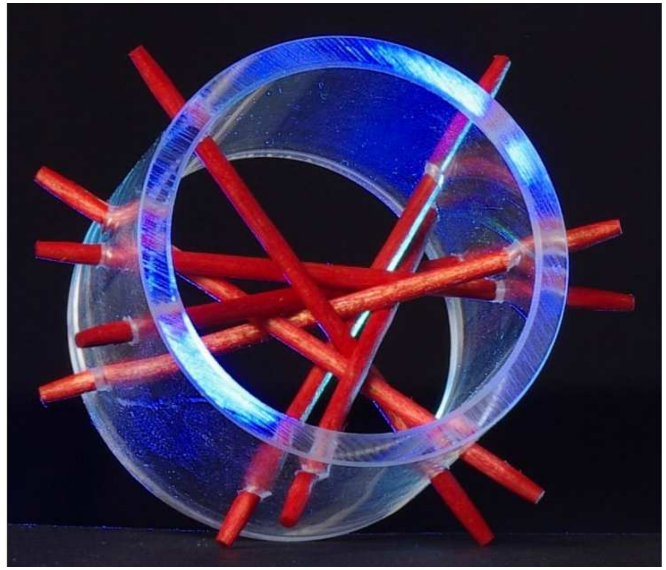
Gardner, M. (1959): The Scientific American Book of Mathematical Puzzles and Diversions, Simon and Schuster, New York, pp. 110–115.

Main references to touching cylinders

Littlewood, J.E. (1968): Some problems in real and complex analysis, Heath Mathematical Monographs, Raytheon Education, Lexington, Massachusetts.

Ogilvy, C.S. (1962): Tomorrow's math: unsolved problems for the amateur, Oxford Univesity Press, New York, 1962.


Thank you for your attention.

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