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Robust reverse logistics network design

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Abstract Recycling waste materials has become increasingly important recently both for economic and environmental reasons. In order to efficiently operate the backward flow of the materials, a basic challenge is to design the proper reverse logistics network containing the collection points, test centers and manufacturing plants. This paper studies the supply network of waste wood, which has to be collected in dedicated accumulation centers, and transported to processing facilities. We focus on the facility location of processing centers and propose mathematical models that take economies of scale and robustness into account, including a novel approach based on bilevel optimization. We also give a local and tabu search method for the solution of the problem. Test results are presented for both the robust and non-robust case using instances based on a real-life dataset.

Keywords Facility location · Robust optimization · Economies of scale · Reverse logistics for wood recycling

1 Introduction and motivation

With the recent increase in the importance of environmental awareness, more stress is being put to on the end-of-life recovery and reuse of resources in supply chains. Activities that aim to recover resources from their final destination are integrated by the field of reverse logistics (Dekker et al., 2004). The goal of the reverse logistics is to use these end-of-life resources either to produce

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further value or to dispose of them properly, usually through a complex recovery process consisting of the stages of repair, reuse, refurbish, remanufacture, retrieve, recycle, incinerate and landfill. Reverse logistics methods can also be integrated into the conventional process of supply chains, forming so-called closed-loop supply chains that account for both forward and reverse flows of resources (Kazemi et al., 2019).

Wood is an extremely versatile raw material with application fields ranging from paper production and packaging to the building industry. Moreover, wooden products can be reused and recycled after their original function becomes obsolete. According to data collected by the Horizon 2020 BioReg Project (Cocchi et al., 2019), the EU countries collectively produced between 40-60 million tonnes of yearly wood waste in the past ten years. Recovery rates of this depend on both the country and the type of wood waste, but it can be seen that there is room for improving the current amounts (Garcia & Hora, 2017).

The amount of research dealing with the management of waste wood has increased over the past years. As an example, the interest can be seen in the furnishing sector, where several studies have been conducted. The paper by de Carvalho Araújo et al. (de Carvalho Araújo et al., 2019) assesses the literature of circular economy in wood panel production. They conclude that while circular economy as a concept is being investigated with regard to waste production in this sector, mainly LCA (life cycle assessment) studies were carried out (Hossain & Poon, 2018; Kim & Song, 2014). Daian and Ozarska (Daian & Ozarska, 2009) studied a sample group of six SMEs in the wood furniture sector of Australia and collected data about the current state of their wood waste and its reuse, recycle and disposal. Based on this, they formulated suggestions on wood waste management. Evaluating the availability of wood waste (and wood biomass in general) is also becoming more and more important, which can be seen from the multiple recent studies that have dealt with this question. Research by Verkerk et al. (Verkerk et al., 2019) and Borzecki et al. (Borzecki et al., 2018) assessed the potential availability of forest biomass from European forests and its spatial distribution, focusing on the hotspots of biomass. Studies comparing waste wood management in selected European countries were also conducted by Garcia and Hora (Garcia & Hora, 2017) and the BioReg Project(Cocchi et al., 2019).

Although similar studies have become more widespread over the past years, the number of papers dealing with the mathematical modelling and optimization of processes in the waste wood supply chain is still scarce. Network design and planning is one of the most studied problem classes in logistics (Govindan et al., 2015). While there have been recent studies into the combined design of the network nodes and their possible links (Rahmaniani & Ghaderi, 2013), it is usually safe to assume for transport problems that the underlying road network already exists. In this case, the most important problem to solve is facility location. The goal of this problem is to find an optimal placement of facilities on a network in order to minimize arising costs, which usually include transportation and opening facilities.

Mathematical models of facility location are extensively studied, see e.g., Chapter 4 in (Dekker et al., 2004). Further variations of the facility location problem (not specific to reverse logistics networks) can be found in (Simchi-Levi et al., 2014). Stochastic variations of the problem can be found in (Verter & Dincer, 1992), which also considers capacity planning as the Capacity Expansion Problem once the facility locations are established. Dasci and Laporte study facility location and capacity acquisition by segmenting a market on the infinite continuous plain with uncertain demand (Dasci & Laporte, 2005). In a recent manuscript, Ahmadi-Javid et al. study a combined facility location and capacity planning problem, where the facilities should serve customers with demand modeled as Poisson processes, which results in a nonlinear model (Ahmadi-Javid et al., 2018). Solution methods for facility location with economies of scales are studied in (Bucci, 2009; Lu, 2010).

Facility location problems usually consider two types of uncertainties; namely, stochastic parameters and disruptions (Chun Peng et al., 2017). An example for the former one is the stochastic demand or cost parameters, see e.g., (Carrizosa & Nickel, 2003). Robust models, on the other hand considers possible changes in the network structure, e.g., expected consequences of random disruptions or targeted attacks by malevolent attackers (Daskin, 2013). Robust facility location is studied in (Cheng et al., 2018).

While general solutions designed for backward biomass streams have been studied in the past (e.g. (Nunes et al., 2020; Sharma et al., 2013)), we only found a handful of papers that focus entirely on waste wood. The reverse logistics network redesign problem for waste wood from the construction industry is investigated in Trochu et al. (Trochu et al., 2018), and a MILP (mixed integer linear programming model) was proposed for its solution. A use-case on a scenario from Quebec, Canada, was also presented. Devjak et al. (Devjak et al., 1994) formulated a mathematical model for optimizing the transportation of wood waste produced in sawmills, but did not present any computational experiments to back up its efficiency. Burnard et al. (Burnard et al., 2015) gave a reverse logistics model for facility location and transportation for waste wood, and presented computational results for a use-case in Slovenia.

In this paper, we consider the facility location problem for transporting waste wood from accumulation centers to processing facilities. Besides transportation, we also study economies of scale as well as the robustness of the network in case of the breakdown of facilities. First, we formulate mathematical models for the problems, and propose both a local and tabu search heuristic method for their solution. The efficiency of these methods is shown on test instances generated using a real-life dataset.

2 Problem definition

In the following subsections we formulate the uncapacitated facility location problem and its extensions.

2.1 Uncapacitated facility location problem

Let $\mathcal{I}$ denote the set of fixed accumulation point locations and $\mathcal{J}$ the set of potential facility locations. Let f_j denote the cost of opening facility j and c_{ij} denote the transportation cost from point i to facility j per m^3. Let u_i denote the annual yield of waste wood from accumulation point $i \in \mathcal{I}$ (in m^3).

The formulation uses two types of binary variables: Y_j is the indicator of opening facility $j \in \mathcal{J}$, while X_{ij} indicates product flow from accumulation point i to facility j. Note that due to uncapacitated facilities, an optimal solution always transports the whole amount of wood from each accumulation point to the closest open facility. The optimization problem is then the following binary integer problem:

$$\min \quad \sum_{j\in\mathcal{J}} f_j Y_j + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} u_i c_{ij} X_{ij} \tag{1}$$

subject to

$$\sum_{j\in\mathcal{J}} X_{ij} = 1 \qquad \forall i \in \mathcal{I} \tag{2}$$

$$X_{ij} \le Y_j \qquad \forall i \in \mathcal{I},\ j \in \mathcal{J} \tag{3}$$

$$Y_j \in \{0, 1\} \qquad \forall j \in \mathcal{J} \tag{4}$$

$$X_{ij} \in \{0, 1\} \qquad \forall i \in \mathcal{I},\ j \in \mathcal{J} \tag{5}$$

The objective function (1) minimizes the total opening and transportation cost, (2) ensures that the wood is transported from each accumulation point, while (3) states that all wood is transported to an open facility. Constraints (4) and (5) state that the variables are binary.

2.2 Economies of scale problem

It is often realistic to assume that the higher the capacity of a facility, the lower its production cost due to the economies of scale (Garcia & Hora, 2017). We consider the following production cost at facility j (based on (Bucci, 2009)): $S_j^b p_j$, where $S_j > 0$ is the total quantity processed at facility j, p_j is the unit production cost at facility j and b is a scale factor, typically -0.35 for manufacturing facilities and between -0.56 and -0.47 in the paper industry. With this modification the objective function of the program becomes non-linear as follows:

$$\min \quad \sum_{j\in\mathcal{J}} f_j Y_j + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} u_i X_{ij} c_{ij} + \sum_{j\in\mathcal{J}} S_j^b p_j S_j \tag{6}$$

The constraints are the same as (2)-(5) with the following additional constraint defining the variable S_j:

$$\sum_{i\in\mathcal{I}} X_{ij} u_i = S_j \qquad \forall j \in \mathcal{J} \tag{7}$$

We still consider solutions where wood from each accumulation point is transported to only one facility, since there exist an optimal solution with this property, see (Dupont, 2008). However, it is no longer true that all wood should necessarily be transported to the closest open facility, for each set of open facilities an assignment problem should be solved to determine the optimal transportation.

2.3 Robust optimization problem

Robust optimization can be modeled as a multi-objective optimization problem, where one objective is minimizing the cost in case of no disruptions, the other is minimizing the cost in case of a disruption. However, we consider only minimizing the cost in case of a disruption instead. More specifically, we consider a solution optimal, if any facility breaks down—i.e., all accumulation points connected with this facility must transport to another facilities—then the resulting cost in the worst case is minimal.

We model this problem as a bilevel optimization: the leader determines which facilities to open, while the follower determines which accumulation point is connected to which facility. The follower's problem assumes a given set of open and undisrupted facilities ($\{\, j \mid Y'_j = 1 \,\}$) and assign the accumulation points to these facilities minimizing the transportation costs:

$$\min \quad \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} u_i c_{ij} X_{ij} \tag{8}$$

subject to

$$\sum_{j\in\mathcal{J}} X_{ij} = 1 \qquad \forall i \in \mathcal{I} \tag{9}$$

$$\sum_{i\in\mathcal{I}} X_{ij} \le Y'_j \qquad \forall j \in \mathcal{J} \tag{10}$$

$$X_{ij} \in \{\, 0, 1 \,\} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J} \tag{11}$$

Note that the follower's problem can be easily solved by transporting all the wood to the closest open facility. Let $G(Y')$ denote the optimal objective value for the follower's assignment problem on the input vector Y'.

Then the leader's problem is to determine the set of facilities to open with the minimal opening cost together with the transportation cost in case of the disruption of exactly one facility:

$$\min_{Y}\left\{\sum_{j\in\mathcal{J}} f_j Y_j + \max_{Y'}\left\{G(Y') \,:\, \sum_{j\in\mathcal{J}} Y'_j + 1 = \sum_{j\in\mathcal{J}} Y_j \wedge Y'_j \le Y_j \quad (\forall j\in\mathcal{J})\right\}\right\} \tag{12}$$

This expresses that facilities $\{\, j \mid Y_j = 1 \,\}$ are opened, but then one of them cannot be used because of a disruption, therefore the transportation has to be determined not using the disrupted facility. The worst case is considered, i.e., when the disrupted facility causes the highest transportation costs. This corresponds to a pessimistic bilevel program.

3 Solution approaches

Solving facility location problems in realistic sizes (i.e., several thousands of accumulation points and possible facility locations) is computationally intractable even without considering economies of scale or robustness. Therefore, similarly to other works in this field, we used metaheuristic algorithms to find quasi-optimal solutions.

3.1 Determining the worst case cost effectively

If economies of scale are disregarded, the optimal solution always transports the wood to the closest open facility. We use this observation to efficiently compute the cost in case of disturbances. Let π_i denote a permutation of the facilities for each i such that $c_{i\pi_{i1}} < \cdots < c_{i\pi_{in}}$, where $n = |\mathcal{J}|$ is the number of facilities. If Y denote the status of the facilities with at least two open facilities, then let $F_i(Y) = \min_k \{Y_{\pi_{ik}} = 1\}$ denote the closest open facility to point i, and let $B_i(Y) = \min_k \{Y_{\pi_{ik}} = 1 \wedge k \neq F_i(Y)\}$ denote the second closest one. If there is a disruption at facility $F_i(Y)$, then the wood from point i should be transported to facility $B_i(Y)$ instead, which means $(c_{iB_i(Y)} - c_{iF_i(Y)})u_i$ additional transportation cost. Therefore in case of a disruption at an open facility j, the additional cost is $CoD_j(Y) = \sum_{i:F_i(Y)=j} (c_{iB_i(Y)} - c_{iF_i(Y)})u_i$. Then the cost increase of disruption in the worst case is simply $\max_{j:Y_j=1} CoD_j$.

Therefore by maintaining the F, B and CoD vectors when the search heuristics open or close a facility, the value of the objective function can be determined efficiently.

3.2 Local search heuristic

We use the neighborhood defined by (Korupolu et al., 2000), which represents the solution only with the set of open facilities. Let $S = \{\, j \mid Y_j = 1 \,\}$ denote the set of open facilities, then the neighborhood of S is $\{ T : |S \setminus T| \leq 1 \wedge |S \setminus T| \leq 1 \}$. From a solution S one can apply three operations to reach a neighbor: (i) open a new facility, (ii) close a facility (in case $|S > 1|$), and (iii) change the status of an open and a closed facility. This neighborhood contains $O(|\mathcal{J}|^2)$ solutions, where $\mathcal{J}$ is the set of potential facilities.

If one intends to solve the robust facility location problem, then instead of the cost defined by (1), the worst case cost should be considered.

3.3 Tabu search heuristic

We have implemented the tabu search based on the approach described in (Sun, 2006) with some modifications. In addition to seeking the minimal cost in case of a disruption, we applied a different medium term memory process as well as different approach for updating the lengths of the tabu lists.

The short term memory process is the following. Let k denote the number of moves since the start of the search and Δz_j^k the cost change by changing facility j's status, i.e., closing if it is open and open if it is closed. The integer vector $\mathbf{t}$ is used to store the last time when the status of the facilities changed, i.e., t_j is the value of k when facility j changed its status last. Let z_0 denote the best objective value in the current search cycle and k_0 denote the time when z_0 was last updated. Let l_0 (l_c) denote the tabu sizes for the open (closed) facilities, i.e., they cannot change status twice during this time interval unless the aspiration criterion is satisfied. These lengths are bound by lower limit l_o^1 (l_c^1) and upper limit l_o^2 (l_c^2). Each move is changing the status of a facility. We choose facility $\bar{j}$, where $\Delta z_{\bar{j}}^k = \min\{ \Delta z_j^k \,|\, \text{facility } j \text{ is not flagged} \}$. A facility $\bar{j}$ is flagged, if the following tabu condition holds: $k - t_{\bar{j}} \leq l_c$ if $Y_{\bar{j}} = 0$ or $k - t_{\bar{j}} \leq l_0$ if $Y_{\bar{j}} = 1$, but does not hold the following aspiration criteria: $z + \Delta z_{\bar{j}}^k < z_0$, where z represents the cost of the current solution. The short term process ends when the solution could not be improved for a specified time, i.e., when $k - k_0 > \alpha_1 n$, where α_1 is a parameter of the search.

After each step the lengths of the tabu lists are updated: if the current solution improved the objective value, then l_0 (l_c) is increased by one, otherwise it is decreased by one to extend the search space.

In the medium term, we changed the frequency based memory process described by (Sun, 2006) and use a wider neighborhood instead. We seek for an open and a closed facility such that if we change their statuses, the total cost decreases the most. Sun states that considering this operation is costly, but our algorithm only use it when the short term process fails to improve the solution, thus providing a trade-off between computation time and solution quality. We have found that this approach performs better on the tested instances.

If the solution can be improved, the search continues with the short term process. The medium term process ends when no improvement can be found.

Finally, the long term process is invoked C times and when invoking the cth time, c moves are made changing the status of facility $\bar{j}$ according to the following criterion: $t_{\bar{j}} = \min\{ t_j \,|\, j = 1, \ldots, n \}$.

3.4 Exact solution of the robust problem

The exact solution can be computed by completely enumerating all possible subsets of open facilities, and for each combination of open facilities, a simple assignment problem must be solved. However, we can apply the following simple bounding procedure to reduce the search space. When the algorithm already has a solution with objective value z, then for any subset of open facilities $\mathcal{S}$ where the sum of opening costs—disregarding the transportation costs—exceeds z, the search can ignore all supersets of $\mathcal{S}$, since they cannot result in a less expensive solution.

This exact method performs well if the opening costs are high compared to the transportation costs, because in those cases finding a good solution can restrict the search to a small number of open facilities. Nevertheless, this exact method works on small problem instances only.

3.5 Bilevel integer program formulation

Considering the formulation of Section 2.3, it can be observed that once the Y' variables are fixed, the X variables are easy to determine to minimize the transportation costs by assigning each accumulation point to the closest open facility. This suggests that a solution of the following constraints determines an optimal assignment.

$$X_{i\pi_{i1}} \geq Y'_{\pi_{i1}} \qquad \forall i \in \mathcal{I} \tag{13}$$

$$X_{i\pi_{ik}} \geq Y'_{\pi_{ik}} - \sum_{t=1}^{k-1} Y'_{\pi_{it}} \qquad \forall i \in \mathcal{I}, k = 2, \ldots, m \tag{14}$$

$$\sum_{j \in \mathcal{J}} X_{ij} = 1 \qquad \forall i \in \mathcal{I} \tag{15}$$

$$X_{ij} \in \{ 0, 1 \} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J} \tag{16}$$

Then, the inner maximization problem of (12) takes the form

$$\max \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} u_i c_{ij} X_{ij} \tag{17}$$

subject to (13)-(16) and the constraints

$$\sum_{j \in \mathcal{J}} Y'_j + 1 = \sum_{j \in \mathcal{J}} Y_j \tag{18}$$

$$Y'_j \leq Y_j \qquad \forall j \in \mathcal{J} \tag{19}$$

Note that this formulation does not include non-linearity in contrast to the usual duality-based formulation (see e.g., (Cheng et al., 2018)).

Using this observation, we search for the optimal solution where exactly k facilities ($\rho_1 < \cdots < \rho_k$) are open: $\sum_{j \in \mathcal{J}} Y_j = k$ and $Y_{\rho_l} = 1$ ($l \in \{ 1, \ldots, k \}$).

Let Y^l denote the vector that differs from Y only in its ρ_lth element and $\{ X^l_{ij} \,:\, i \in \mathcal{I}, j \in \mathcal{J} \}$ the optimal transportation from accumulation point i to facility j using open factories determined by Y^l. Then the optimization problem (12) becomes:

$$\min \quad \sum_{j\in\mathcal{J}} f_j Y_j + z \tag{20}$$

subject to

$$z \geq \sum_{i\in\mathcal{I},j\in\mathcal{J}} u_i c_{ij} X^l_{ij} \qquad \forall l \in \{ 1,\dots,k \} \tag{21}$$

$$X^l_{i\pi_{i1}} \geq Y^l_{\pi_{i1}} \qquad \forall i \in \mathcal{I}, l \in \{ 1,\dots,k \} \tag{22}$$

$$X^l_{i\pi_{is}} \geq Y^l_{\pi_{is}} - \sum_{t=1}^{s-1} Y^l_{\pi_{it}} \qquad \forall i \in \mathcal{I}, s = 2,\dots,m, l \in \{ 1,\dots,k \} \tag{23}$$

$$\sum_{j\in\mathcal{J}} X^l_{ij} = 1 \qquad \forall i \in \mathcal{I}, l \in \{ 1,\dots,k \} \tag{24}$$

$$Y^l_j \leq Y_j \qquad \forall j \in \mathcal{J}, l \in \{ 1,\dots,k \} \tag{25}$$

$$\sum_{l=1}^{k} Y^l_j = (k-1) Y_j, \qquad \forall j \in \mathcal{J} \tag{26}$$

$$\sum_{j\in\mathcal{J}} Y_j = k \tag{27}$$

$$Y_j \in \{ 0,1 \} \qquad \forall j \in \mathcal{J} \tag{28}$$

$$Y^l_j \in \{ 0,1 \} \qquad \forall j \in \mathcal{J}, l \in \{ 1,\dots,k \} \tag{29}$$

$$X^l_{ij} \in \{ 0,1 \} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, l \in \{ 1,\dots,k \} \tag{30}$$

Constraints (22)-(24) are the constraints of the inner optimization problem. Inequality (26) says that if $Y_j = 0$, then all $Y^l_j = 0$, whereas if $Y_j = 1$, then exactly $k-1$ of the Y^l has a 1 in position j. This, along with (25) implies that vectors Y^l are all different, they are not bigger than Y (coordinatewise), and they have $k-1$ coordinates of value 1, all other coordinates being 0.

This formulation considers a fixed number of open facilities, therefore it should be solved for all possible (or realistic) k values.

4 Numerical study

Based on the industrial dataset of 1839 accumulation points and possible facility locations, we generated test sets containing 50, 100 and 500 locations, five different test cases for each set. Then we computed the solutions assuming different facility opening costs from the realistic 5 million to 1000. The solutions were computed using the local search, the tabu search, and when possible, the exact solver. For the tabu search we used the same parameters as (Sun, 2006): $l_c^1 = l_o^1 = 10$, $l_c^2 = l_o^2 = 20$, $C = 5$ and $\alpha_1 = 2.5$.

Table 1 contains the average results over the five test sets. The non-robust solutions aim at minimizing the total opening and transportation cost indicated in the cost column, while robust solutions aim at minimizing the worst case cost (WCC), i.e., the total opening cost and transportation costs in case of a facility disruption.

We have estimated the production cost for the facility location model with the economies of scale, however, we have found that for realistic cases (large number of accumulation points, large facility opening costs, few open facilities) the economies of scale does not influence the solution. Therefore the non-linearity of the problem was not considered in the results presented below, which resulted in more tractable problems.

The following indicators are included in the table: the cost of disruption (CoD) is the additional cost in case of a disturbance ((WCC-cost)/cost), the price of robustness (PoR) is the difference between the robust solution and the non-robust one ((robust cost - non-robust cost)/non-robust cost) and the benefit of robustness (BoR) is the difference in case of a disruption ((non-robust WCC - robust WCC)/non-robust WCC). This latter indicator cannot be interpreted when the non-robust solution contains only one opened facility, i.e., when in case of a disruption the whole network fails.

The rows labelled with "OPT" denote the average costs of the optimal solutions. For the non-robust problem, it is computed by the the FICO XPRESS Solver using the formulation in Section 2.1, and for the robust problem the optimum is computed by solving the bilvel programming formulation of Section 3.5.

Table 1 contains the results of the solutions considering 50 locations. The following observations were made:

- For the opening costs between 1 million and 5 million, the exact solutions could be computed for the non-robust, and the robust variants as well, and both the local search and the tabu search could find the optimal solution in every case.
- Changing the opening costs in a wide range (above one million) did not change the solutions. That means that the uncertainty of the exact opening cost does not matter too much.
- For the 4 largest facility opening costs, the non-robust solutions contain only one opened facility, therefore they are quite vulnerable for disruptions. Adding one more factory to improve robustness is quite expensive, increasing the required budget by 36-76%.
- Considering 1000 as the opening cost, the tabu search resulted in better solution both for robust and non-robust cases in one case out of five, therefore the last two rows are separate in order to differentiate the two approaches. The robust version of the problem could not be solved with the exact solver.

- For an extremely low opening cost, large number of facilities are opened and even the non-robust solution offer some robustness. However, the robustness can be improved relatively inexpensively (for less than 0.4% of the budget) and in case of a disruption this can result in more than 10% saving in the additional costs.
- In each cases, either the local search or the tabu search could find the optimal solution for the uncapacitated facility location problem without robustness.

Table 2 contains the results of the solutions considering 100 locations. The following observations were made:

- For opening costs 5 million and 2.5 million local search and tabu search resulted in the same results as the exact solver. The non-robust solutions in these cases always contain only one open facility and adding robustness by opening more facilities are quite costly.
- For opening costs 1.6 and 1 million, the non-robust solutions contain one or two open facilities. The WCC and BoR values are the averages over the valid values. For these problems the solution of the local search and the tabu search often differ and it varies which performs better.
- For opening cost 1000, the tabu search performed better in one case. It can be observed that increasing robustness in this case is quite inexpensive, but the achieved benefit is also lower that in the 50 facility case.
- For only one problem instance neither the local search nor the tabu search heuristics could find the optimal solution for the uncapacitated facility location problem without robustness.

Table 3 contains the results of the solutions considering 500 locations. The following observations were made:

- With this size of solution space the result of the local search and the tabu search often differ and on average the tabu search performs slightly better.
- Most of the non-robust solutions contain two or more open facilities. Optimizing for robustness increases the cost usually under 20%, therefore as the problem size growths, it becomes relatively less expensive to provide robustness. However, in case of the disruption robustness can save at least 10% of the additional cost, when the opening cost is above one million.
- For five problem instances neither the local search nor the tabu search heuristics could find the optimal solution for the uncapacitated facility location problem without robustness. Four of these five cases have 1000 as the facility opening cost.

As a conclusion it can be observed that including robustness is the most important when the number of opened facilities are low.

5 Conclusions

In this paper, we studied the facility location problem in the reverse logistics network of waste wood. This network considered the accumulation centre for waste wood as well as the processing facilities where they have to be transported. The traditional facility location was extended with the consideration of economies of scale and robustness against the breakdown of facilities. We formulated mathematical models for these problems including a novel approach based on bilevel optimization, and also presented a local and tabu search heuristic method for their solution.

We tested the efficiency of the proposed methods on instances generated using a real-life dataset. Different facility opening costs were considered, and robust and non-robust solutions were examined in every case. While economies of scale seemed to have no influence on the solutions in the case of realistic cost parameters, robustness turned out to be significant when the number of opened facilities was low. In the case of a larger number of opened facilities (which usually happened with unrealistically low facility costs) even the non-robust solutions contained some inherent robustness.

While the heuristic method gave the same solutions for instances with a smaller number of locations (where they mostly found the optimal solution), the tabu search had a slight edge over the local search for larger instances. However, we were not able to obtain exact solutions for these instances with a large number of locations, and working on mathematical programming methods to help the solution of the model will be part of our future work.

As a future work, we intend to further study the integer program formulation of the bilevel robust facility location model and use it for computing lower bound on the cost. In addition, we are going to examine the delivery planning problem in the network designed by the facility location optimization.

Table 1 Average performance using 50 locations

Opening	Non-robust			Robust					
	Open	Cost	WCC	Open	Cost	WCC	CoD	PoR	BoR
5000000 (OPT)	1	6,452,010	-	2	11,349,166	11,494,142	1.28%	75.98%	-
5000000 (LS/TS)	1	6,452,010	-	2	11,349,166	11,494,142	1.28%	75.98%	-
2500000 (OPT)	1	3,952,010	-	2	6,349,166	6,494,142	2.31%	60.81%	-
2500000 (LS/TS)	1	3,952,010	-	2	6,349,166	6,494,142	2.31%	60.81%	-
1666666 (OPT)	1	3,118,677	-	2	4,682,499	4,827,476	3.14%	50.34%	-
1666666 (LS/TS)	1	3,118,677	-	2	4,682,499	4,827,476	3.14%	50.34%	-
1000000 (OPT)	1	2,452,010	-	2	3,349,166	3,494,142	4.42%	36.81%	-
1000000 (LS/TS)	1	2,452,010	-	2	3,349,166	3,494,142	4.42%	36.81%	-
1000 (OPT)	40.8	46,360	102,481						
1000 (LS)	40.6	46,377	102,498	41.4	46,534	93,605	102.20%	0.34%	10.57%
1000 (TS)	40.8	46,360	102,481	41.6	46,518	93,588	102.21%	0.34%	10.58%

Table 2 Average performance using 100 locations

Opening	Non-robust			Robust					
	Open	Cost	WCC	Open	Cost	WCC	CoD	PoR	BoR
5000000 (OPT)	1	7,579,528	-	2	12,383,253	12,594,382	1.72%	63.58%	-
5000000 (LS)	1	7,579,528	-	2	12,383,253	12,594,382	1.72%	63.58%	-
5000000 (TS)	1	7,579,528	-	2	12,383,253	12,594,382	1.72%	63.58%	-
2500000 (OPT)	1	5,079,528	-	2	7,383,253	7,594,382	2.90%	45.67%	-
2500000 (LS)	1	5,079,528	-	2	7,383,253	7,594,382	2.90%	45.67%	-
2500000 (TS)	1	5,079,528	-	2	7,383,253	7,594,382	2.90%	45.67%	-
1666666 (OPT)	1	4,246,195	-	2		5,927,715			
1666666 (LS)	1.2	4,327,338	8,102,164	2.2	5,703,829	6,083,148	6.33%	32.85%	5.87%
1666666 (TS)	1	4,246,195	-	2	5,612,645	5,998,162	6.67%	32.95%	-
1000000 (OPT)	1.4	3,510,617		2		4,594,382			
1000000 (LS)	1.6	3,554,200	5,663,822	2.2	4,237,162	4,616,481	8.76%	19.82%	15.35%
1000000 (TS)	1.4	3,533,251	6,062,458	2.2	4,242,211	4,628,458	8.85%	20.56%	17.01%
1000 (OPT)	77.6	90,535	118,734						
1000 (LS)	77.6	90,535	118,734	78.2	90,703	113,882	25.67%	0.18%	4.05%
1000 (TS)	77.6	90,535	118,734	78.6	90,829	113,859	25.49%	0.32%	4.07%

Table 3 Average performance using 500 locations

Opening	Non-robust			Robust					
	Open	Cost	WCC	Open	Cost	WCC	CoD	PoR	BoR
5000000 (OPT)	1.6	19,242,002							
5000000 (LS)	2	19,497,959	31,080,579	2.4	23,068,454	25,278,632	9.79%	18.57%	17.72%
5000000 (TS)	1.6	19,307,357	33,319,075	2.4	23,062,160	25,421,665	10.31%	19.78%	20.13%
2500000 (OPT)	2.4	14,263,693							
2500000 (LS)	2.2	14,372,353	25,267,673	3.8	16,318,181	19,023,002	17.26%	13.69%	22.33%
2500000 (TS)	2.6	14,304,504	21,116,645	3.8	16,336,745	18,676,174	14.55%	14.38%	11.35%
1666666 (OPT)	3.2	11,900,321							
1666666 (LS)	3	11,952,078	18,065,126	4	13,464,150	15,561,839	15.60%	12.69%	13.48%
1666666 (TS)	3.4	11,905,914	17,393,652	4.8	13,380,271	15,323,262	14.60%	12.36%	11.43%
1000000 (OPT)	4.4	9,496,951							
1000000 (LS)	4.4	9,496,951	13,729,015	6	10,402,176	11,881,160	14.26%	9.60%	12.61%
1000000 (TS)	4.4	9,522,457	13,678,493	5.8	10,429,309	11,946,057	14.65%	9.48%	11.80%
1000 (OPT)	282	396,016							
1000 (LS)	281.4	396,159	454,042	281.60	396,163	453,068	14.38%	0.00%	0.24%
1000 (TS)	282	396,054	453,937	283	396,198	452,907	14.33%	0.04%	0.25%

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