

# Coordination in Production Networks

PhD Thesis

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## Declaration

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Péter Egri  
Budapest, December 2008

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# List of Abbreviations

$AC_q$	Average cost heuristic according to quantity
$AC_x$	Average cost heuristic according to time
BOM	Bill-Of-Materials
CDF	Cumulative Density Function
EOQ	Economic Order Quantity
ERP	Enterprise Resource Planning
JIT	Just-In-Time
MRP, MRP II	Material Requirements Planning/Manufacturing Resource Planning
LP	Logistics Platform
MMFE	Martingale Model of Forecast Evolution
MTO	Make-to-Order
MTS	Make-to-Stock
SCM	Supply Chain Management
PDF	Probability Density Function
VMI	Vendor Managed Inventory
WIP	Work-in-Progress
WWr	Wagner – Whitin with Run-Out

# Notations

## Centralised newsvendor model

$c_s$	setup cost
$c_p$	production cost per unit
$\phi$	probability density function (PDF) of the demand
$\Phi$	cumulative density function (CDF) of the demand
$m$	expected value of the demand
$b$	parameter of the logistic distribution, proportional to its standard deviation
$\xi$	realised demand
$s$	minimal production quantity
$q$	production quantity (decision variable)
$Z$	total cost (objective function)

## WWr model

$n$	length of the horizon
$F_1, \dots, F_n \geq 0$	forecasted demand
$c_s$	setup cost
$h$	inventory holding cost per piece per period
$c_p$	production cost per piece (cost of obsolete inventory per piece)
$\eta \in \{1, \dots\}$	period of the run-out <sup>1</sup> (random variable)
$x_0, \dots, x_n$	production quantities (decision variables)
$I_0, \dots, I_n$	inventory at the end of periods (auxiliary variables)
$Z$	objective function

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<sup>1</sup>This can also be interpreted as the length of the remaining product life.

## Decentralised newsvendor model

$\xi$	realised demand
$m$	expected value of the demand
$b$	parameter of the logistic distribution, proportional to its standard deviation
$c_0$	unit price of a component
$c_1$	unit compensation price
$m'$	communicated expected value of the demand (decision variable of the customer)
$b'$	communicated parameter of the logistic distribution (decision variable of the customer)
$q$	production quantity (decision variable of the supplier)
$P(m', b', \xi)$	payment

## Rolling horizon coordination model

$n$	length of the forecast horizon
$n'$	length of the stability horizon ( $n' \leq n$ )
$F_{i,j}$	forecast for period $i$ made in period $j$ ( $j + 1 \leq i \leq j + n$ )
$\xi_i$	realised demand in period $i$
$c_0$	unit price of a component
$c_1$	unit compensation price
$c_2$	compensation price for run-out possibility
$\alpha_i$	discount factor of period $i$
$\eta_j \in \{1, \dots\}$	period of the run-out estimated in period $j$
$e_i$	error of period $i$
$d_j$	deviation of the forecast generated in period $j$
$P_k$	payment for the period $k$

## Forecast sharing games

$\theta, \hat{\theta} \in \Theta$	real and communicated parameters of the forecast
$c, \hat{c}$	real and communicated cost function
$v$	income function
$f$	choice function
$u, u_s, u_r$	utility functions
$t, t_1, t_2$	payment (transfer) functions
$x \in \mathcal{K}$	production plan
$\xi \in D$	realised demand

## Simulation environment

$n$	length of the forecast horizon
$F_{t,t+i}$	forecast made in period $t$ for demand in period $t+i$
$d$	long-run average demand
$\varepsilon_{t,t+i}, \varepsilon_t$	forecast update and forecast update vector
$\mathcal{F}_t$	knowledge in period $t$ (filtration)
$\Sigma$	covariance matrix



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# Chapter 1

## Introduction

“Every kind of peaceful cooperation among men is primarily based on mutual trust and only secondarily on institutions such as courts of justice and police.”

Albert Einstein

In the past decades the circumstances of the industrial production have dramatically changed. The increasing customer expectations require ever shorter delivery times, customised products and extremely high service levels. This taut situation boosts competition between manufacturing enterprises, which inspires them to work out new ways towards achieving more efficient production. In parallel, the new paradigm of *production networks* has emerged, which nowadays refers to cross-company relations [119]. In this introductory chapter I briefly present the ongoing trends in manufacturing, point out why scientific research is crucial to support the advance of new attitudes towards the exposed problems, and I also pose some challenges for mathematics, economics and informatics, which I would like to answer for some special but practically relevant cases in this work.

### 1.1 Production: Current Issues and Dilemmas

Due to today’s continuously changing market conditions, manufacturing enterprises are facing more difficult challenges than ever before. In spite of the still existing uncertainties of the environment—such as demand fluctuation, resource failures, scrap production, procurement delays—, customer expectations are persistently growing and manufacturing

must fulfil their needs to remain competitive. Nowadays, customers seldom accept shortages or backlogs and in addition, they often want to customise the product characteristics themselves. The widely accepted and utilised Total Quality Management (TQM) principle states that all expressed and unexpressed wishes of the customer should be satisfied and the most significant manufacturers act upon this management philosophy, which phenomenon is usually referred to as *customer-oriented* or *demand centric* attitude [55].

Naturally, there exist several paradigms to answer the current challenges all with their own advantages and disadvantages. The *craft production*—whose golden age was before the 20th century—allows large variety of products, but requires complicated, time-consuming manufacturing processes, which are also expensive. *Mass production*—the main paradigm in the 20th century—achieves higher efficiency with standardised products, exploiting economies of scale and (semi-)automated processes, but gives up the wide product scale. In the last few decades the new paradigm of *mass customisation* has arisen, which tries to combine the advantages of the previous two approaches by offering a larger variety of products made of standardised components with mass production technology. As it has turned out, while this new paradigm offers some solutions to some of the problems, it also poses new questions [97, 98, 99].

The key issues can be characterised by a set of dilemmas: one has to find acceptable trade-offs between conflicting objectives such as (i) running efficient production in large batches or small inventory-related costs, (ii) holding more inventories or using more frequent transportation, (iii) choosing the faster or the cheaper technology or transportation alternative, (iv) offering wide variety of products or reducing product inventories, and finally, (v) offering high service level or low prices to customers [97].

Traditional ways of improving efficiency—such as decreasing setup costs by applying new technologies, shortening lead-times by following the so-called *lean* initiative, combining *push* and *pull* supply as well as applying *delayed differentiation* (pushing customisation downward<sup>1</sup> in the supply chains)—are still important, but usually not enough. Sustaining growth and competitiveness nowadays can be achieved only through a cooperative attitude between enterprises as well as through the transition from factory automation to network automation [99]. This means not only automated data exchange between enterprises, but also increasing supply flexibility, i.e., allowing contracts with flexible order quantities [25].

<sup>1</sup>According to the standard nomenclature, *downward* refers to the direction towards the customers, while *upward* means toward suppliers.

One of the most subtle challenges in production networks is *managing inventories* appropriately [69]. In the second half of the 20th century, the Just-In-Time (JIT) production paradigm became very popular, since it promised the elimination of inventories, which were considered passive elements of the business creating only expenses but no value [16]. However, this “zero inventory” concept could rarely be realised in practice due to its difficult introduction into existing production systems and to the high expectations of JIT production—unvarying demand, negligible setup cost/time, and so forth. Accordingly, the original Toyota-approach is strongly based on aggressive marketing strategies in order to smooth the demand and avoid changes, which is the opposite of what Wal-Mart applies and calls its “always low prices” policy [52].

Satisfying demand directly from production is often impossible, because production and supply lead-times are much longer than the acceptable delivery times for the customers and the stock-out situations not only cause loss of profit but also of customers [22]. In order to provide high service levels toward end customers, inventories are essential. In addition, the manufacturing uncertainties also have to be considered, which can originate from three sources: (i) the internal processes (e.g., machine break down), (ii) the demand (e.g., sudden demand increase) and (iii) the supply (e.g., late delivery) [103]. These can again be handled by keeping buffers not only of capacities but also of materials and end-products. The third reason for keeping inventories is to exploit economies of scale, i.e., to divide the fixed part of the cost (e.g., setup, delivery) among more products in order to decrease the average cost.

Although inventories and thus Make-to-Stock (MTS) production are necessary, the decision about inventory levels can only be based on fluctuating and uncertain *forecasts* [118]. In addition, due to unforeseen changes of demand, stocks of products with short life-cycles (e.g., customised packaging materials) may easily become obsolete, which causes not only significant financial losses for the enterprises, but also serious waste of material, labour, energy and environmental resources. Recently, increasing societal pressure came forward against this kind of environmental harm and the paradigm of *competitive sustainable manufacturing* (CSM) arose, which aims at changing technology and productivity considering ecological and biological capacities, too [49].

Therefore it is still actual and extremely important to concentrate on inventory-related problems; that is why it is one of the main topics also in my dissertation. Of course MTS production is not the only possibility and the chosen approach has to match with

the problem characteristics; there is no one-size-fits-all solution. Fig. 1.1 summarises the most common types of the order fulfilment. It is necessary to study the market conditions, differentiate products according to the demand volume, variety, variability and choose the appropriate answer for the given situation, which results typically in a hybrid approach [23]. However, in this work I concentrate chiefly on customised components with uncertain life-cycle, whose demand therefore can suddenly cease.

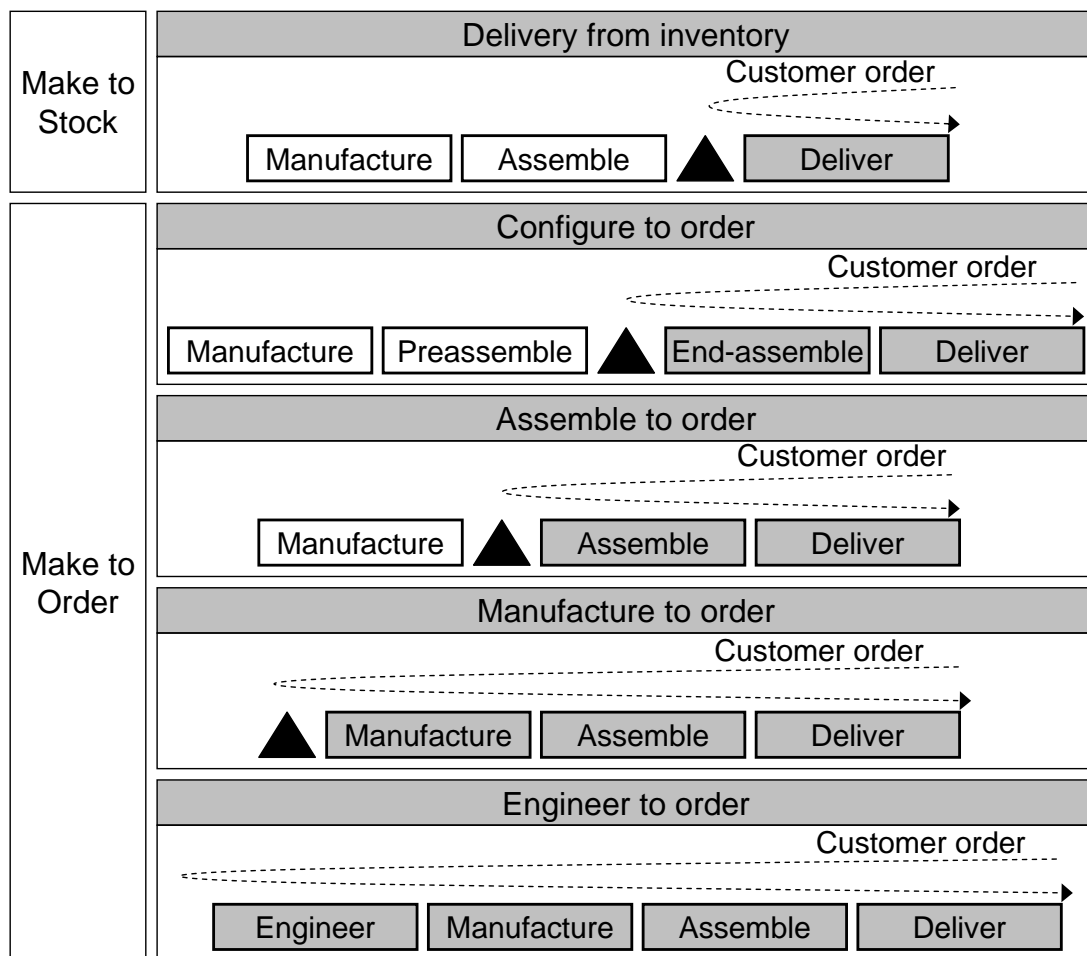


Figure 1.1: Make-to-Stock and Make-to-Order approaches (Source: [3] p. 95.).

## 1.2 Cooperation in Production Networks

Consumer goods are mainly produced in a long process of multiple steps, which are often carried out by separate, *autonomous* and rational production enterprises, linked by *supply chains*. Since the uncertainty is amplified due to safety stocks as we traverse upwards the chains (the so-called *bullwhip effect* [59]), decentralisation leads to suboptimal overall system performance called *double marginalisation*, which can be interpreted as the symptom of the prisoners' dilemma in supply chains [105].

Hence, in production networks inventory management is even more problematic than in the centralised case. As previous studies have shown (see Chapter 2), the resultant of the locally optimal decisions usually leads to suboptimal network performance, since the objectives of the autonomous decision makers are not aligned with any global objective [1]. A network-wide solution emerges from the interaction of local decisions. This is essentially a *distributed planning problem*: network members would like to exercise control over some future events based on information what they know at the moment for certain (about products, technologies, resource capabilities, sales histories) and only anticipate (demand, resource and material availability).

The theoretical solution to this problem is to appoint a *central decision maker*, whom every participant has to share all relevant information. The resulted planning task is rather complex in itself, since the information about the future is still uncertain, and in addition, different, conflicting objectives (e.g., service level and operation efficiency) should be considered. However, this *centralised coordination* approach is practically unrealisable. Several intermediate settings are also conceivable between the two extremes of the completely distributed and centralised planning. Pibernik and Sucky call these approaches as *partially centralised coordination* and they also introduce a measure for centralisation [82]. This is a general model of describing stages of cooperation; their paper regards only the master planning task, though. These different cooperation stages can also be illustrated in a range of colours from cold blue to hot red [35]. In a real production network several types of relationships are combined in order to appropriately face the challenges of the different market characteristics [120].

The currently accepted direction for resolving the problems points towards extended coordination and cooperation along the supply chains, thus the paradigm of *production in networks* has emerged [119]. This is especially true for the case of customised com-

ponents, since they cannot be procured with auctions on the short-term—which is usual on *matching markets*. Instead, mass customisation necessitates long-term strategic partnerships, clear regulation of responsibilities and *vertically integrated* supply chains. It is widely accepted that tight cooperation also results in more efficient production, facilitates technology sharing and helps mutual growth [65].

Several practical initiatives have taken this approach, like the Vendor Managed Inventory (VMI), the Quick Response (QR), the Efficient Consumer Response (ECR) or the Collaborative Planning, Forecasting and Replenishment (CPFR) programme, to name a few examples. In this dissertation I consider the *VMI business model*, where the supplier takes full responsibility of managing the one-point inventory so that the customer does not have to possess component buffers at all [97]. However, the main reason for applying VMI in the practice is the market power of the customer, and not the mutual interest.

The theory of *contracting* aims at supporting the cooperation and developing arrangements for aligning the different objectives of the partners. Contracts are protocols that control the flows of information, materials (or service) and financial assets alike. In general, a contracting scheme should consist of the following components [63]:

- i.) local planning methods which consider the constraints and objectives of the individual partners,
- ii.) an infrastructure and protocol for information sharing, and
- iii.) an incentive scheme for aligning the individual interests of the partners.

The appropriate planning methods are necessary for optimising the behaviour of the production network. The second component should support the information visibility and transparency both within and among the partners and facilitates the realisation of *real-time enterprises*. Information is often uncertain, but if it is further distorted or delayed, it corresponds to a car whose control panel indicates always a few days earlier fuel level [72]. Finally, the third component should guarantee that the partners act upon to the common goals of the network.

A contract is said to achieve *channel coordination*, if thereby the partners' optimal local decisions lead to optimal system-wide performance. My present work deals with these three issues of operating cooperative production networks.

## 1.3 Motivation

The industrial motivation of this work comes from a large-scale national industry-academia RTD project aimed at realising *real-time, cooperative enterprises* [74, 75]. The participating industrial partners form a complete *focal* network: a central assembly plant with several external and internal suppliers. The assembler produces altogether several million units of low-tech consumer goods per week from a mix of thousands of products. The ratio of the customised products follows the *80/20 Pareto-principle*: they give 80% of the product spectrum, but only 20% of the volume. The setup costs are significant and since customised products are consumed slower, their smaller lot-sizes involve higher average setup costs. Service level requirements are extremely high: some retailers suddenly demand the delivery of products in large quantities, even within 24 hours, and if the request is not fulfilled on time, they cancel the order (i.e., backlogs are not allowed). This causes not only lost sales, but also decrease of goodwill and perhaps lost customers. In order to deal with the uncertainty efficiently, cooperative attitude is present in the network whereupon my models are built.

Due to the strong industrial background of my research, my goal was also to bridge the gap between theory and practice. I intended to develop precise mathematical models and information technology infrastructure considering conditions in real production networks, together with such efficient algorithms that support decision making and help estimating the possible consequences of the decisions. I considered the criteria of realisability and applicability all along my work, but at the same time I did not give up the exact and solid mathematical principles. All in all, the models should capture

- i.) market uncertainty,
- ii.) local decision-making at enterprises,
- iii.) information asymmetry,
- iv.) long-term relations and planning horizon,
- v.) integrability with existing information systems and



vi.) simplicity of models and solutions (as far as possible)<sup>2</sup>.

In addition, in my models I assume rational, risk neutral—expected value maximiser—decision makers. Such behaviour can be expected from decision support systems, but human decision makers rarely act upon this “ideal” approach. Considering bounded rationality and risk aversion is therefore a possible further research direction in this field.

As it turned out, the developed models and applied concepts can be used in other industrial sectors, too. My research is continuing with a network situated in the automotive industry which aims at shifting toward a *customise-to-order* approach. Although it basically differs from the consumer goods industry, the fundamental goals and problems are surprisingly similar. Furthermore, such circumstances are being reported also from other industrial fields—like pharmaceutical and high-tech—including increased variety of products, strict standards, high quality requirements and short product life-cycles.

My work therefore fits into the series of Hungarian research in frames of the National Research and Development Projects (NKFP), such as Digital Factories and VITAL; as well as to the EU’s Framework Programmes (FP) for Research and Technological Development.

## 1.4 Organisation of the Dissertation

In accordance with the introductory problem statement, I compiled a *roadmap* to cooperative planning that I followed throughout my research, see Fig. 1.2. The first phase is creating an organisational model of the networked enterprises, which can be used for identifying and stating the operational problems and which serves as a basis for the further work. The second phase is the design of specific planning algorithms, cooperation mechanisms and information sharing concept models; and finally, the developed algorithms can be implemented into real applications. In this work, I concentrate chiefly on the second phase, which requires a formal modelling and problem solving approach.

This thesis is organised into four further chapters. In Chapter 2, I overview the state-of-the-art in the three different fields related to my research: enterprise and supply chain modelling, lot-sizing, and channel coordination. Chapter 3 introduces novel extensions

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<sup>2</sup>Similarly to the *Ockham’s razor* principle. Simplicity is especially important for small and medium enterprises (SMEs) that are unable to apply complex solutions.

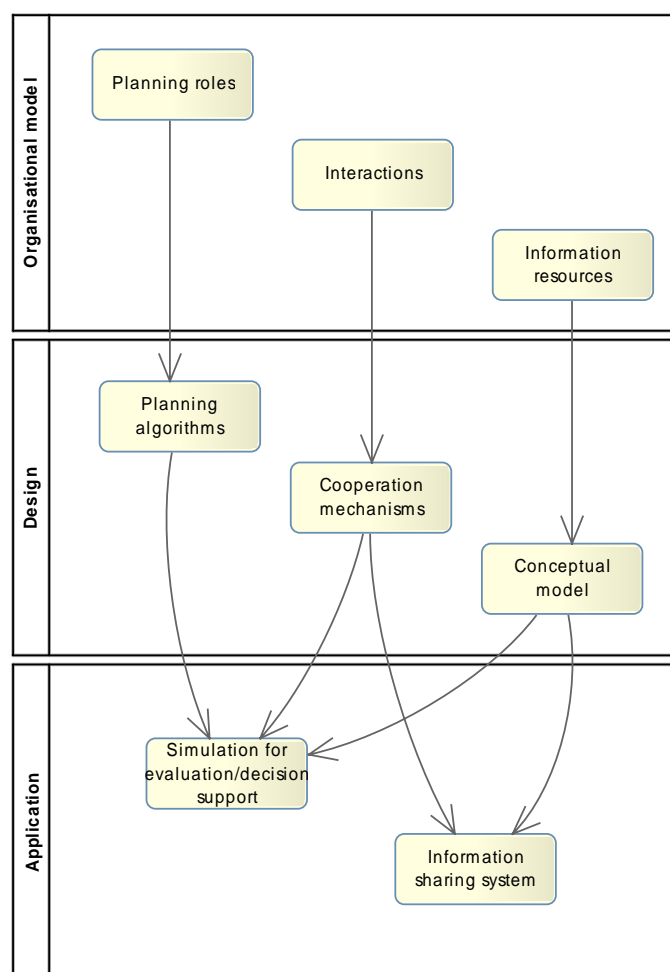


Figure 1.2: Roadmap to cooperative planning.

and solutions of two classical lot-sizing models, namely the newsvendor and the Wagner – Whitin. Chapter 4 studies the previous models in a decentralised setting and presents such compensation contracts that provide channel coordination. Finally, in Chapter 5, I shortly demonstrate some implemented software applications for illustrating the results of my research.

Throughout the thesis I present some numerical examples that were partially derived from real industrial historical data in order to test how the proposed algorithms could perform in practical situations. Other simulations were based on large amount of random data, which give more insights into the general properties of the described solutions.

# Chapter 2

## Literature Review

In this chapter after a general introduction to supply chain management, I briefly overview two fields related to my dissertation: the centralised inventory management models and the coordination models for decentralised supply chains.

### 2.1 Supply Chain Management and Advanced Planning Systems

Manufacturing systems modelling research has proposed several *business process modelling* methodologies and tools over the last few decades [114]. The most common, so-called *semi-formal* techniques integrate easily understandable graphical representations with formal theoretical background. Beyond the models designed specifically for manufacturing systems—e.g., CIMOSA, IDEF3, ARIS—the UML notation originated from object-oriented software technology is also widely used for enterprise modelling purposes. Nowadays, the state-of-the-art approach is the Business Process Modelling Notation (BPMN), which is aimed at being a common understanding for all stakeholders and bridging the communication gap between process design and implementation [115]. BPMN is defined by the Object Management Group (OMG)—the same consortium which is responsible for the UML and CORBA standards among others.

In production networks every enterprise has basically similar tasks, although they differ in the inner organisational structure, processes, complexity, dimensions and realisation. When two enterprises are linked by a supply chain—and in absence of centralised

coordination and lateral connections this is the dominant link—, they join the corresponding processes with each other. In this way, a  $\text{tier}_n$  company is linked only with  $\text{tier}_{n+1}$  (supplier) and  $\text{tier}_{n-1}$  (customer) enterprises, thus every inter-enterprise relationship is bilateral, which is easy to implement and control. In this case, also the cooperation can be only bilateral and the operation of the whole network emerges from these cooperative agreements.

In order to extend the process modelling to the network level, the Supply-Chain Council has developed the Supply-Chain Operations Reference (SCOR) model, which provides a unique framework for linking business processes, performance metrics, best practices and technology features into a unified structure [92]. The SCOR model consists of a chain of companies, each having two parallel, opposite flows of goods: the *source-make-deliver* manufacturing and the *return* reverse logistics flows. Behind these functions, there is a complex planning process which controls these flows, see Fig. 2.1.

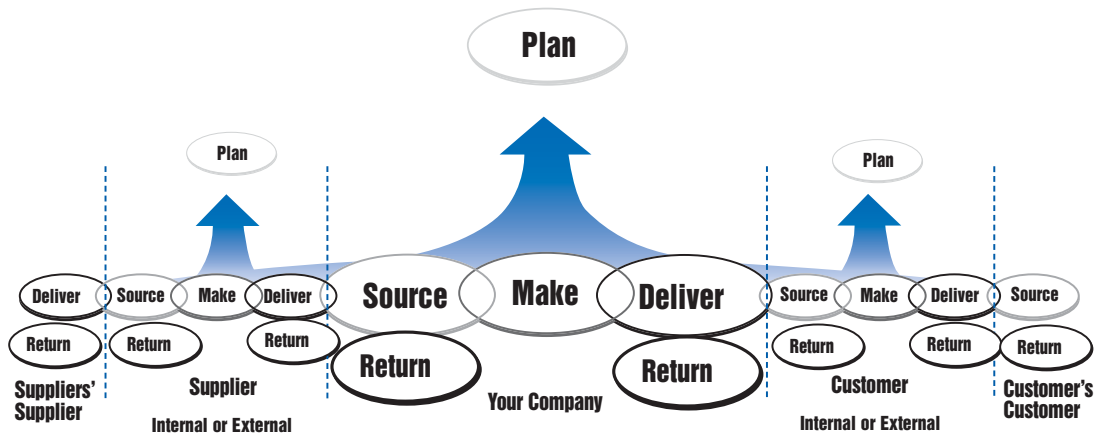


Figure 2.1: Processes of the SCOR model. (Source: [92] p. 3.)

Based on the SCOR model, several reference models have been proposed to unfold the details of the planning process. One of these is the *planning matrix*, which decomposes the planning functions into the commonly used software modules of the so-called the Advanced Planning Systems (APS) [50, 100]. This model—like complex artificial and natural systems often—builds up a hierarchical structure in order to be able to deal with the complexity of problems efficiently. For an analogy, consider the human brain, where the temporal perception and cognitive control tasks compose five different levels: strategic, segmented

tactical, maneuver, short-term integration and synchronisation levels [104]. Each level differs in its temporal frame (granularity) and horizon. They are linked by a feedback-control cycle which defines the relationship between the subproblems. Like the human brain, such models can be regarded pluralistic systems with several different, often redundant or even inconsistent views of the very same environment, which also apply different “algorithms”. The right choice of the appropriate submodel depends on the actual task to be solved.

In the planning matrix the hierarchy is separated into three proactive planning levels: *strategic* (long-term), *tactical* (mid-term) and *operational* (short-term), see Fig. 2.2. Note that this is a hierarchy of tasks and independent from the organisation structure (e.g., pyramidal, network). To continue the analogy, the hierarchy of human cognitive tasks does not imply any hierarchy of the individual neurons.

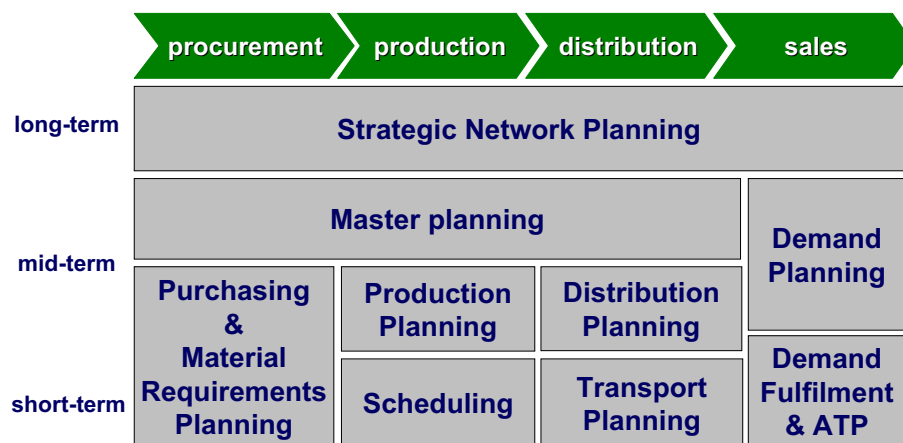


Figure 2.2: Planning matrix<sup>1</sup>. (Source: [100] p. 579.)

The horizon of the strategic level usually covers several years. The goal here is to design the network on the long term, which involves decisions about core competencies, choosing from available suppliers (or sometimes even from customers) and adjust capacities to the planned yield. Since the problem in this level is too complex to be completely modelled and the decisions have consequences in the long run, it is generally supervised by human experts. During the planning process, several possible *frame plan* scenarios are generated and evaluated both with the help of decision support systems and with

<sup>1</sup>ATP is the abbreviation of Available-to-Promise.

negotiations between enterprises. On the tactical level the objective is to plan cost efficient production on a medium term—approximately one year—with one week as the time unit. This level should create demand forecasts for the horizon and make corresponding plans for the yield, production, capacity usage, inventories, supply and distribution. On the operational level the main goal is to realise the medium-term plans. The horizon here is only few weeks long, the planning cycle is daily and the granularity of plans are often less than an hour, but maximum one day. This level should plan the demand fulfilment, transportation, schedule production and ensure necessary materials. Moreover, the delivery of components and products should be planned often in JIT manner. In order to adapt to changing circumstances, the planning tasks on these two latter levels should be performed on a rolling horizon basis, i.e., cyclically the previous plans have to be revised and modified according to the current situation.

The feedback mechanism in the brain is functioning well—even though we do not fully understand it—, but in enterprises the relationships between these levels are often ill-defined and sometimes completely disregarded. Inappropriate communication between planning tasks can lead to suboptimal efficiency in an enterprise, let alone in the whole network. A common example is when the sourcing department on the strategic level regards only the unit price and chooses suppliers with the lowest bids. On tactical and operational levels however, this can cause enormous inventories and logistic costs. Another typical case is when inconsequential models of production planning and scheduling are applied, which lead to either infeasible problems or idle capacities on the operational level [112]. In order to manage these issues, it is not enough to focus on independent planning functions (*structural view*), but they should be regarded as coherent processes (*dynamic view*) [126].

A recently popular and promising approach for modelling complex systems is the *multiagent technology* offering (i) a design metaphor, (ii) technologies for handling interactions and (iii) simulation tools alike [68]. The field of multiagent systems originated from the artificial intelligence research, but while the latter is interested in the behaviour of an intelligent artifact, multiagent systems deal with the emergent behaviour of a society consisting of intelligent agents. An advantage of this approach is that it can include results of a number of other disciplines, e.g., distributed systems, artificial intelligence, game theory and social sciences. Accordingly, it offers several approaches for cooperative distributed problem solving—such as the Contract Net Protocol—but unfortunately most of these solutions assume *benevolence*, i.e., the agents must implicitly share a common goal, which

often does not hold in the reality [122].

Agents provide a natural metaphor in many complex environments, such as the manufacturing and the supply chains [76, 77]. Although multiagent systems are usually less efficient compared to centralised solutions, they are more flexible, easier to understand and implement. In addition, in certain settings centralised approaches are impossible. By now, there is a common understanding that various requirements of networked manufacturing can really be met by autonomous, embodied, communicative and eventually cooperative agents.

Still, the number of deployed multiagent systems that are already running in real industrial environments is surprisingly small. According to a survey made in 2006, even in the “ideal” field of supply chain management, only half a dozen deployed applications could be found that were in everyday use [76]. An important reason for this is that in the behaviour of a multiagent system there is always an element of *emergence* which can be a serious barrier to the practical acceptance of agent-based solutions. Industry needs safeguards against unpredictable behaviour and guarantees regarding reliability and operational performance.

However, the agent metaphor is useful not only for system design, but also for *simulation* with at least three different purposes: (i) simulation for decision making, (ii) simulation for evaluating planning technologies and (iii) simulation for education [88].

## 2.2 Lot-Sizing

While the golden age of inventory research was in the 1950s, the recently changed market conditions have induced paradigm change and the need for new models [16]. In order to remain competitive on global markets, today’s production must be *customer-oriented*, which means that customer demand must be satisfied at high service level with short lead-times. These main requirements—which are specified by the long-term *strategic management*—must be achieved on lower levels by the *tactical* and *operational management*, which need new models and tools for optimisation [24].

As it was previously mentioned, production is typically based on uncertain finished good forecast, which can be prepared using several statistical methods [45] and therefore the demand uncertainty can be expressed in terms of standard deviation. Unfortunately, this information is usually distorted by human factors [37]. In addition, when the production is planned in the medium term, the uncertainty of information disappears or is transformed

to *safety stock* margins, because most practical planning systems cannot handle stochastic problems. The result of the planning process is a discrete plan of production quantities, respecting the capacity and technological constraints. This is also regarded as the basis of the dependent “component consumption forecast” and the economic purchase plan can be determined from this component forecast using appropriate *lot-sizing* methods. This metamorphosis of demand-related information is illustrated in Fig. 2.3.

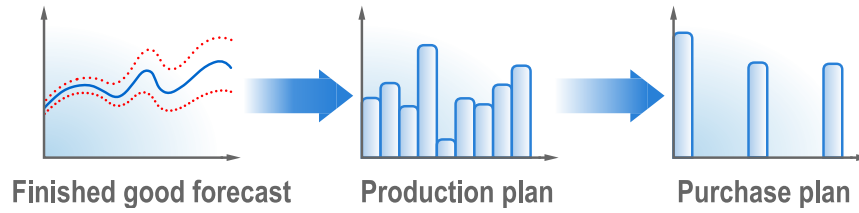


Figure 2.3: The transformation of demand-related information.

The general inventory planning problem can be briefly characterised in the following way: the demand is given in a medium-term *planning horizon* with an uncertain demand and the production (or procurement) should be planned in such a way which satisfies the demand—with a prescribed service level—and involves minimal total cost. The total cost in standard models can include (i) fixed (setup or ordering) cost, (ii) unit price, (iii) inventory holding cost and (iv) shortage penalty. I extended this list with (v) the cost of obsolete inventory, see Chapter 3 for further details.

The lot-sizing models can be classified according to the following criteria [85]:

**Location**<sup>2</sup>. From this viewpoint *single-* and *multi-location* models exist. The latter type includes also transportation planning, which results in a more complex problem.

**Level**. *Single-level* models disregard product structure, while *multi-level* variants include this structure, therefore raw product<sup>3</sup> (RPI), work-in-progress (WIP) and finished goods (FGI) inventories are considered alike. These problems are usually handled with Material Requirements Planning/Manufacturing Resource Planning (MRP/MRP II) methods in practical situations, which often apply some single-level lot-sizing algorithm at each level.

<sup>2</sup>In [85] “facility” is used instead.

<sup>3</sup>According to the nomenclature, RPI is used for component inventory.



**Item.** *Single-item* models focus on the decision about one specific product without considering the relationship among different products. *Multi-item* models however, study these relations, e.g., joint setup costs or required common capacitated resources.

**Capacity.** *Uncapacitated* models assume that all resources are available without limits, while *capacitated* models consider limited resource (or sometimes inventory) capacities .

**Demand.** Demand can be classified along two dimensions. On one hand, it can be *deterministic* or *stochastic*. While in most real situations demand is uncertain, thus considering stochastic demand is closer to the reality, these models are too complex, therefore they are usually transformed into approximate deterministic problems<sup>4</sup>. On the other hand, demand can have *static* or *dynamic* nature, which expresses that the demand quantity is considered to be constant or can be different from time to time.

In addition, models can be differentiated according to further characteristics, such as length of the horizon, backlogging or lead-time properties. The detailed description of the models reviewed below—if no other reference is indicated—can be found in [5, 45, 58, 121].

### 2.2.1 Classical Static Models

One of the first papers about mathematical approaches of lot-sizing introduced the famous *Economic Order Quantity* (EOQ) model [42]. This model assumes deterministic static demand on infinite horizon, which should be satisfied without backlogs, uncapacitated resources and zero lead-time. Two types of costs are considered: the fixed and the unit holding cost. Like in all models without lost sales, due to the fact that all demand should be satisfied, the production cost is constant independently of decision variables, thus can be omitted from the objective function. The optimal trade-off between the fixed and holding costs is given by the EOQ square root formula. Based on this simple model, several variants have been developed by relaxing some assumptions, e.g., considering continuous instead of instantaneous production results in the *Economic Production Lot* (EPL) model.

The widely studied *newsvendor model* omits the assumption of deterministic demand and considers the stochastic problem in one period. In this case, the planner has to

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<sup>4</sup>Theoretically, other approaches for modelling uncertainty—e.g., fuzzy sets—are also conceivable.

determine the lot-size before the demand would be realised. The decision can be based only on a forecast, thus either shortage or excess may occur and both incur a cost that is proportional to the deviation from the lot-size. The linear cost for the shortage is usually explained with loss of goodwill, with penalty engagement or with a more expensive production mode (e.g., overtime, outsourcing). In this case, the optimal lot-size can be expressed using the inverse of the cumulative density function (CDF) of the demand, therefore strictly monotonically increasing CDF yields a unique optimum. Most papers assume the *normal distribution*—which choice is reasoned with the *central limit theorem*—, in spite of its drawbacks: even negative demands have some probability and it forces the probability density function (PDF) to be symmetric [101]. Thus in extreme cases, also negative lot-sizes can be resulted as optimal.

When no fixed cost is involved, it is not rational to apply large lots instead of one-at-a-time fashion production or procurement. This situation is handled by the *base stock model*, which includes stochastic demand and fixed lead-time. The solution of the model determines an optimal *base stock level* respecting a specified *service level* (usually an upper bound on the probability of shortage). The base stock inventory handling policy states that the on-hand inventory plus the open order minus the backlogs should be equal to the base stock level.

When fixed cost is included to the base stock model, we get the  $(Q, r)$  model, where  $Q$  is the lot-size and  $r$  is the *reorder point*: when the inventory level decreases to  $r$ , quantity  $Q$  should be produced or ordered. The advantage of this model is that the lot-size is constant, but it requires, that the demand occurs in one-at-a-time fashion—otherwise the inventory level may not be equal to  $r$ . A variant of this approach is the  $(s, S)$  model, which solves this problem: it proposes that when the inventory level reaches or goes below  $s$ , it should be filled up to  $S$  (*order-up-to-level*). These two models assume *continuous review*, i.e., products can be ordered at any time point. When this does not apply and orders can be given only periodically in specific time points—e.g., once per month—then the  $(R, S)$  *periodic review* policy can be used, which means that in every  $R$  time interval the inventory should be filled up to level  $S$ .

A variant of the problem raised by the *economics of shortage* assumes uncertain supply and focuses on the task of determining the optimal safety stock level; this became known as the *Hungarian inventory control model* [83].

### 2.2.2 Classical Dynamic Models

The basic model and solution of the deterministic dynamic lot-sizing was published in 1958 by Wagner and Whitin [116]. In their paper, a finite, discrete horizon is assumed and the demand is given in each period of the horizon. To minimise the total cost consisting of inventory holding and fixed setup costs, the planned lot-size should be determined for each period, which can be done with a backward induction algorithm in  $\mathcal{O}(n^2)$  time (where  $n$  is the length of the horizon). When the method was invented, that time complexity was considered too high for practical applications, therefore several heuristics were developed—e.g., the Silver–Meal approach [95]—, which are frequently used in MRP systems. Much later, in the early 1990s, three groups of researchers proved independently that this problem can be solved in  $\mathcal{O}(n)$  (even the generalisation of the model in  $\mathcal{O}(n \ln n)$ ) time; further discussion and references can be found for example in [5].

The deterministic lot-sizing problem also has several extensions and variants. For example, Zangwill studies the situation when backlogs—negative inventories—are allowed, where the inventory holding and the shortage costs are combined into a piecewise concave inventory cost [127]. This model can be solved also with a dynamic programming algorithm, and in case of linear production cost, its time complexity is only the double of the no backlogging case's.

The more realistic versions of the deterministic lot-sizing problems including e.g., capacity constraints and sequence-dependent setups are usually NP-hard, thus exact solutions of such problems—even with efficient specialised algorithms—are usually applicable only on relatively small instances, but not in industrial sizes [4, 54]. Therefore numerous approximation algorithms and heuristics are applied to provide quasi-optimal solutions or to consider practically important special cases [47].

The basic dynamic model for the stochastic case was developed in 1959 by Herbert Scarf [91]. The solution of this model applies a *stochastic dynamic programming* formulation for determining  $(s_t, S_t)$  reorder points and order-up-to levels for each period  $t$  on the horizon. Scarf pointed out that the recursive formulation of the expected total cost satisfies the so-called *K-concavity* property, and this can be used to prove the optimality of the solution. However, the existing algorithms for determining  $(s_t, S_t)$  levels are complex and time consuming.

Since the stochastic problem is hard to solve and the component forecasts usually come

from production planning system which are unable to handle stochastic problems, almost always deterministic demand is considered in practical applications. In order to deal with uncertainty, the following approaches can be used:

**Safety buffers.** The stochastic demand can be turned into a deterministic one by considering its expected value plus a *safety stock* based on the standard deviation. Furthermore, *safety capacities* and *safety lead-times* are also frequently applied.

**Rolling horizon planning.** Demand forecasts are generated periodically in such a way that two consecutive forecasts of length  $n$  are overlapping in  $n - 1$  periods. In the overlapping period, the newer forecast updates the values of the previous forecast based on more recent information.

**Time fences.** Time fences constrain (or define) the flexibility of the demand in particular intervals of the horizon [45]. One widely used variant of this concept is called *frozen period*, which fixes the demand for the first few periods, thus it cannot be modified later.

Combining these approaches provides useful heuristics for handling dynamic lot-sizing problems. However, choosing parameters for these models—such as length of the horizon [113] and length of the frozen period [64]—greatly affects the approximation of the optimal solution.

Recently, on markets of customised mass products, the short life-cycle of the products also had to be considered. One of the few papers studying this problem is [51], which extends Scarf's  $(s_t, S_t)$  model with stochastic product life-cycle, transforms it into a deterministic form and solves the resulted problem approximately with a modified Wagner–Whitin algorithm. This is akin to my approach described in Section 3, where also some important differences between the two models will be shown.

The models reviewed above consider only the problem of managing inventory at only one location<sup>5</sup>. The seminal paper of Clark and Scarf extended Scarf's original  $(s_t, S_t)$  model to the cases of sequential supply chains and tree-shaped networks [20]. In spite of some strict assumptions of this model justified by the original military background of the research—such as a central planner of the network and lack of fixed setup cost at all but

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<sup>5</sup>In [127] a deterministic multi-echelon model is also described, but it was published after the Clark and Scarf paper.

the lowest level—it prompted the modern inventory theory of supply chain management. Several channel coordination papers presented in the next section are clearly influenced by this multi-echelon inventory model.

## 2.3 Channel Coordination

In order to demonstrate the problem of decentralised planning, I present a simple example here based on [94]. Let us consider a supply chain with one supplier and one customer serving a market with uncertain demand for the end-product. The customer creates a demand forecast (the density function) and then orders components from the supplier. The profit of the supplier becomes linear in the order quantity, consisting of the difference between the wholesale price and the production cost:

$$\text{supplier's profit} = (\text{wholesale price} - \text{production cost}) * \text{order quantity}.$$

On the other hand, the profit function of the customer is more complex: ordering too much results in obsolete inventory, which can have some salvage value (even negative). Thus the customer's profit can be expressed in the following way:

$$\begin{aligned} \text{customer's profit} = & \text{Min}(\text{order quantity}, \text{demand}) * \text{retail price} \\ & + \text{Max}(\text{order quantity} - \text{demand}, 0) * \text{salvage value} \\ & - \text{order quantity} * \text{wholesale price}. \end{aligned}$$

Since the demand is stochastic, the customer wants to maximise its expected profit. Fig. 2.4 shows the profits of both parties—with some definite parameters—as well as their sum, i.e., total supply chain profit. Note that this latter is independent from the wholesale price.

As the figure shows, the customer's optimal order quantity is 700, which yields € 42,080 profit on the customer's and € 59,500 on the supplier's side, i.e., totally € 101,580. However, ordering 900 would result in € 112,770 total profit, which is € 11,190 more than in the previous case, clearly not the optimal decision for the customer, though.

The goal of *channel coordination* is to achieve the optimal efficiency on the supply chain level—e.g., by inspiring the customer with discounts to increase order quantity—, so thus the extra profit can be shared between the partners. This can be possible, because as

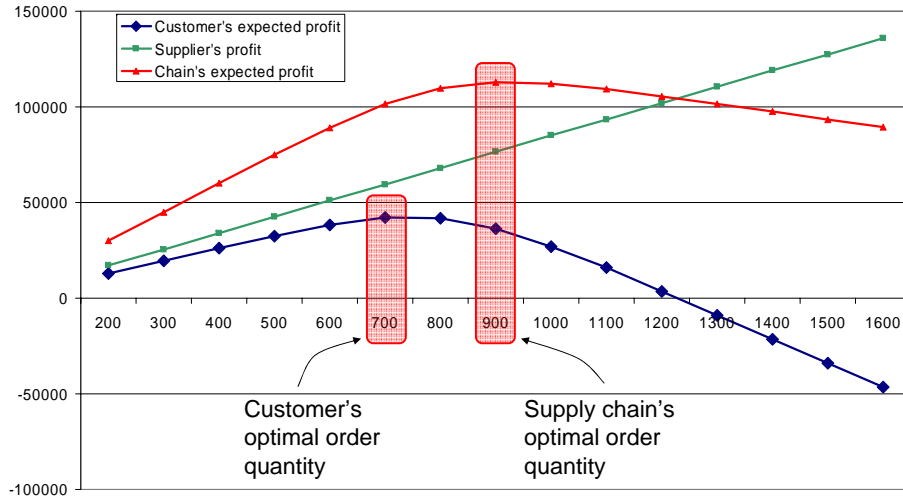


Figure 2.4: Example for suboptimal channel performance. (Source: [94])

we have seen, the total supply chain profit is independent from the wholesale price which modifies only the individual profit functions.

In the above example all parameters were considered to be common knowledge, therefore both partners can compute the expected profits, optimal order sizes and share the surplus fairly. In real situations this is usually not true; some parameters (e.g., the production cost or the demand forecast) are *private information* of the partners, which makes the coordination problem much more difficult. In such cases there is an *information asymmetry* which can be resolved by *information sharing*, but the partners can have incentives to share untruthful information in order to maximise their profits. Although channel coordination is achievable also in these cases with special contracts, they usually cannot guarantee arbitrary profit sharing.

The general method for studying coordination mechanisms consists of two steps. At first, one assumes a *central decision maker* with complete information who solves the problem. The result is a so-called *first-best solution* which provides bound on the obtainable system-wide performance objective. In the second step one regards the *decentralised problem* and designs such a contract protocol that approaches or even achieves the performance of the first-best solution.

### 2.3.1 Classification of the Models

An early review of supply chain contracts can be found in [107]. In this paper supply chain management is defined as the extension of the classic multi-echelon inventory theory with the ideas of decentralisation (multiple decision makers), asymmetric information and new manufacturing and logistic paradigms, such as delayed differentiation and outsourcing. The study also provides a taxonomy for classifying contracts, which consists of eight different contract types. The authors pointed out however, that these classes are not disjoint. Another classification can be found in [66], where the different contract types are categorised according to the *leader*, i.e., the partner, who designs them. This taxonomy does also not define disjoint classes. Therefore I present below a set of aspects which generalises these taxonomies by allowing classification along multiple viewpoints. This approach resembles the scheme presented in [12], albeit that review focuses on models where the market price is a decision variable, I consider exogenous prices instead. The different viewpoints can be defined as follows:

**Horizon.** Most of the related models consider either *one-period horizon* or *two-period horizon with forecast update*. In the latter, the production can be based on the preliminary forecast with normal production mode or on the updated forecast with emergency production, which means shorter lead-time, but higher cost. These models are extensively discussed e.g., in [93]. In addition, the horizon can consist of *multiple periods* and it can be even *infinite*.

**Number of products.** Almost all models regard only one product. Handling more products in gross is necessary in case of technological or financial constraints, like capacity or budget limits.

**Demand characteristic.** Generally, the demand is considered *stochastic*, although some models assume *deterministic* demand.

**Risk treatment.** In most of the models the players are regarded to be *risk neutral*. This means that they intend to maximise their expected profit (or minimise their expected costs). However, some studies regard *risk averse* players who want to find an acceptable trade-off considering both the expected value and the variance of the profit. Risk aversion is widely applied in the financial researches.

**Shortage treatment.** The models differ in their attitude towards stockouts. Most authors consider either *backlogs*, when the demand must be fulfilled later at the expense of providing lower price or *lost sales* which also includes some theoretical costs (e.g., loss of goodwill, loss of profit, etc.). Some models include a *service level constraint*, which limits the occurrence or quantity of expected stockouts. Even the *100% service level* can be achieved with additional or emergency production (e.g., overtime, outsourcing) for higher costs.

**Parameters and variables.** This viewpoint shows the largest variations in the different models. The main decision variable is quantity-related (*production quantity, order quantity, number of options*, etc.), but sometimes *prices* are also decision variables. The parameters can be either constant or stochastic. The most common parameters are related to costs: *fixed (ordering or setup) cost, production cost* and *inventory holding cost*. These are optional; many models disregard fixed or inventory holding costs. There exist numerous other parameters: prices for the different contracts (see details later), salvage value, shortage penalty, lead-time, etc.

**Basic model.** Most of the one-period models apply the *newsvendor model*. On two-period horizon, this is extended with the possibility of two production modes. On a multiple period horizon the *base-stock*, or in case of deterministic demand the *EOQ* models are the most widespread.

**Technological constraints.** Generally, technological constraints are completely disregarded in the coordination literature. However, in real industrial cases resource capacity, inventory or budget constraints can be relevant.

**Solution technique.** In the basic models—and most papers study these—the optimum of the objective function can be determined with simple algebraic operations. However, in case of more complex models and further constraints, more powerful solution techniques may be required, like mathematical programming, dynamic programming, constraint programming, and, in the last resort, heuristics or metaheuristics.

**Number of players.** We focus on the *two-player* case and call the players *supplier* and *customer*. There are also extensions of this simple model: the *multiple customers with correlated demand* and the *multiple suppliers with different production parameters*. Multi-echelon extensions are also conceivable, however, sparse in the literature.



**Information structure.** Some papers study the *symmetric information* case, when all of the players know exactly the same parameters. This approach is very convenient for cost sharing, since all players know the incurring system cost. The *asymmetric* case, when there is an *information gap* between the players is more realistic, but poses new challenges. The asymmetry typically concerns either the cost parameters or the demand forecast. The demand and the forecast are usually considered to be qualitative, limited to only two possible values: high and low. Some papers also study the *cost of information sharing*, but this problem has decreasing importance nowadays, thanks to the widespread electronic data interchange solutions.

**Decision structure.** The decision making roles of the players depend on the specified decision variables. However, there is a more-or-less general classification in this aspect: forced and voluntary compliance. Under *forced compliance* the supplier is responsible for satisfying all orders of the customer, therefore he<sup>6</sup> does not have the opportunity to decide about the production quantity. Under *voluntary compliance*, the supplier decides about the production quantity and he cannot be forced to fill an order. This latter is more complex analytically, but I agree with the conclusion of [8]: “[...] forced compliance violates the original premise for studying supply chain contracting: that no one firm controls all supply chain actions. [...] Firm commitments are undesirable because they restrict the system’s ability to respond to evolving information.” Even so, several papers assume that the supplier decides about the price and then the customer decides the order quantity.

**Game theoretic model.** From this point of view the models can take *cooperative* or *non-cooperative* approaches [57, 87, 105]. The cooperative approach studies, how the players form coalitions therefore these models are usually applied on the strategic level of network design. Other typical form of cooperative games involves some *bargaining* framework, e.g., the Nash bargaining model. The non-cooperative approaches usually apply the sequential *Stackelberg game* model, where one of the players, the *leader* moves first and then the *follower* reacts. Both cases—the supplier or the customer as the Stackelberg leader—are widely studied in the literature. In case of information asymmetry, a similar sequential model is used and it is called *principal – agent* setting.

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<sup>6</sup>According to the widespread notation in the literature, I refer customer as *she* and the supplier as *he*.

The study of the long-term supply relationship as a *repeated game* is a promising new research field [84].

**Contract type.** This aspect also provides many possibilities, some widespread variations are briefly described below. Besides, there exist several combinations and customised approaches, too.

**Two-part tariff.** In this case the customer pays not only for the purchased goods, but in addition a fixed amount called *franchise fee* per order. This is intended to compensate the supplier for his fixed setup cost.

**Sales rebate.** This contract specifies two prices and a quantity threshold. If the order size is below the threshold, the customer pays the higher price, and if it is above, she pays a lower price for the units above the threshold.

**Quantity discount.** This resembles to the sales rebate contract, but there is no threshold defined, but the customer pays a wholesale price inversely proportional to the order quantity.

**Buyback/return.** With these types of contracts the supplier offers that he will buy back the remaining obsolete inventory at a discounted price. This supports the sharing of inventory risk between the partners. A variation of this contract is the *backup agreement*, where the customer gives a preliminary forecast and then makes an order less or equal to the forecasted quantity. If the order is less, she must also pay a proportional penalty for the remaining obsolete inventory. Buyback agreements are widespread in the newspaper, book, CD and fashion industries.

**Quantity flexibility.** In this case the customer gives a preliminary forecast and then she can give fixed order in an interval around the forecast. Such contracts are widespread in several markets, e.g., among the suppliers of the European automotive industry.

**Revenue sharing.** With revenue sharing the customer pays not only for the purchased goods, but also shares a given percentage of her revenue with the supplier. This contract is successfully used in video cassette rental and movie exhibition fields. It can be proved, that the optimal revenue sharing and buyback contracts are equivalent, i.e., they generate the same profits for the partners.

**Options.** The options contracts are originated from the product and stock exchange.

With options contract, the customer can give fixed orders in advance, as well as buy rights to purchase more (call option) or return (put option) products later. The options can be bought at a predefined *option price* and executed at the *execution price*. This approach is a generalisation of some previous contracts.

In Fig. 2.5 and Fig. 2.6 the above aspects are presented in a structured way.

### 2.3.2 One- and Two-Period Models

An extensive discussion of the coordination contracts can be found in [9]. This review mostly focuses on the newsvendor model, but several extensions of the basic problem are also considered. Another, more general and recent survey can be found in [63]. In the following two subsections—which are divided along the information structure—I overview some previous works related to my research, in order of their appearance.

#### Models with Symmetric Information Structure

Barnes-Schuster et al. [2] study a two-period model with forecast update. The authors propose an option model and show that it is a generalisation of the most common coordination contracts. The demand can always be satisfied with emergency production, therefore no shortage occurs at the supplier's side. On the customer's side however, if her order and executed options do not cover the demand, she has to pay penalty. There are several differences between this work and my model: Barnes-Schuster et al. disregard fixed costs and information asymmetry, and in addition, their model assumes that the customer limits the flexibility of the supply chain with the orders and options.

Cheng et al. [15] study a one-period newsvendor problem and propose a Stackelberg game with option contracts. In their model, the supplier is the Stackelberg leader who decides about prices, then customer orders and buys options before the selling period. Only one production mode is considered and the compliance is forced: the supplier must produce all quantity for satisfying firm orders and bought options.

Cachon [10] studies a one-period newsvendor model, where the production must be realised before the selling period due to the long lead-times. The paper compares two different contracts: *push* and *pull*. In the former, the customer submits a “prebook order”

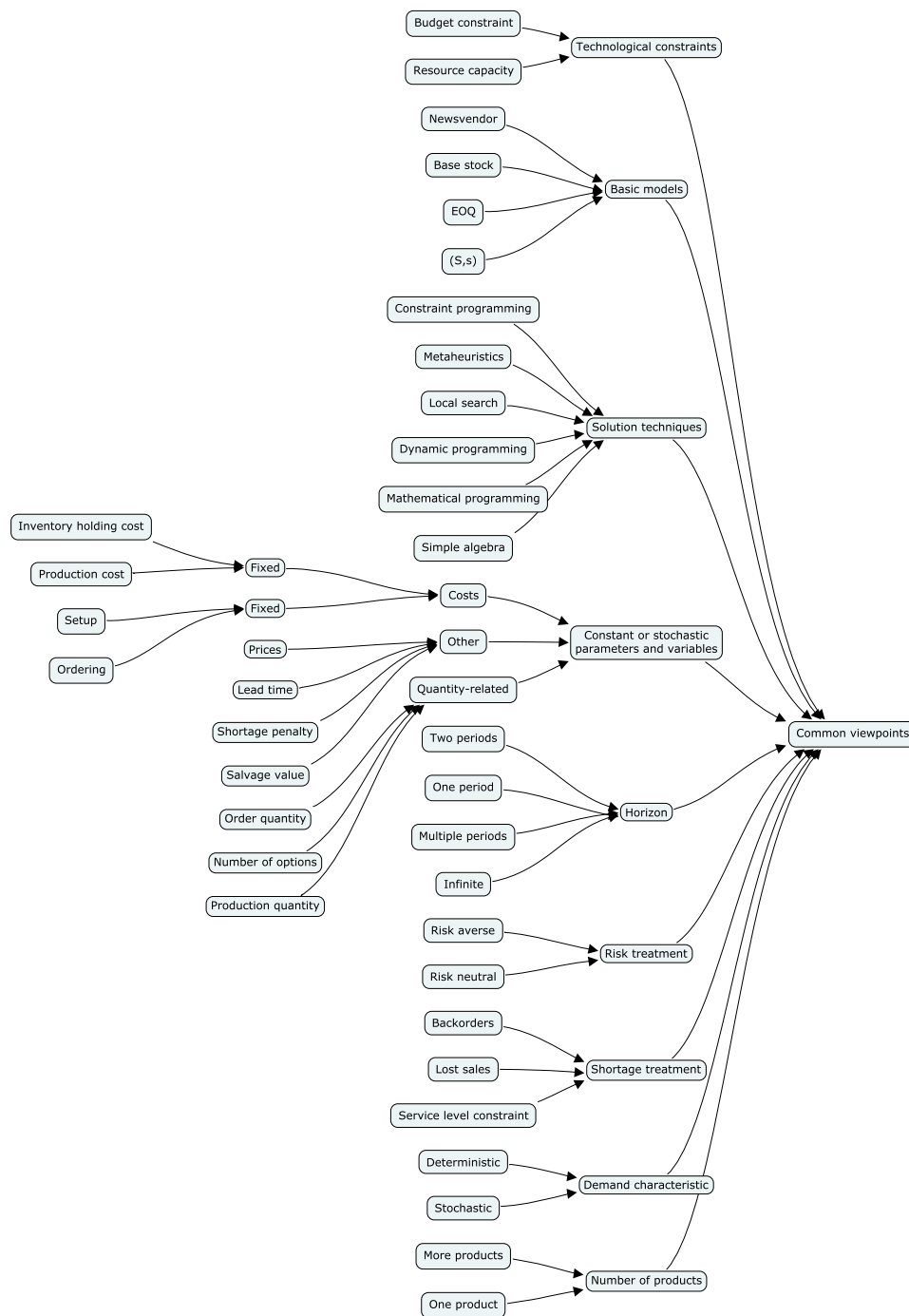


Figure 2.5: Overview of common aspects of coordination models.

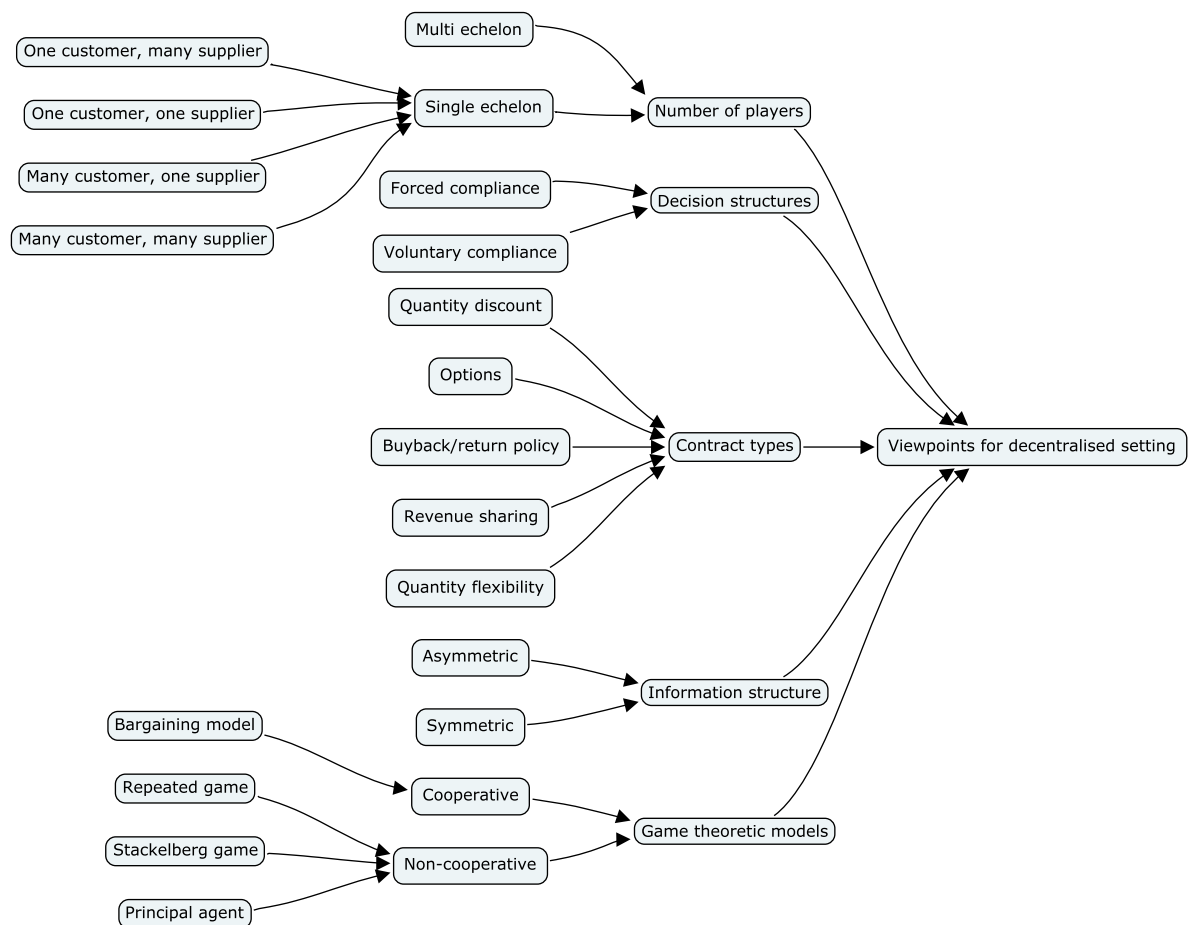


Figure 2.6: Overview of aspects of decentralised coordination models.

and the supplier must fulfil it. In the latter, the supplier must produce before getting any order and the customer submits an “at-once order” only at the beginning of the selling period. In fact, Cachon proposes a combination of these approaches: the *advance-purchase discount* contract. Here the customer submits both prebook and at-once order, but this latter with a higher wholesale price. It is shown that this contract achieves channel coordination, and by adjusting the two wholesale prices the overall profit can be arbitrary allocated. The main disadvantage of the model (pointed out by the author) is the lack of fixed cost. If an additional shipment cost is introduced for the at-once order, the combined contract no longer coordinates the channel.

Wang et al. [117] analyse the decisions of a supplier and a customer whether or not to join a business-to-business (B2B) e-market with annual subscription and per unit transaction fees. They consider markets with long lead-times, therefore the customer has to order in advance which can cause either obsolete inventory or lost sales. The authors study the benefits of using a return policy, which is shown to be appropriate for coordinating the channel and under reasonable conditions it performs better on e-markets than on traditional ones.

Lee and Chu [61] present the most common practical business relations, where the supplier is responsible for the inventory handling. They formulate models for both the traditional and the new business relations based on the newsvendor problem and they also extend the model with service level guarantee. The authors derive necessary and sufficient conditions, under which it is worth changing the traditional business model to the new one for both players. However, they do not determine optimal supply chain performance and thus they do not study channel coordination.

Liu et al. [67] study a two-period problem, where the supplier decides the prices and the customer generates orders. The paper proposes a combined contract with two wholesale prices and buyback policy and shows, that this can coordinate the channel and can allocate supply chain profit arbitrarily. This study also disregards fixed cost and allows shortages.

Koulamas [53] considers the revenue sharing contract and models it with a Stackelberg game, where the supplier defines the price. In the revenue sharing contract, the customer orders for a unit price and also shares a percentage of the realised profit after the sales. This contract is proved to coordinate the channel and strictly increases the expected profit of the customer. The author also derives conditions which guarantee that the expected profit of the supplier will not decrease.

Chen et al. [14] also study a two-period problem, where the supplier decides about the production quantity in the first period and the customer decides the order quantity in the second. The authors propose a *bi-directional return policy*, where the customer partially compensates the supplier for the overproduction in the first period, while the supplier buys back obsolete inventory at the end of the second period. This contract also coordinates the channel and allows arbitrary profit allocation, but disregards fixed costs and allows shortages.

Sabbaghi et al. [86] present a capacity constrained newsvendor problem and prove the surprising result that in this case even the simple linear wholesale price can coordinate

the channel. The model is then generalised also to the multiple suppliers setting. This study provides promising perspectives for the research of capacity constrained coordination, which should be analysed also with asymmetric information.

### Models with Asymmetric Information Structure

These problems are generally studied by the *economy with asymmetric information*, a discipline spawned from game theory [87]. Following the traditional nomenclature, when the information asymmetry affects a decision variable (i.e., the action of the agents cannot be observed), this raises a *moral hazard problem* and when the asymmetry affects an external parameter, it is called an *adverse selection problem*.

The main model is called the *principal – agent model*, where the decisions are sequential. When the player with the incomplete information is the leader, it is called *screening model*. In this case, the aim of the leader is to design such a *menu of contracts*, from which the follower's rational choice is optimal for the leader. On the other hand, when the well-informed player is the leader, it is a *signalling model*. Such models are used, when the leader can offer such a contract that guarantees the truthful revelation of its private information. This should be in the interest of the leader, because without truthful revelation the so-called *adverse selection* may cause *market failure* (i.e., no deal at all) which is suboptimal for both players.

The generalisation of screening and signalling models is the terrain of *mechanism design* (*inverse game theory* or *game engineering*). Here the main goal is that given a system-wide optimal strategy tuple, one must design such a game where the given strategy tuple is an equilibrium.

In the supply chain contracting theory the asymmetry is related either to the demand forecast or to the cost factors. Both screening and signalling models are studied for such problems. In the practice however, the menu-of-contracts approach is rarely used.

Cachon and Lariviere [8] study the two-period case, when the information asymmetry affects the qualitative demand forecast, which can be either high or low. The customer who is better informed, signals the expected demand and the supplier must reserve capacity. Here, the customer has obviously an incentive to inflate its forecast. Both forced and voluntary compliance are modelled, and although forced compliance is more efficient in this case, it is not preferred due to its centralisation of decisions.

Li et al. [62] model the case when also the qualitative demand forecast is known only by the customer, but the supplier is the leader. The supplier offers a menu of contracts consisting of firm orders, options and combined contracts. The authors identify cases when the combined contract is dominant. One further speciality of this model is that the price of the end product is stochastic.

Çınar and Bilgiç [19] study within the newsvendor framework the effect of asymmetric information on the inventory handling cost of the customer. The supplier is the leader, who offers a menu of firm order and option contracts. They assume forced compliance, show the existence (but not the uniqueness) of the equilibrium and derive the conditions for channel coordination. This paper also contains an excellent literature review.

Ülkü et al. [108] consider the one supplier and multiple customers case. They assume that all demand can be satisfied with additional emergency production, but this means higher unit production cost without any fixed cost. The authors examine whether the supplier or the customers have to take the responsibility and consequences of the decision. They conclude, that the situation where the decision and the risk is at the supplier's side (so-called *risk pooling*) is always desirable for the customers, but can be inefficient on the system level. They also study such a contract which helps avoid the so-called *double marginalisation* effect.

Chu and Lee [17] study a newsvendor problem where the supplier decides about inventory levels, while the customer is better informed about the expected market demand. They consider the cost of information sharing, but assume, that if any information is shared, it is truthful without further incentives. They do not focus on different contract and channel coordination, but the conditions, under which information sharing is rational at all.

Zhou [128] analyses the case, when the random demand depends on the retail price as well as on a parameter known only by the customer. In this setting, the supplier is the leader who can offer different types of price discount contracts. Although these contracts cannot coordinate the channel, the author examines the *channel efficiency*, i.e., the approximation of the first-best solution. The paper also contains comparisons of the supplier's and customer's profits.

Lutze and Özer [70] consider the situation where the customer's shortage cost (or service level) is a private information, but has a known distribution. The described model assumes static stochastic demand, no setup cost and allows backlogs. The authors study the be-



haviour of the supply chain applying *promised lead-time contracts*—i.e., when the supplier guarantees on-time delivery of arbitrary orders after a given lead-time—and compare the performance both with the full information and the centralised control case.

### 2.3.3 Models with Longer Horizon

The studies of the longer horizon problems usually apply either the EOQ or the base-stock model. As far as I know, the dynamic, rolling horizon version of forecast sharing is not yet studied. Furthermore, I have found only a few papers that consider asymmetric information and models the problem mostly as a repeated newsvendor games. However, in order to gain an insight into the existing solution approaches, some related models are reviewed below in order of their appearance.

Lee and Whang [60] study an infinite-horizon case where the periodic demands are identically and independently distributed. They assume that unsatisfied demand is backlogged and disregard fixed costs, therefore the first-best solution is a multi-echelon version of the base-stock model. The paper emphasises the importance of performance measurement schemes and proves, that a compensation scheme consisting of four parts—transfer pricing, consignment, backlog penalty and shortage reimbursement—can coordinate the channel in this case.

Cachon [6] considers an infinite horizon model with exponentially distributed demand-arrival times, lost sales and applies the base-stock solution. The traditional one-period contract types are studied in the paper and they are proved to be unable to coordinate the channel, as well as every other one-parameter contracts. Cachon proposes combining inventory sharing with a lost sale transfer for channel coordination and shows, that this will increase the supply chain inventory level in order to decrease the number of shortages.

Cachon and Fischer [7] consider a single-supplier, multiple customers market with static stochastic demand and backlogging. They found that the value of information sharing in this case is limited and did not study further the possibilities of coordination. They concluded that information sharing is more beneficial when the demand is fluctuating and the capacities are practically unconstrained thus supply can be flexible adapted to demand.

Corbett [21] studies two different problems with static stochastic demand-arrival times, setup cost and backlogs applying the  $(Q, r)$  model. In one case, the information asymmetry is about the supplier's setup cost and in the other, about the backlogging cost of the

customer. For both problems, the author examines screening solutions, i.e., the principal offers a menu of contracts. In both cases, the principal must know a priori distribution about the private information of the agent. The results of the analysis show that the optimal menus of contract can decrease inventory levels in a supply chains, but cannot coordinate the channel.

Caldentey and Wein [11] consider a full information model with Poisson-distributed demand-arrival times, no fixed setup costs, voluntary compliance and backlogs, which setting is handled by base-stock policy. They show that in this situation a cost-sharing agreement can coordinate the channel, furthermore it increases the service level of the system comparing to the case without contracting.

Xu [123] regards the dynamic version of the multi-period supply chain model, where the demand of each period can have different distributions, but all knowledge is symmetric. The author proposes a cancellation contract, i.e., the customer can cancel a portion of the order for a linear penalty. A dynamic programming algorithm is used to derive the optimal policy in the finite horizon case. The infinite horizon case with static demand distribution and discounted cost is also considered, which can be solved with the help of the Bellman equation. Although cancellation contract cannot coordinate the channel, it is easy to implement and beneficial for both parties.

Chu et al. [18] model an assembly system with two suppliers providing complementary components with different lead-times. The paper assumes that there is no fixed cost and the demand can be backlogged, therefore base-stock model is appropriate for this situation. The possible Stackelberg games are examined and the authors also propose a channel coordinating mechanism for this specific system.

Gupta and Weerawat [41] study the supply problem for customised mass products, where the demand-arrival times follow a Poisson distribution. They consider three variants of the problem: one follows the assumptions of the base-stock model, one offers guaranteed service level (in terms of lead-time) and the last one assumes lost sales. The authors propose a two-part revenue-sharing scheme (with a linear and a non-linear part), which is proved to coordinate the channel.

Sarmah et al. [89] review existing coordination approaches applying the EOQ model and a quantity discount contract. The authors point out an important limitation of the existing models concluding “to make coordination successful, faith between the parties and true revelation of information is necessary which model builder should take into consideration

in their model in future”.

Kwak et al. [56] model long-term replenishment contracts assuming VMI applying the  $(Q, r)$  policy. In their model the supplier has full information, while the customer is not familiar with the cost parameters of the supplier. The authors assume exogenous wholesale price, thus the decision variables are only the reorder point ( $r$ ), the order quantity ( $Q$ ) and the length of the horizon. The paper contains a *buyer-driven* model, where the customer decides about all parameters, and a *supplier-driven* model, in which the supplier is responsible for determining the order quantity. The studied long-term contract cannot coordinate the channel, but its performance is compared to the centralised case in order to analyse the efficiency gain.

Yao et al. [124] study the deterministic static demand case with fixed costs, therefore the EOQ model is applied. They compare the traditional supply with the VMI setting and examine the benefits of the latter both for the supplier and for the customer. Channel coordination contracts are not considered in this paper.

Ren et al. [84] models the newsvendor setting on a longer horizon as a repeated game. The demand is considered to be *scaled random* (i.e., random high or random low), whose distribution is known by the supplier only with an additional white noise, therefore the customer has to share her forecast with him. The authors propose measuring the forecast quality and compensating the supplier for the inappropriate forecast. However, due to the stochastic nature of the demand, this measurement does not reflect the customer’s truthfulness and furthermore, it is suboptimal for both parties. Instead of this approach, the authors present a review strategy, which means credibility checking in a longer time interval with a tolerance zone. This coordinates the channel by inspiring the parties to prefer long-term benefits to short-term interests. Another important element of their model is that they consider the short product life-cycles by including a (static) probability of run-out into their model.

Sarmah et al. [90] study the coordination problem assuming static, deterministic demand and setup cost which is handled by the EOQ model. The main aim of the paper is to provide guaranteed profit targets for the parties and share the surplus above that level. Two different coordination mechanisms are presented, the credit option and the discount contracts, which are shown to be equivalent under certain conditions. The authors point out in the conclusion that disregarding the problem of truthful information sharing makes the model unadaptable in practical situations, therefore future research have to take aim

at the challenge of relaxing the full information assumption.

Yue and Raghunathan [125] models the effects of applying returns policy in a repeated purchasing situation. In this setting, the supplier is the leader, he has to determine the wholesale price, then the customer decides about the retail price and order quantity. The realised demand is stochastic—both high-low qualitative and continuous cases are considered—, price sensitive and unknown to the supplier. The paper does not study whether the returns policy coordinates the channel or not; only compares it to the traditional purchasing without returns.

Frascatore and Mahmoodi [36] consider a repeated newsvendor game with symmetric information, static demand and no setup cost, where the obsolete buffer of one period is not lost, but can be consumed later. In their model, the supplier is the Stackelberg leader, who determines the wholesale price, then the customer orders. The authors first introduce a long-term contract and proves, that when the length of the contract horizon increases, the channel performance approaches the optimal (coordinated) behaviour, but never achieves it. They also presents a penalty contract, where the supplier must pay penalty in case of shortages. This second approach can coordinate the channel, but provides zero profit for the supplier.

## Chapter 3

# Extended Models of the Lot-Sizing Problem

Planning tasks of enterprises are usually categorised according to their horizons into three levels: long-term, medium-term and short-term, as it was described in Section 2.1. In this chapter I chiefly focus on the medium-term planning problem, but at the end, I examine its relationship with the short-term level.

When make-to-order production is not possible, manufacturers have to plan their productions based on forecast. These forecasts are usually created from historical statistics—which process is sometimes called as “guesstimation” or “driving by the rear-view mirror”, referring to its uncertain results. I consider that the component demand is derived from the production plan, since supply must be aligned with the production instead of the finished good sales. Components should be produced in large batches in order to decrease the setup cost, but this comes together with an increase of inventory levels and the risk of obsolete inventory. The solutions should provide acceptable trade-off between these conflicting objectives. My novel models described below consider single-item and no inventory or capacity limits.

### 3.1 Newsvendor Model with Possible Emergency Production

The general form of the *one-period decision problem* is the following: the decision maker has to determine the value of a variable  $q$ , then  $c(q, \xi)$  cost arises, where  $\xi$  is a random variable with known distribution. The risk neutral decision maker intends to minimise its expected cost. In the context of inventory planning of perishable goods, this model is called the newsvendor model (see Section 2.2).

The standard model disregards the setup cost: it considers per unit left over cost—if the demand is below the produced quantity—and per unit shortage cost—if the demand exceeds the produced quantity. However, when the inventory is filled by manufacturing instead of ordering, then the setup cost must be included in the calculation. In my model, service level has the highest priority, hence it follows that the manufacturer has to satisfy all demand. If the produced quantity is below the actual demand, then it can only be satisfied by an *emergency production* which also involves an additional setup. Thus, due to the incurring setup cost, I modified the newsvendor model in a non-trivial way. To the best of my knowledge, all previous variants of the newsvendor model disregard such fixed cost because of the complexity of the resulted problem. The main parameters of my model are as follows [34]:

- $c_s$  setup cost,
- $c_p$  production cost per unit,
- $\phi$  probability density function (PDF) of the demand,
- $\Phi$  cumulative density function (CDF) of the demand,
- $\xi$  realised demand,
- $s$  minimal production quantity,
- $q$  production quantity (decision variable) and
- $Z$  total cost (objective function).

#### 3.1.1 The Optimal Lot-Size

I consider continuous consumption, therefore certain amount must be produced at the beginning in order to avoid shortage, i.e.,  $q > 0$ . Practically, I assume  $q \geq s$ , where  $s > 0$  is an appropriate minimal production quantity, wherewith the probability of the obsolete

inventory  $\Pr(\xi < s)$  is acceptably small. I also assume that if the originally produced quantity is not enough, then at the time of the emergency production the final demand quantity is already known.

The model considers fixed setup cost, where the eventual emergency production comes also together with an extra setup. This way the total cost will consist of four parts:

- i.) the certain setup cost:  $c_s$ ,
- ii.) the production cost for satisfying the actual demand:  $c_p\xi$ ,
- iii.) the value of obsolete left over products<sup>1</sup>:  $c_p \max(q - \xi, 0)$  and
- iv.) the cost of additional setup:  $c_s\delta(\xi - q)$ , where

$$\delta(\xi - q) = \begin{cases} 0 & , \text{ if } \xi - q \leq 0 \\ 1 & , \text{ if } \xi - q > 0 \end{cases} . \quad (3.1)$$

The expected total cost in function of the production quantity becomes:

$$\mathbb{E}[Z(q)] = c_s + c_p\mathbb{E}[\xi] + c_p\mathbb{E}[\max(q - \xi, 0)] + c_s\mathbb{E}[\delta(\xi - q)]. \quad (3.2)$$

**Proposition 3.1** *The derivative of the expected total cost function is*

$$\frac{d\mathbb{E}[Z(q)]}{dq} = c_p\Phi(q) - c_s\phi(q). \quad (3.3)$$

**Proof.** Using the definition of the expectation value one can express:

$$\mathbb{E}[\max(q - \xi, 0)] = \int_{-\infty}^q (q - x)\phi(x)dx = q\Phi(q) - \int_{-\infty}^q x\phi(x)dx \quad (3.4)$$

and

$$\mathbb{E}[\delta(\xi - q)] = \int_q^{\infty} \phi(x)dx = 1 - \Phi(q). \quad (3.5)$$

From these expressions the statement of the proposition follows.  $\square$

Determining the root of this derivative function is not as easy as in the standard model, because it is not invertible in the general case, therefore I focus on a special probability

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<sup>1</sup>The production cost of the remaining inventory is considered to be lost.

distribution. The normal distribution leads to a problem where it is hard to determine the root analytically, hence I regard the *logistic distribution*, whose PDF is similar to the PDF of the normal distribution, but has longer tails. In Fig. 3.1 the PDFs of the logistic and normal distributions with the same expectation value and variance can be seen; with solid and dashed curves respectively.

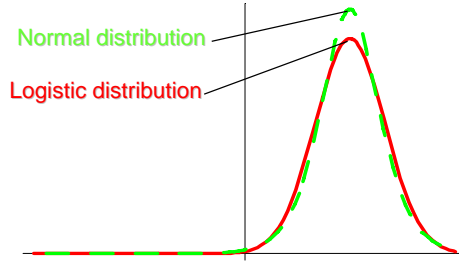


Figure 3.1: Comparison of the PDFs.

The PDF of the logistic distribution with parameters  $m$  and  $b$  is

$$\phi(x) = \frac{e^{\frac{m-x}{b}}}{b \left(1 + e^{\frac{m-x}{b}}\right)^2}, \quad (3.6)$$

and it has the expectation value and variance  $m$  and  $\sigma^2 = \frac{1}{3}\pi^2 b^2$ , respectively. It can be seen that the logistic distribution has a simpler form than the normal distribution, but otherwise they have similar properties.

**Theorem 3.2** *There exists an optimal lot-size  $q^*$  which minimises the expected total cost iff  $b < \frac{c_s}{c_p}$ . In this case, the optimal lot-size is unique:*

$$q^* = m - b \ln \left( \frac{bc_p}{c_s - bc_p} \right). \quad (3.7)$$

**Proof.** Substituting the PDF and CDF of the logistic distribution into Proposition 3.1 we get:

$$\frac{dE[Z(q)]}{dq} = c_p \frac{1}{1 + e^{\frac{m-q}{b}}} - c_s \frac{e^{\frac{m-q}{b}}}{b \left(1 + e^{\frac{m-q}{b}}\right)^2}. \quad (3.8)$$



This should equal to zero, therefore simplifying the equation leads to

$$e^{\frac{m-q^*}{b}} = \frac{bc_p}{c_s - bc_p}. \quad (3.9)$$

After taking the logarithm of this equation  $q^*$  can be expressed as Eq. 3.7, which has a real solution iff the argument of the logarithm is positive. Since both  $b$  and  $c_p$  are positive, this yields the condition  $b < \frac{c_s}{c_p}$ .

This  $q^*$  is a local minimum place of the expected total cost, as it follows from the *second derivative test*:

$$\frac{d^2\mathbb{E}[Z(q^*)]}{dq^2} = \frac{c_p(c_s - bc_p)^2}{bc_s^2} > 0. \quad (3.10)$$

Furthermore, the derivative 3.8 is a continuous function with only one root, thus  $q^*$  is also the global minimum place.  $\square$

This optimal lot-size gives a balance between the risk of obsolete inventory and the additional setup. It can be either more or less than the expectation value, depending on the variance and cost parameters, see Sect. 3.4.1 for a specific example.

### 3.1.2 Discussion

As an illustration, taking a particular industrial example with  $m = 65553$ ,  $c_s = 45331$  and  $c_p = 3.29$ , the shape of the expected total cost function can be seen in Fig. 3.2.

The percentage numbers express the relative deviation ( $r$ ) from the expected demand, i.e., the  $b$  parameter is determined as  $b = \frac{\sqrt{3}rm}{100\pi}$ . When the deviation is low (e.g.,  $r = 10\%$ ), then an incorrect lot-size causes significant raise in the expected total cost. The shape of this curve can be explained in the following way:

- i.) there is a unique optimum,  $q^*$ , given by Eq. 3.7,
- ii.) decreasing  $q$  starting from the optimum increases the probability of the additional setup cost, however, the expected obsolete inventory is decreasing, therefore the function is bounded, and
- iii.) increasing  $q$  starting from the optimum decreases the expected additional setup cost, but the expected obsolete inventory increases arbitrarily.

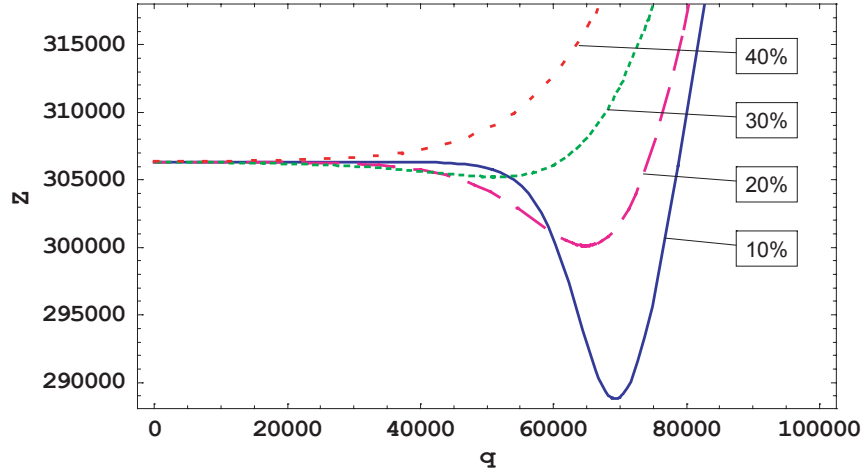


Figure 3.2: Deterioration of the cost function.

As the diagram shows, the minimal expected total cost grows together with the relative deviation. The curve with  $r = 40\%$ —where  $b \geq \frac{c_s}{c_p}$ —is *degenerated* in the sense that it has no positive optimum; the  $s$  minimal quantity should be produced to avoid immediate shortage.

Using this model one can also express the *cost for being present on an uncertain market*. If the demand was certain, the total cost would be  $c_s + mc_p$ , without additional setup and obsolete left over. The value of

$$\Delta Z = \mathbb{E}[Z(q^*)] - (c_s + mc_p) \quad (3.11)$$

thus can be interpreted as the *cost of uncertainty*. Fig. 3.3 demonstrates this kind of cost, using the same  $m$ ,  $c_s$  and  $c_p$  parameters as in the previous example and let  $r$  range in the  $(0, 30\%]$  interval.

## 3.2 Wagner – Whitin Model with Run-Out

In this section I study the dynamic lot-sizing problem on a medium-term horizon. It is an exogenous property of the modern markets that the product life-cycles are short and uncertain, thus demand for a product can suddenly cease and this *run-out* produces obsolete inventory. This happens more frequently in case of the non-standardisable (customised)

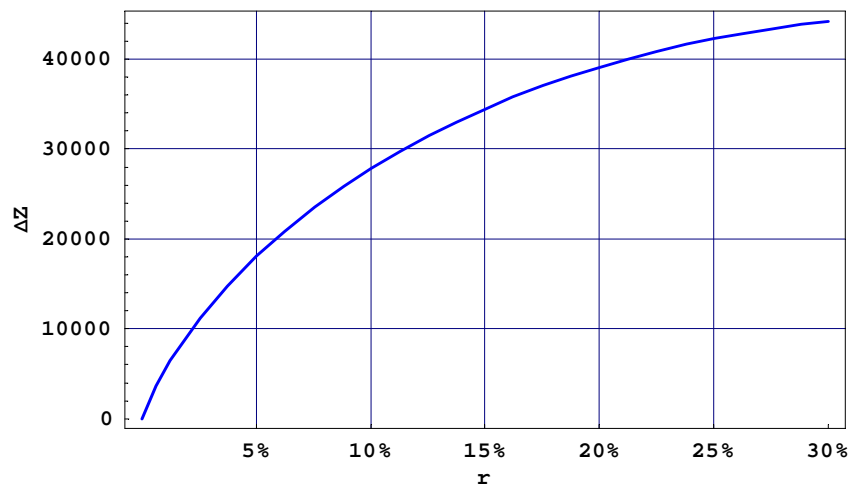


Figure 3.3: Cost of the uncertain market.

packaging materials, where *design changes* are common. The run-out must be taken into account, because obsolete products cause significant financial loss for the manufacturers let alone the waste of environmental resources. This phenomenon is illustrated in Fig. 3.4 with data coming from a real Enterprise Resource Planner (ERP) system. Note that in what follows I consider the total inventory level and disregard various components of the inventory.

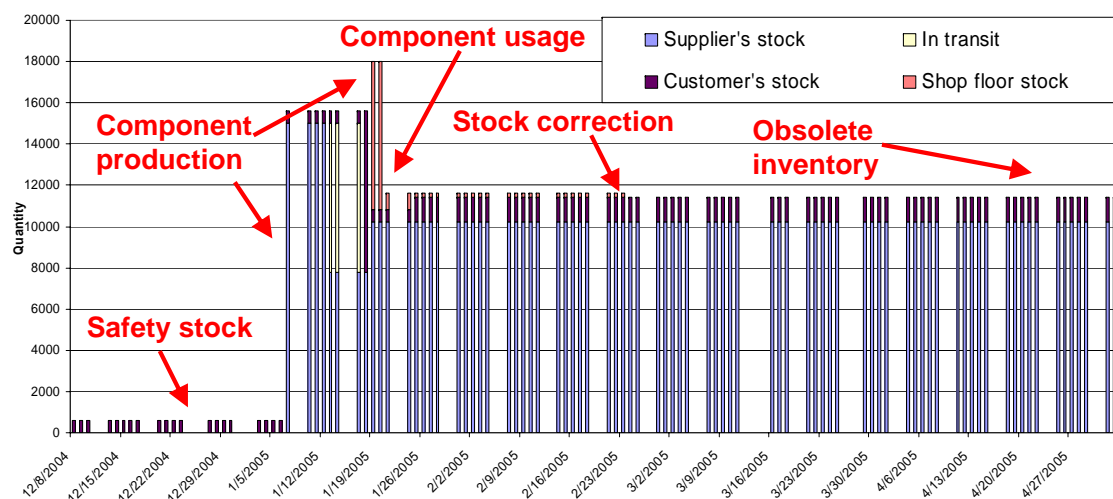


Figure 3.4: An example for the run-out and obsolete inventory.

To handle this problem, I build upon the standard Wagner–Whitin model and generalise it to the case when the demand can cease in an arbitrary period of the planning horizon [29]. The properties of the model denoted with WWr according to the aspects presented in Section 2.2 are as follows: single-echelon, single-item, discrete time scale, finite horizon, no backlogs, deterministic demand forecast, possibility of run-out, one period lead-time, no capacity constraint and minimising expected cost (setup, inventory holding and obsolete inventory). The parameters in my model are as follows:

$n$	length of the horizon,
$F_1, \dots, F_n \geq 0$	forecasted demand,
$c_s$	setup cost,
$h$	inventory holding cost per piece per period,
$c_p$	production cost per piece (cost of obsolete inventory per piece),
$\eta \in \{1, \dots\}$	period of the run-out <sup>2</sup> (random variable),
$x_0, \dots, x_n$	production quantities (decision variables) and
$I_0, \dots, I_n$	inventory at the end of the periods (auxiliary variables).

I also consider one period production time, i.e., the quantity produced in one period is available in the next one.

### 3.2.1 The WWr Model

Without loss of generality it can be assumed that the inventory is zero initially. I also assume that  $F_1 > 0$  (otherwise the decision can be delayed). These assumptions imply that production is necessary in the actual period (period 0), i.e.,  $x_0 > 0$ .

The goal is to construct such a production plan  $(x_0, \dots, x_n)$  which satisfies the forecasted demand in every period and minimises the expected cost consisting of the setup cost in period 0 plus the expected cost on the horizon. This latter is the sum of the possible setups plus the inventory holding costs (assuming linear consumption within a period) if no run-out happens, and the cost of obsolete inventory in case of run-out of each period of the horizon. Formally:

$$\mathbb{E}[Z] = c_s + \sum_{i=1}^n \left( \Pr(\eta > i) \left( c_s \delta(x_i) + h \left( I_{i-1} - \frac{F_i}{2} \right) \right) + \Pr(\eta = i) c_p I_{i-1} \right), \quad (3.12)$$

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<sup>2</sup>This can also be interpreted as the length of the remaining product life.

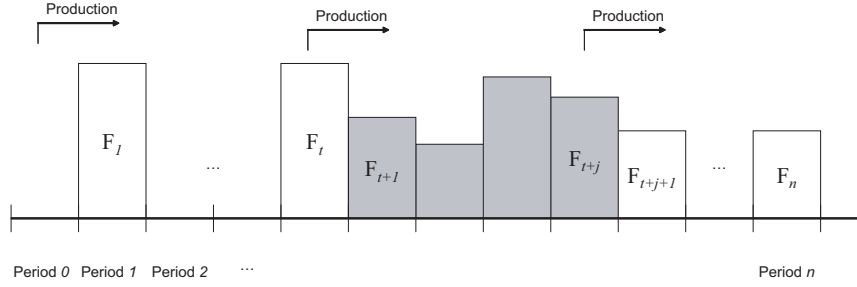


Figure 3.5: Forecasted quantities on the planning horizon of WWr.

where

$$\delta(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}. \quad (3.13)$$

The stochastic programming formulation of the model is as follows:

$$\min \mathbb{E}[Z] \quad (3.14)$$

s. t.

$$I_0 = x_0 \quad (3.15)$$

$$I_i = I_{i-1} - F_i + x_i \quad (i \in \{1, \dots, n\}) \quad (3.16)$$

$$I_{i-1} \geq F_i \quad (i \in \{1, \dots, n\}) \quad (3.17)$$

$$x_i \geq 0 \quad (i \in \{0, \dots, n\}). \quad (3.18)$$

Eq. 3.15 expresses that we start with an empty inventory, i.e., the inventory at the beginning of the horizon equals the production in period 0. Eq. 3.16 states that the planned inventory at the end of a period equals to the inventory at the beginning of the period plus the production minus the demand of that period. Eq. 3.17 guarantees that the demand can be always satisfied from inventory. Eq. 3.18 constrains the production quantities to be non-negative.

The following basic property of this problem—whose proof is analogous to the one which can be found in most textbooks—will be useful:

**Proposition 3.3 (Wagner–Whitin property)** *Given a  $(x_0, \dots, x_n, I_0, \dots, I_n)$  optimal solution of the (3.14–3.18) stochastic program. Then*

- i.)  $I_n = x_n = 0$
- ii.)  $(I_{i-1} - F_i)\delta(x_i) = 0 \quad (i \in \{1, \dots, n\})$ .

Hence the search space can be narrowed to the feasible solutions with the Wagner–Whitin property. The latter complementarity condition states that either the inventory level would be zero at the end of period  $i$  and thus the production is allowed, or the inventory level would be positive and the production quantity is zero. This equation can be interpreted as the lack of *speculative motives*, i.e., it is always preferable to produce at a later period rather than producing earlier and holding stock.

### 3.2.2 Algorithm for Solving the WWr Model

Let us assume that we produce in period  $t$  for the next  $j$  periods. This implies that

- i.) the product has not run out until period  $t$ , which has a probability  $\Pr(\eta > t)$  and
- ii.)  $I_t = x_t = \sum_{i=t+1}^{t+j} F_i$  (from the Wagner–Whitin property).

Then the expected inventory holding cost in period  $i \in \{t+1, \dots, t+j\}$  can be expressed as

$$H(t, j, i) = \Pr(\eta > i \mid \eta > t)h \left( I_{i-1} - \frac{F_i}{2} \right) = \Pr(\eta > i \mid \eta > t)h \left( \sum_{k=i}^{t+j} F_k - \frac{F_i}{2} \right), \quad (3.19)$$

and the cost of expected obsolete inventory is

$$O(t, j, i) = \Pr(\eta = i \mid \eta > t)c_p I_{i-1} = \Pr(\eta = i \mid \eta > t)c_p \sum_{k=i}^{t+j} F_k. \quad (3.20)$$

The expected cost of producing in period  $t$  for  $j$  periods including the setup cost is therefore

$$C_{tj} = c_s + \sum_{i=t+1}^{t+j} (H(t, j, i) + O(t, j, i)). \quad (3.21)$$

Exploiting the Wagner–Whitin property, the optimal cost for periods  $\{t, \dots, n\}$  ( $Z_t$ ) can be computed with the following recursion:

$$Z_n := 0 \quad (3.22)$$

$$Z_t := \min_{j \in \{1, \dots, n-t\}} \{C_{tj} + \Pr(\eta > t+j \mid \eta > t)Z_{t+j}\} \quad (t \in \{0, \dots, n-1\}). \quad (3.23)$$

This expresses that we are searching a particular  $j$  value having the following property: if we produce for the next  $j$  periods and for the rest of the horizon we achieve the already known minimal cost—as long as no run-out happens during the  $j$  periods—then the cost on the  $\{t, \dots, n\}$  horizon will be minimal. The optimal  $x_0, \dots, x_n$  production quantities achieving the minimal cost thus can be determined in two phases:

- i.) determine the possible  $x_t$  values, i.e., if we produce in period, we have to produce exactly  $x_t$  and
- ii.) determine the periods in which we have to produce; in other periods  $x_t$  must be set to zero.

The first phase can be computed by backward induction using recursion (3.22-3.23). Let us define the following functions that will be used in the algorithm:

$$\begin{aligned}
 V(t, j) &= C_{tj} + \Pr(\eta > t + j \mid \eta > t) Z_{t+j} \\
 &= c_s + \frac{1}{\Pr(\eta > t)} \sum_{i=t+1}^{t+j} \left( \Pr(\eta > i) h \left( \sum_{k=i}^{t+j} F_k - \frac{F_i}{2} \right) + \Pr(\eta = i) c_p \sum_{k=i}^{t+j} F_k \right. \\
 &\quad \left. + \Pr(\eta > t + j) Z_{t+j} \right)
 \end{aligned} \tag{3.24}$$

is the argument of Eq. 3.23, and

$$Q(t, j) = \sum_{i=t+1}^{t+j} F_i \tag{3.25}$$

is the production quantity in period  $t$  for the next  $j$  periods. The WWr algorithm can be seen in Fig. 3.6.

The *compute*( $t, j$ ) procedure computes  $V(t, j)$  and  $Q(t, j)$  functions and stores them in variables  $V$  and  $Q$ . For each  $t$  the algorithm determines the optimal  $j$  value, which necessitates the computation of  $V(t, j)$ —which can be done in linear time—thus all in all the running time of the algorithm is  $\mathcal{O}(n^3)$ .

### 3.2.3 Special Probability Distributions of Run-Out

My original formulation of the WWr model considered that in the beginning of each period the product runs out with probability  $p$ , i.e., the run-out probability has *geometric*

---

```

variable  $V$ 
variable  $Q$ 

procedure WWR
   $Z_n \leftarrow 0$ 
   $x_n \leftarrow 0$ 
  for  $t \leftarrow n - 1$  to  $0$  by  $-1$  do                                      $\triangleright$  First phase
    call  $\text{compute}(t, 1)$ 
     $Z_t \leftarrow V$ 
     $x_t \leftarrow Q$ 
    for  $j \leftarrow 2$  to  $n - t$  do
      call  $\text{compute}(t, j)$ 
      if  $V < Z_t$  then
         $Z_t \leftarrow V$ 
         $x_t \leftarrow Q$ 
      end if
    end for
  end for

   $Q \leftarrow x_0$                                                           $\triangleright$  Second phase
  for  $t \leftarrow 1$  to  $n$  do
     $Q \leftarrow Q - F_t$ 
    if  $Q = 0$  then
       $Q \leftarrow x_t$ 
    else
       $x_t \leftarrow 0$ 
    end if
  end for
end procedure

procedure COMPUTE( $t, j$ )
   $V \leftarrow 0$ 
   $Q \leftarrow 0$ 
  for  $i \leftarrow t + j$  to  $t + 1$  by  $-1$  do
     $Q \leftarrow Q + F_i$ 
     $V \leftarrow V + \Pr(\eta > i) * h * (Q - F_i/2) + \Pr(\eta = i) * c_p * Q$ 
  end for
   $V \leftarrow V + \Pr(\eta > t + j) * Z_{t+j}$ 
   $V \leftarrow c_s + V / \Pr(\eta > t)$ 
end procedure

```

---

Figure 3.6: The WWR algorithm.



*distribution.* In the previous section I presented the generalised version enabling arbitrary distribution. The geometric distribution is inappropriate when the run-out probability should grow the farther we look into the future. For this reason, in [73] the Poisson-distribution was proposed to model the run-out probability. In what follows, I compare three different distributions, which are possible alternatives for characterising run-out. Table 3.1 summarises some properties of the distributions, while Fig. 3.7 shows an example of the run-out probabilities in each period.

Table 3.1: Comparison of the distributions.

	Geometric ( $p$ )	Uniform ( $N$ )	Poisson ( $\lambda$ )
$\Pr(\eta = i)$	$(1 - p)^{i-1}p$	$\begin{cases} \frac{1}{N} & \text{if } i \leq N \\ 0 & \text{if } i > N \end{cases}$	$\frac{\lambda^{i-1}}{(i-1)!}e^{-\lambda}$
$\mu$	$\frac{1}{p}$	$\frac{N+1}{2}$	$\lambda + 1$
$\sigma^2$	$\frac{1-p}{p^2}$	$\frac{(N-1)(N+1)}{12}$	$\lambda$
$\Pr(\eta > i)$	$(1 - p)^i$	$\begin{cases} 1 - \frac{i}{N} & \text{if } i \leq N \\ 0 & \text{if } i > N \end{cases}$	$1 - e^{-\lambda} \sum_{k=0}^{i-1} \frac{\lambda^k}{k!}$

Note that the  $V(t, j)$  and  $Q(t, j)$  functions can be determined using a recursion:

$$V(t, 1) = c_s + \frac{1}{\Pr(\eta > t)} \left( \Pr(\eta > t+1)h\frac{F_{t+1}}{2} + \Pr(\eta = t+1)c_p F_{t+1} + \Pr(\eta > t+j)Z_{t+j} \right), \quad (3.26)$$

$$V(t, j+1) = V(t, j) + \frac{F_{t+j+1}}{\Pr(\eta > t)} \left( h \sum_{i=t+1}^{t+j} \Pr(\eta > i) + \frac{h}{2} \Pr(\eta > t+j+1) + c_p \sum_{i=t+1}^{t+j+1} \Pr(\eta = i) + \Pr(\eta > t+j+1)Z_{t+j+1} - \Pr(\eta > t+j)Z_{t+j} \right) \quad (3.27)$$

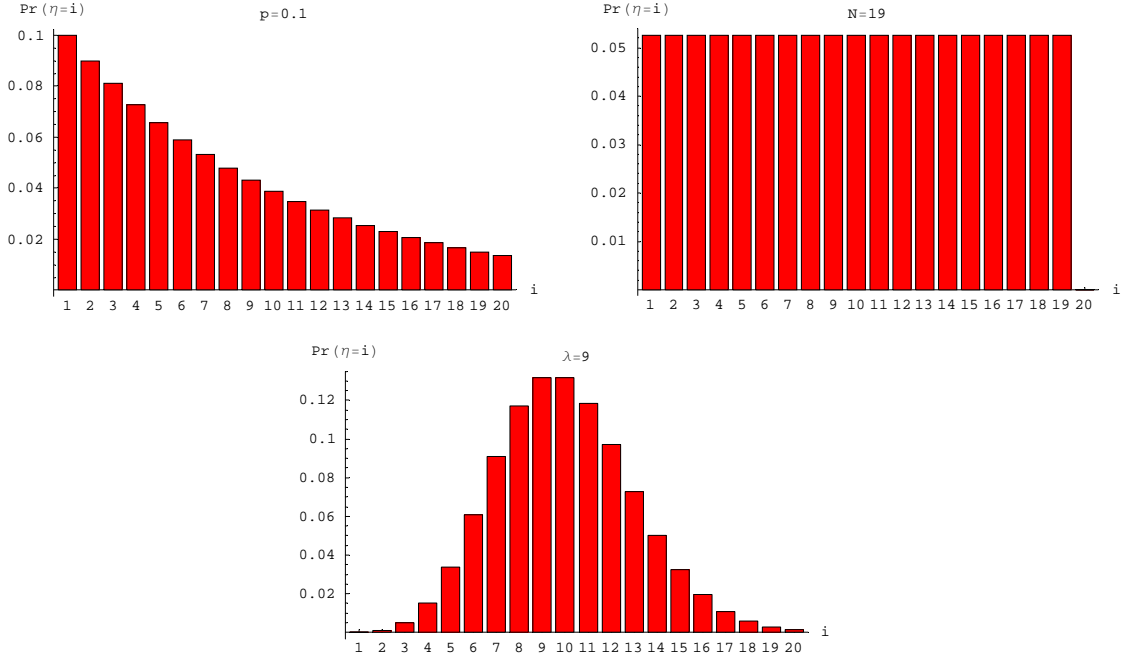


Figure 3.7: Example of the geometric, uniform and Poisson distributions with the same expected value.

and

$$Q(t, 1) = F_{t+1}, \quad (3.28)$$

$$Q(t, j+1) = Q(t, j) + F_{t+j+1}. \quad (3.29)$$

Using geometric and uniform distributions,  $\text{compute}(t, j)$  procedure can be simplified by calculating  $V$  and  $Q$  based on their previous values in constant time, therefore the algorithm can be modified to run in  $\mathcal{O}(n^2)$  time.

### 3.2.4 Heuristics

The more remote a forecast is, the more uncertain it is<sup>3</sup>—this reasonable hypothesis was confirmed by extensive analysis of historical industrial data. Based on this observation, the first idea was to disregard the less trusted remote forecasts and plan only the starting

<sup>3</sup>This is usually referred to as the *third law of forecasting* [45].

segment of the horizon. Therefore I developed two heuristic methods which minimise the expected average cost—both per time unit and per piece—in the first segment of the horizon [32, 109]. These approaches turn the problem into a continuous form, but in other aspects, they resemble to the Silver–Meal heuristic [95]. They assume that the lot-size can cover an arbitrary front fragment of the forecast horizon which minimises the expected average cost either by the length of the expected consumption period or by the produced quantity, see Fig. 3.8.

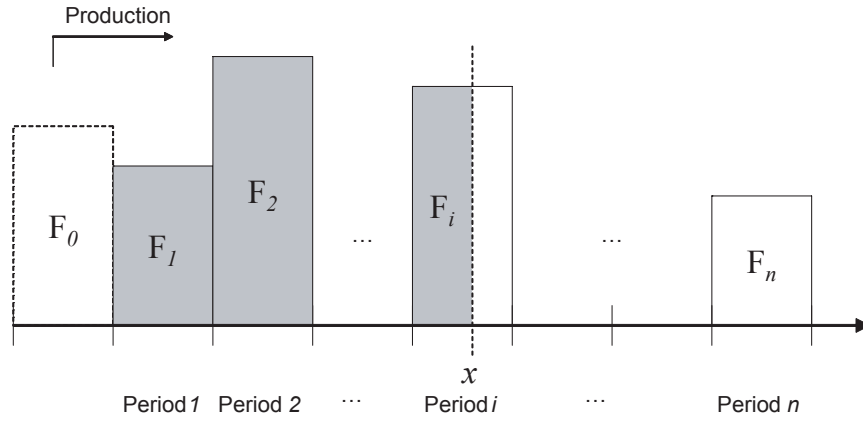


Figure 3.8: Planning horizon of the heuristics.

I modified the notations here to match this approach: let  $S_k := \sum_{l=1}^k F_l$  be the accumulated forecast of the first  $k$  periods and  $q(x) := S_{i-1} + yF_i$  is the production quantity, where  $i := \lfloor x \rfloor + 1$  and  $y := \{x\}$  (here  $\lfloor x \rfloor$  means the integer, and  $\{x\}$  the fractional part of  $x$ ). This expresses, that we produce all quantities of the first  $(i - 1)$  periods, and the  $y$  proportion of the demand of period  $i$ . The expected decrease of the inventory level can be seen in Fig. 3.9.

If one does not consider run-out and assumes linearly decreasing inventory within a period, then the expected inventory holding cost in the first  $l$  ( $l < i$ ) time unit is:

$$H(l, x) = h \sum_{k=1}^l \left( q(x) - S_{k-1} - \frac{F_k}{2} \right), \quad (3.30)$$

where  $q(x) - S_{k-1}$  is the opening inventory of the period  $k$ , and  $\frac{F_k}{2}$  expresses the linearly consumption within the period. Hence, the expected storage cost with run-out can be

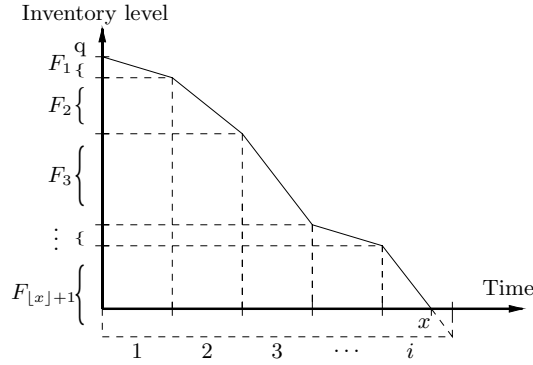


Figure 3.9: Expected inventory level.

expressed as:

$$H(x) = \sum_{k=1}^i (\Pr(\eta = k)H(k-1, x)) + \Pr(\eta > i) \left( H(i-1, x) + h \frac{y^2 F_i}{2} \right). \quad (3.31)$$

If the product is still saleable in period  $i$ , both the storage cost of the first  $(i-1)$  periods and the storage cost of the remaining fraction<sup>4</sup> incur. The cost of the obsolete inventory can be computed in a similar manner:

$$O(x) = c_p \sum_{k=1}^{i-1} (\Pr(\eta = k)(q(x) - S_{k-1})) + c_p \Pr(\eta = i)yF_i. \quad (3.32)$$

Thus we obtain piecewise continuously differentiable average cost functions

$$A_x(x) = \frac{c_s + H(x) + O(x)}{x} \quad (3.33)$$

and

$$A_q(x) = \frac{c_s + H(x) + O(x)}{q(x)}. \quad (3.34)$$

They can be minimised by searching through the roots of the their derivative and the borders of the periods.

As it has turned out, the heuristics have several disadvantages comparing with the WWr algorithm: (i) they cannot estimate the number of setups on the horizon, (ii) disregarding a part of the available information can lead to significant inefficiency and (iii) they sometimes

<sup>4</sup>The quantity  $yF_i$  is consumed only during  $y$  period.

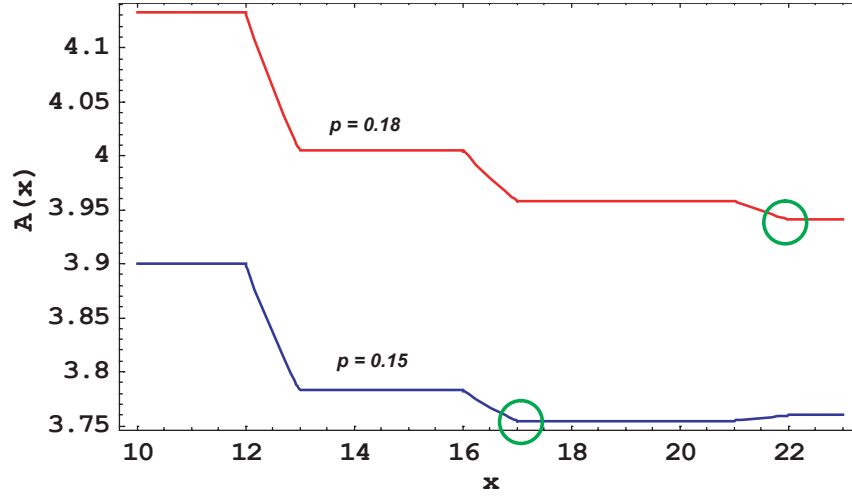


Figure 3.10: Anomaly of the heuristics.

behave unreasonably: increasing the probability of run-out can cause higher lot-size, an example can be found in Fig. 3.10.

The lines show average costs in case of different run-out probabilities considering geometric run-out distribution. If  $p = 0.15$ , producing the demand of 17 time units would minimise the average cost, while the higher  $p$  suggests 22 time units. According to the experiments, this phenomenon occur rarely, and when run-out probability is relatively high ( $p > 0.13$ ). Note that such anomalies are known in the field of operations management; see e.g., the *nervousness syndrome* in the widely used MRP method, when a decrease in the demand leads to an infeasible situation [45].

### 3.3 Practical Considerations

In this section I briefly mention some additional topics, which cannot be considered scientific results, but are essential in practical applications of the previous models. Firstly, I propose a framework for determining which model should be applied in specific situations. The presented simple rules were created by studying actual industrial problems and discussing them with industrial experts—which also confirmed the practical applicability of the theoretical models. Secondly, I regard the problem when no estimations for the product life-cycle are available. Finally, I briefly present the role of safety stocks in allowing 100%

service level and in coupling medium- and short-term planning levels.

### 3.3.1 Choosing the Appropriate Method

There are two fundamentally different situations: (i) the fact of the run-out and its date are known a priori and (ii) run-out can occur with a certain probability, but no further details are known. The first run-out situation also covers two cases: the *rolling* and the *instant change*. When the change is rolling, the stored products can be still sold, hence obsolete inventories are not created. However, at instant change on one hand there is the risk of producing obsolete inventory, while on the other hand the necessity of additional production due to incorrect forecasts. In the second case the possibility of run-out must also be considered, since a greedy inventory policy can lead to significant obsolete inventories.

In case of the unknown run-out date, both the WWr and the heuristic policies can be applied. One practically useful property of WWr in contrast to the heuristic ones is that it can approximate the number of setups on a longer horizon in advance. In order to handle also the dynamically changing forecasts, I propose a rolling horizon approach, i.e., the algorithm should be run in every period with the most actual data. Note that if we use WWr on a rolling horizon, only the first production quantity ( $x_0$ ) is important, therefore the second phase of the algorithm in Fig. 3.6 can be skipped.

In case of known run-out date, it should be distinguished whether the date is near or far. It can be considered near, if the previous methods (especially WWr) suggest that all forecasted quantities have to be produced immediately in one lot. If the date is far, then the standard Wagner–Whitin algorithm can be applied.

However, if the date is near, the horizon can be considered one period long, hence in this case I propose using the extended newsvendor model which minimises the expected total cost allowing additional emergency production. The advantage of this approach is that the optimal lot-size can exceed the total forecasted quantity, thus an extra setup may be avoided. Fig. 3.11 summarises the proposed selection from the models, while Table 3.2 compares their main properties.

### 3.3.2 Sensitivity and What-If Analysis

While forecasts and most of the cost parameters are easily accessible in existing transactional ERP systems, the estimates of product life-cycles are sometimes not available.

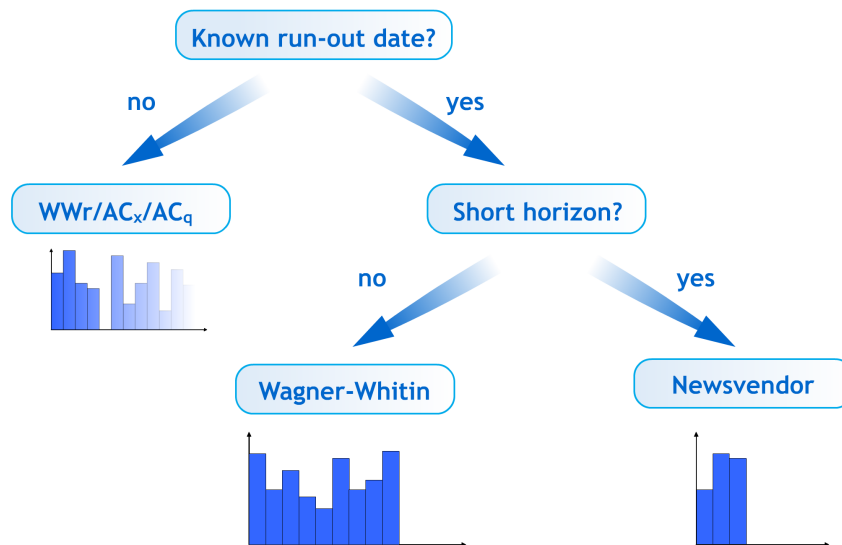


Figure 3.11: Selecting the appropriate model.

Table 3.2: Summary of the lot-sizing methods.

	$AC_x, AC_q$	$WWr$	Newsvendor
Length of the horizon	$n$	$n$	1
Minimises expected	average cost	total cost	total cost
Model	continuous	discrete	continuous
Lot-size can exceed total forecast	no	no	yes
Gives number of setups	no	yes	1 or 2
Uncertainty	run-out	run-out	demand quantity

However, it turned out that applying the WWR model with some real-life production plans where planned manufacturing of a product is sparse and involves large volumes—quantity is almost everywhere zero—the methods are not too sensitive to the uncertainty. To measure this sensitivity, an interval around the parameters can be examined instead of only a single value. Fig. 3.12 illustrates an example of the sensitivity analysis using the  $AC_q$  heuristic with geometric run-out distribution. It can be seen that the lot-size is a step function, i.e., a small modification of the  $p$  parameter rarely affects the optimal solution.

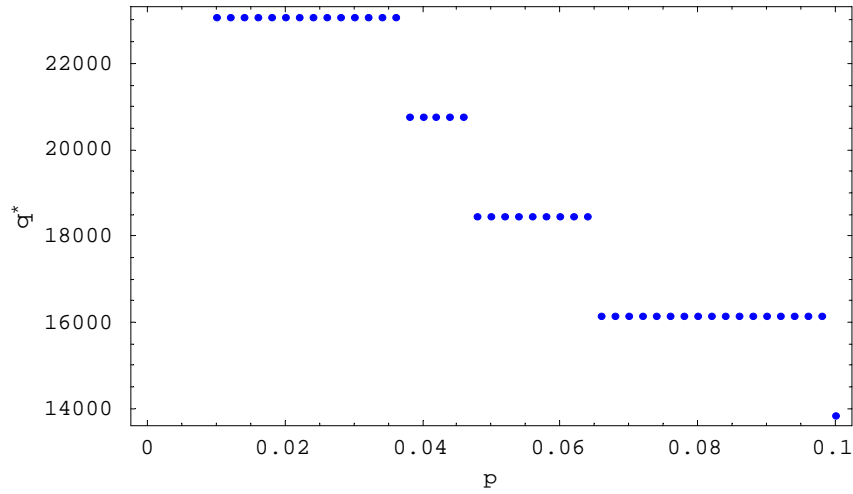


Figure 3.12: Sensitivity analysis of the parameter.

The implemented algorithms were included in a pilot lot-sizing application, where the following approach was used: the algorithms run with different run-out parameters in parallel, and from these *scenarios* the program calculates a measure for the *robustness*—in the meaning as the opposite of sensitivity—of the result. The less robust the proposed lot-size, the more care is needed from the human experts who evaluate and reconsider the results.

### 3.3.3 Safety Stocks

An important assumption of the WWr model is that the demand should be satisfied without backlogs. Hence, as part of the inventory, safety stock is needed in order to avoid short-term stock-outs. However, the stock level should not be more than necessary, because that would conflict with the *lean manufacturing* principle and may result in obsolete inventory.

The safety stock level can be adjusted by at least four policies:

**Fixed value.** Simple, not adaptive method. E.g., zero can be used for rare components, while frequent components can have slightly higher inventory levels, which facilitate the flexibility of the production.

**Forward coverage.** The safety stock is based on the forecasted demand of the next few time units. The length of the regarded period can be revised in an adaptive man-



ner. This approach is suitable when demand fluctuation is caused by demand shifts between periods.

**Forecast error.** The safety stock is based on the length of throughput time of components and the standard deviation of historical forecasts. This approach is proper, when fluctuations are mostly caused by inserted unexpected orders.

**Hybrid.** The combination of the previous methods.

Using average value of the forecasted demand on a longer horizon could be dangerous, since large, but far quantities lead to unnecessarily high safety stock levels, while in case of high short-term demand, shortage may occur even so. After consulting with the industrial partners, we have agreed on an acceptable default rule of calculating safety stock as the maximum demand of the next few weeks—namely four weeks in our case. This allows for the planners certain modifications (optimisation) in the production plan without violating the continuity of production.

After deciding the size of the safety stock, one can easily determine whether to produce or not: if the available inventory of the next week (current inventory minus the planned consumption of the actual week) is not less than the demand of the next week ( $F_1$ ) plus the safety stock, then no production is needed. If production is necessary, then the forecast data should be temporarily modified in order to comply with the preconditions of the WWr model:  $F_1$  should be increased with the quantity of the safety stock and decreased with the available inventory, while further forecasts should be decreased with the safety stock. This way the net demand is not changing,  $F_1$  will be positive and initial inventory can be considered zero. After these modifications, the WWr algorithm or the heuristics can be used to compute the appropriate lot-sizes.

## 3.4 Experiments

In this section I present some results of experimenting with the described algorithms. The industrial examples are related to packaging material supply, which are typically customised components and therefore high volatility is especially typical for them. In these case the setup costs are much larger—almost 15.000 times larger—than the unit production costs, since materials are relatively cheap, but the setup involves washing out the paints from the machines in addition to changing the offset plate and the cutter tool.

Some of the results presented below were made with the *InventoSim* simulation system (see Chapter 5); other analyses were carried out with the *Order Planner*, a pilot lot-sizing application I implemented as an Excel add-in. Fig. 3.13 shows an example input sheet for the system. The *SapNo* column contain the material identifiers<sup>5</sup>; the *Restage* is a flag for denoting foreseen run-outs; the *Setup* and *Price* contain the  $c_s$  and  $c_p$  parameters, respectively; *Holding Cost* is an approximation used as the  $h$  parameter in the WWr model; *Stock* is the actual inventory level minus the required material for the fixed tasks (thus it can be negative); the *Safety Stock* is optional, if left blank, the programme determines it using some default rules presented in Sect. 3.3.3; the *Length* denotes the length of the forecast horizon; then the forecasted quantities follow. The applied algorithm is determined using the decision tree in Fig. 3.11.

	A	B	C	D	E	F	G	H	I	J	K	L
	SapNo	Restage	Setup	Price	Holding Cost	Stock	Safety Stock	Length	1	2	3	4
2	1779280	0	40065.8	2.46	0.01	-3970		1	8343			
3	1896542	0	32192	2.46	0.01	-2024		13	11124	0	0	0
4	1910096	0	91886.7	1.14	0.01	-2224		23	0	0	0	0
5	1911021	0	40210	2.52	0.01	-13420		2	0	24720		
6	1946641	0	89093	0.92	0.01	19161		24	11124	38934	0	0
7	1911019	0	4532						0	34608		
8	2014179	0	290						23.6	67053	34978.8	35967.6
9	1969682	1	5526						3605			
10	1989659	1	4599						362.4	92.7	648.9	1936.4
11	1989665	1	4604						3605	7704.4	3007.6	3017.9
12	1989668	1	453						0	5768	0	2997.3
13	1958480	1	453						0	0	4326	0
14	1966709	1	445						0	0	4326	0

Figure 3.13: Order Planner.

### 3.4.1 Experiments with the Extended Newsvendor Model

In Table 3.3, I summarise the proposed lot-sizes of the newsvendor model with  $c_s$ ,  $c_p$  and  $m$  parameters taken from an industrial database and relate to components near to the end

<sup>5</sup>The material numbers are encoded.

of their life-cycles.

Table 3.3: Some results of the newsvendor model.

$c_s$	$c_p$	$m$	r=5%	10%	15%	20%	25%	30%
55269.5	3.15	7152	8035	8640	9137	9562	9933	10260
45997.25	3.29	36733	39316	40330	40627	40366	39600	38333
46046.5	3.29	50899	53979	54781	54451	53152	50864	47433
45892	3.29	38323	40967	41965	42204	41845	40939	39483
45331	3.29	65553	68970	69290	67899	64841	59730	51389
44541	3.29	19807	21538	22464	23055	23407	23564	23548

Since the  $b$  parameters are not available in the ERP system of the industrial partner, I decided to take a series of values, compute the results for each such *scenario* and compare their results. I took certain percentages ( $r$ ) of the forecasted demand as the standard deviation and derived the  $b$  parameters from them. As a graphical presentation, an extended series of the second row of Table 3.3 can be seen in Fig. 3.14.

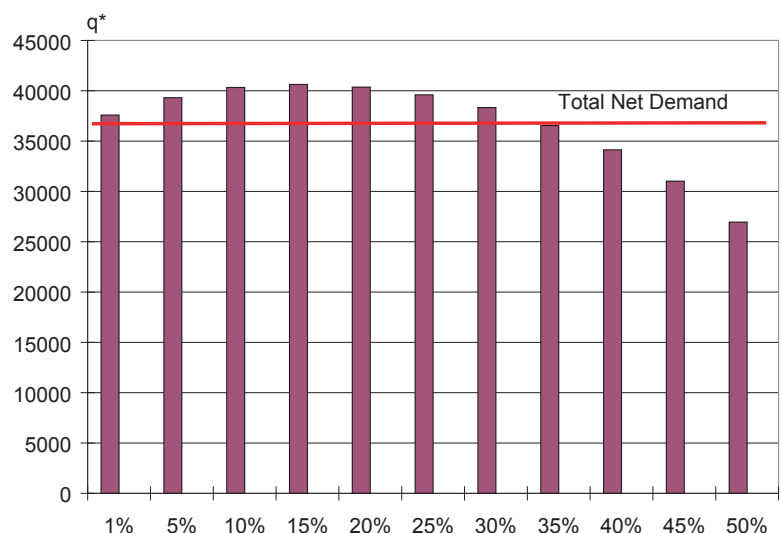


Figure 3.14: The effect of uncertainty on the optimal lot-size.

The series of the optimal lot-sizes can be explained in the following way: if there was no uncertainty, the lot-size would equal to the demand. As the uncertainty increases, it is

better to increase the lot-size in order to avoid the additional setup. However, when the uncertainty reaches a certain threshold, the expected cost of obsolete inventory reaches the expected cost of the additional setup, therefore the optimal lot-size starts to decrease. If one increased the uncertainty parameter further until  $b$  reaches  $c_s/c_p$ , the model would not be able to provide the optimal lot-size. Nevertheless, as the uncertainty grows, more attention must be paid by the human experts.

### 3.4.2 Experiments with the WWr Model

I tested the WWr algorithm on almost 10000 components based on real industrial data from the Logistics Platform (LP)—see Sect. 5.1 for details about the system. The planning horizon was theoretically 52 periods (weeks) long, but in several cases the end of the plan contained zero values, thus the average period length can be considered approximately 39 periods. With such problem sizes I simulated the complete inventory histories, which took about one hour. This means that for every components several lot-sizing decisions (several runs of the WWr) had to be made. In a real planning situation however, only 10-20% of the components has to be produced per period, and each lot-sizing decision requires only one run on the WWr. Hence, the complete lot-size planning can be done in a few minutes—even quicker by optimising the database access—, which is abundant for a weekly planning task. Note that this speed is due to the disregarding of capacity constraints.

I compared the factual inventory levels<sup>6</sup> with the simulated inventory levels, an example of which can be seen in Fig. 3.15.

In fact, it is difficult to evaluate and compare the results, due to the following reasons:

- i.) The exact cost parameters (setup, production, inventory holding) are not available in the LP, I could only estimate them.
- ii.) The types of the components (customised or standardised), and therefore information about the uncertainty is not directly available.
- iii.) The types of the procurements (order-based or VMI) are also not available.
- iv.) It is the suppliers' responsibility to register their inventory levels in the system in case of VMI, but for the time being, most of this information is rarely or never updated.

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<sup>6</sup>I considered the total inventory level in the supply chain, i.e., the sum of the customer's and supplier's inventory.

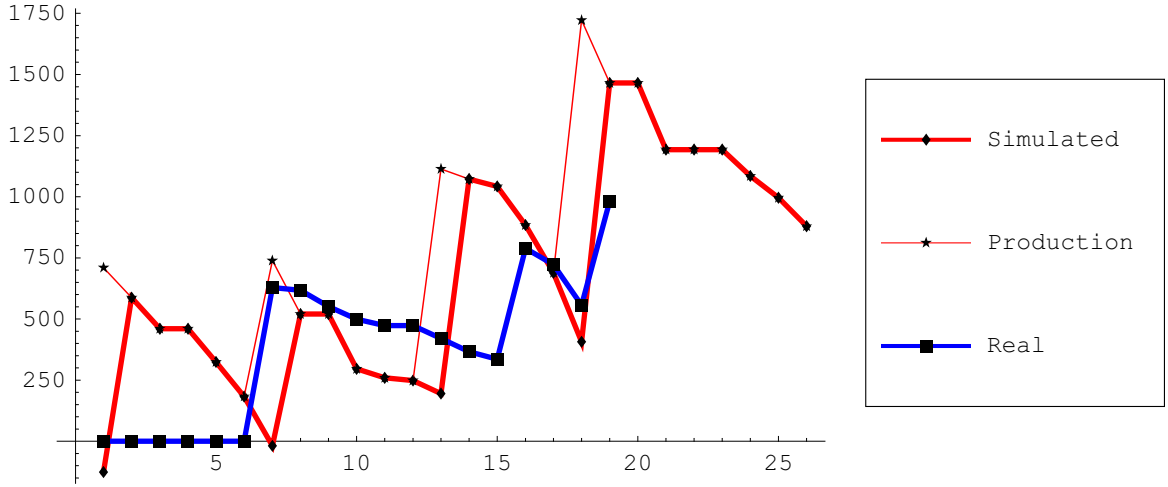


Figure 3.15: An example of the inventory level history.

- v.) The changes in the inventory levels are often not in accordance with the consumption data. The main causes of these discrepancies are the unregistered scrap production and the delayed tracing of inventory levels. This phenomenon can be observed also in Fig. 3.15.
- vi.) The component procurement or production is unknown in the system, thus an increase in the inventory level might be only a data correction, and not an effect of a setup and production.
- vii.) The number and size of the shortages are unknown, since the system contains only the production and the inventory levels that are naturally always non-negative.
- viii.) It is not registered in the system, whether and when a component run-out happens. The absence of the forecasts or the unvarying inventory levels can denote run-out, but they are sometimes signs only of temporary pauses in the demand.

Even so, analysing the resulted charts helped to detect and classify some glitches in the information consistency and the planning processes.

In the example in Fig. 3.15, I used the following estimated, but realistic parameters:  $c_s = 50000$ ,  $c_p = 3$  and  $h = 0.01$ . I considered the geometric run-out distribution with  $p = 0.02$ , the safety stock as the maximal forecast of the next five weeks, i.e.,  $\max(F_2, \dots, F_6)$ ,

since quantity  $F_1$  has to be already in the inventory. If a shortage still happens, the demand is backlogged and can be satisfied later.

I also run simulations based on randomly generated forecasts with parameters computed from the real cases. For example, in the case of the component presented in Fig. 3.15, the average demand is 113, and the fluctuation in the forecasts is 88%, approximately. I ran the simulation on 1000 different forecasts generated with these parameters<sup>7</sup>, while every other parameter was left unchanged. The summary of the results can be seen in Table 3.4 in terms of the average, standard deviation, minimum and maximum value of some relevant characteristics. Note that although in all of the simulations exactly two setups were optimal, on the real forecast four setups were proposed. This can be explained with the fact that the real forecasts have varying horizon length—sometimes only one or two periods—while in the simulations I used the maximal forecast horizon length ( $n = 22$ ). Furthermore, the real forecast are not so even as the generated ones assuming uniform distribution.

Table 3.4: Summary of the simulation runs.

Statistics	AVG	STD	MAX	MIN
Number of setups	2	0	2	2
Average lot-size	2359.98	211.34	3045.5	1654
Maximal lot-size	2555.22	257.26	3575	1751
Minimal lot-size	2164.75	257.15	3023	1123
Average inventory	1487.63	147.73	1900.36	1076.92
Maximal inventory	2524.83	249.79	3413	1740
Minimal inventory	256.37	73.21	484	75
Number of shortages	0	0	0	0
Average demand	113.13	12.76	153.12	70
Maximal demand	225.15	16.14	297	169
Cost	114579.04	1301.70	118793.79	110262.21
Average cost per period	4583.16	52.06	4751.75	4410.48
Average cost per total demand	41.00	4.48	63.71	30.58

<sup>7</sup>I considered that the product does not run out and the only uncertainty in the demand is the 88% fluctuation.

I also examined the effects of the inappropriately estimated uncertainty of the life-cycle. For this purpose, I chose 1153 materials from the database—where the number of inventory increments (estimated number of setups) were between two and six—, determined their average demand and forecast fluctuation, and used them as a basis for simulation. The length of life-cycles was assumed to have geometric distribution with  $p = 0.02$ , but the WWr algorithm ran with different parameters in order to study the changes in the occurrent cost. It has turned out that using  $p = 0.01$  or  $p = 0.03$  instead of  $p = 0.02$  resulted in an approximately 2% increase of the cost in average. For further details, see Sect. 4.5

## 3.5 Summary

In this chapter I presented novel extensions of two classical lot-sizing models. The first one is the one-period newsvendor model that I modified in order to satisfy the demand with minimal cost—even if it necessitates setting up an additional production. The second model is based on the deterministic uncapacitated single-item lot-sizing problem first formalised and solved by Wagner and Whitin. I introduced a stochastic variable into this model, the length of the product life-cycle, and showed how the original algorithm should be modified to solve the new formalism. I also suggested a simple decision rule for choosing the appropriate model facing practical problems. Finally, I discussed some practical issues of the models and illustrated them by examples taken from an industrial case study of packaging material supply.

The newsvendor model with additional setup cost is a completely new approach to one-period lot-sizing problems; I do not know any similar models in the literature. The solution—even the existence or the uniqueness—is not as straightforward in general case as with the standard model, therefore I focused on the special case of the logistic distribution, which is also an unprecedented idea in the newsvendor problem.

Modifying the Wagner–Whitin model to the case of products with short life-cycle is also scarce in the literature, I have found one model similar to my approach, though. The most important difference between WWr and the model I refer to as “Jeppesen” [51] is that my model considers also the cost of remaining obsolete inventory, which is therefore included in the objective function. Secondly, I assume deterministic forecast coming from a production planner system. Jeppesen is a stochastic model which is transformed to a deterministic one in order to solve it approximately. Thirdly, since Jeppesen is stochastic,

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it plans the production in terms of the  $(s_t, S_t)$  model, i.e., if a plan is created, it can be used until the end of the horizon. I consider the forecast changing from period to period, thus I propose using WWr with rolling horizon approach: in the beginning of every period, it should be run on the most actual data. Finally, I recommend determining safety stock only for the actual period in advance of planning the production quantities, while Jeppesen determines safety stocks ( $s_t$  values) after planning. All in all, I consider my model a new research contribution.



# Chapter 4

## Channel Coordination Models

In the following, I extend the previous models to a more realistic situation: a two-echelon decentralised supply chain with autonomous enterprises and asymmetric information—consisting of an end-product manufacturer in the customer’s role and a component supplier. The decentralisation—let alone the information asymmetry—can lead to suboptimal overall system performance, materialised in more obsolete inventory or unnecessary additional setups compared to the solutions presented previously. The goal of this chapter is to offer such channel coordination mechanisms, wherewith the rational operations of the partners lead to the theoretical optimum on the system level by minimising the effects of the uncertainties.

*Manufacturing uncertainty* can be defined as the difference between the amount of required and available information [78]. In their paper Mula et al. adopt a classification which distinguishes two types of uncertainty: (i) *environmental uncertainty* stems from the unpredictable markets (e.g., demand, supply) and (ii) *system uncertainty*, which comes from the production processes (e.g., yield, failures, lead-times, quality). But there is another possible source of uncertainty: the (iii) *planning process uncertainty*, when the generated plan is not appropriate; I call this as *unnecessary uncertainty*. Such phenomenon occurs, when human decision makers distort forecast as studied e.g., in [37]. Since the production plan of a tier<sub>*n*</sub> enterprise is the basis of the component forecast for the tier<sub>*n*+1</sub> suppliers, uncertainties of the plan spread upwards in supply chain. In addition, they not only spread, but also grow, usually referred to as the *bullwhip effect* [59].

Sometimes the planners have lack of motivation for determining the best available forecast or have incentives even for distorting them. The main underlying motivations are

the followings:

**Incentive for inflating demand.** If the planners are measured by the eventual shortage, then they tend to overplan demand and forward too optimistic plans towards suppliers.

**Incentive for deflating demand.** If the planners are rewarded for overperforming the plans, then they tend to underestimate the demand and forward too pessimistic plans toward suppliers. Sometimes similar situation occurs due to forcing the lean manufacturing paradigm at an extreme level.

In both cases, the suppliers may be aware of these biases of the manufacturer and may not accept the demand information without critique. They try to correct the forecast based on past experiences, but this cannot completely restore the quality of the forecast. The selfish distortion of information will necessarily lead to additional operational costs. Instead of these inappropriate measures, the planners should be evaluated by *planning imprecision*, which would inspire them to create as accurate plans as possible and thus avoid unnecessary uncertainty.

All along this chapter I assume *one-point-inventory* between the manufacturer and the supplier, practically in a VMI setting. The component forecast, which is derived from the manufacturer's medium-term production plan is the basic input for the supplier's lot-sizing problem. This forecast is uncertain as we have seen, but should represent the best knowledge of the manufacturer. Actual orders (so-called *call-offs*) can be given only for one period ahead for accounting purposes, therefore they must be satisfied with *just-in-time (JIT) delivery* from stock. By using the VMI approach, the responsibility of the supplier is increased: he can arbitrarily determine the production quantity, but this goes hand in hand with the whole inventory risk, i.e., possessing the occurrent obsolete inventory. This risk should be compensated by the proposed contracts, which enable the network being *flexible* for satisfying the demand, since it does not bound the service level with ex ante commitments.

On the supplier's side, infinite capacity is assumed henceforward. It is also assumed that cost and price parameters are constant, only the production quantities are decision variables and they should be determined by the supplier. In the following models the demand forecast is known by the customer, who signals it towards the supplier. The

forecast is not a simple qualitative information (i.e., high or low) as in several theoretical studies, but is given either by a distribution function or by discrete component forecast on a rolling horizon.

Throughout this chapter I assume that the entire demand should be fulfilled by the supplier, who therefore provides 100% service level. Similarly to the famed *zero defects* principle of the Total Quality Management, this requirement is hard to realise in practice, but has similar motivation: if an acceptable fault tolerance is defined, it is usually realised as a self-fulfilling prophecy.

Since the production quantity is determined by the supplier, the following simple axiom holds, which is the bottom of my novel coordination mechanisms.

**Axiom 4.1 (Basic axiom of channel coordination.)** *If*

- i.) there is no information distortion at the information sharing (the signaling is truthful),*
- ii.) this is a common knowledge and*
- iii.) the payment is independent from the supplier's decision,*

*then the supplier is facing the centralised lot-sizing problem with all required information, hence his rational decision will be also optimal on the system level.*

Note that in contrast to standard *games with incomplete information*, no common knowledge about the distribution of the private information (so-called *type* of the player) is assumed—which means omitting the *Harsanyi Doctrine* [79]. In this case that would lead to a probability distribution (belief) of the demand probability distribution (forecast), which seems practically inapplicable.

In contrast, I present such coordination mechanisms wherewith the truthful forecast sharing is rational for the customer, but no other private information need to be shared—neither the cost parameters of the supplier nor his production quantity. This makes the issue of cost and profit sharing difficult, therefore it is excluded from this dissertation and can be a further direction of my research. However, the models guarantee optimal supply chain performance assuming rational partners.

## 4.1 Coordinating the Decentralised Newsvendor Model

In the decentralised newsvendor model, the customer is familiar with the end-product market, thus she estimates the demand ( $m$  and  $b$  parameters of the logistic distribution). A component is produced by the supplier, who knows the actual production and setup costs ( $c_p$  and  $c_s$ ). The lot-sizing decision should be made by the supplier, who has to plan and schedule his own production, manage the inventory and provide 100% service level towards the customer. The proposed protocol of the supply process is as follows (see also Fig. 4.1):

- i.) The customer signals forecast information towards the supplier, but this may differ from the real values, therefore I denote these parameters with  $m'$  and  $b'$ .
- ii.) The supplier decides about the lot-size ( $q$ ) and produces this quantity.
- iii.) The customer faces the demand ( $\xi$ ), calls-off this quantity from the supplier.
- iv.) The supplier delivers  $\min(\xi, q)$  items instantly. If  $\xi < q$ , the obsolete inventory remains at the supplier; but if  $\xi > q$ , the supplier has to start an emergency production for  $\xi - q$  items and deliver them as soon as possible.
- v.) The customer pays according to the payment function described below.

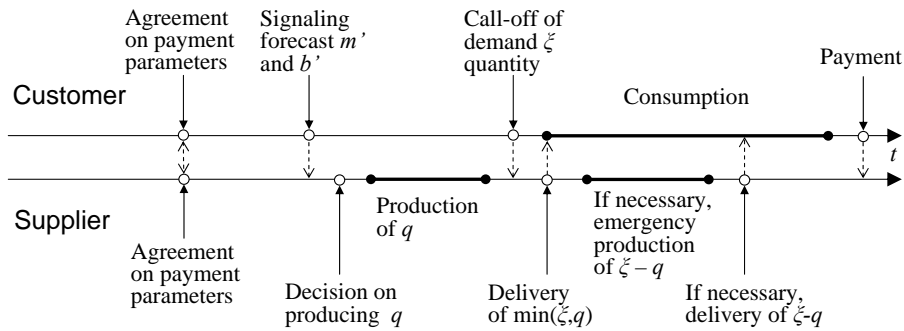


Figure 4.1: Protocol of the newsvendor supply process.

I emphasise the assumption that the finally realised demand ( $\xi$ ) is known by both partners in the end of the process. I present below such a payment function, which guarantees

that in the above protocol communicating  $m' = m$  and  $b' = b$  is the unique optimum for the customer, hence due to Axiom 4.1 it coordinates the channel [30, 31].

The main idea is that with VMI the supplier not only offers products, but also *flexibility* as a *service*. Accordingly, a composite payment function should be constructed: the customer must pay not only (i) for the quantity called-off, but also (ii) for the deviation from the forecast, as well as (iii) for the forecast uncertainty. This payment compensates the supplier for the eventual obsolete inventory or the additional setup. The parameters of the decentralised newsvendor model are as follows:

$\xi$	realised demand,
$m$	expected value of the demand,
$b$	parameter of the logistic distribution, proportional to its standard deviation,
$c_0$	unit price of a component,
$c_1$	unit compensation price,
$m'$	communicated expected value of the demand (decision variable of the customer),
$b'$	communicated parameter of the logistic distribution (decision variable of the customer),
$q$	production quantity (decision variable of the supplier) and
$P(m', b', \xi)$	payment.

The proposed payment function in general form is the following:

$$P(m', b', \xi) = c_0 \xi + \frac{c_1}{b'} d(m', \xi) + f(b'), \quad (4.1)$$

where  $d(m', \xi)$  is the difference between the communicated and the realised demand and  $f(b')$  is a monotonically increasing compensation term for uncertainty of the demand. Note some properties of this function:

- i.) it depends only on commonly known parameters,
- ii.) the first term in the payment is independent from the decision variables and
- iii.) if the customer communicates higher uncertainty (larger  $b'$ ), it will pay less for the deviation (second term), but more for the uncertainty (third term) and vice versa.

Deviation from the forecast can be measured e.g., by the simple difference, the absolute difference or the squared difference. I have found that the first two measures are inappropriate for channel coordination with the proposed payment function, thus I chose the latter:  $d(m', \xi) = (m' - \xi)^2$ . For this case, I have derived such an  $f(\cdot)$  compensation function, wherewith the payment function inspires the customer towards truthful information sharing, hence it coordinates the channel.

**Theorem 4.2** *If the demand  $\xi$  is a random variable from the logistic distribution with parameters  $m$  and  $b$  and the payment function is*

$$P(m', b', \xi) = c_0 \xi + \frac{c_1}{b'} (m' - \xi)^2 + c_1 \frac{\pi^2}{3} b', \quad (4.2)$$

*then the expected payment is minimal iff  $m' = m$  and  $b' = b$ .*

**Proof.** The expected payment is the following:

$$\mathbb{E}[P(m', b', \xi)] = c_0 m + \frac{c_1}{b'} \mathbb{E}[(m' - \xi)^2] + c_1 \frac{\pi^2}{3} b'. \quad (4.3)$$

The expected difference can be computed using the definition of the expected value:

$$\mathbb{E}[(m' - \xi)^2] = \int_{-\infty}^{\infty} ((m')^2 + x^2 - 2m'x) \phi(x) \mathbf{d}x = (m')^2 + \mathbb{E}[\xi^2] - 2m'm. \quad (4.4)$$

The term  $\mathbb{E}[\xi^2]$  can be expressed from the following basic property of the variance:  $\sigma^2 = \mathbb{E}[\xi^2] - m^2$  utilising the variance of the logistic distribution:  $\sigma^2 = \frac{\pi^2 b^2}{3}$ . Then the expected payment becomes:

$$\mathbb{E}[P(m', b', \xi)] = c_0 m + \frac{c_1}{b'} \left( (m')^2 + m^2 - 2m'm + \frac{\pi^2 b^2}{3} \right) + c_1 \frac{\pi^2}{3} b'. \quad (4.5)$$

In order to minimise the expected payment, its partial derivatives must equal to zero:

$$\frac{\partial \mathbb{E}[P(m', b', \xi)]}{\partial m'} = \frac{c_1}{b'} (2m' - 2m). \quad (4.6)$$

This equals zero iff  $m' = m$ , independently from choosing  $b'$ . Furthermore

$$\frac{\partial^2 \mathbb{E}[P(m', b', \xi)]}{\partial (m')^2} = 2 \frac{c_1}{b'} > 0, \quad (4.7)$$

i.e., according to the *second derivative test*, this is a minimum. For computing the other partial derivative, I already exploit that  $m' = m$ :

$$\frac{\partial \mathbb{E}[P(m', b', \xi)]}{\partial b'} = c_1 \frac{\pi^2}{3} - c_1 \frac{\pi^2 b^2}{3(b')^2}. \quad (4.8)$$

This equals zero iff  $b' = b$ , and in this case the second derivative is also positive:

$$\frac{\partial^2 \mathbb{E}[P(m', b', \xi)]}{\partial (b')^2} = c_1 \frac{\pi^2 b^2}{3(b')^3} > 0. \quad (4.9)$$

□

## 4.2 Coordinating the Decentralised Rolling Horizon Model

*Incentive alignment* by compensation for truthful information sharing which can synchronise the decentralised decision making was proposed in [96]. This empirical study mostly focuses on different benchmarks of collaboration and verifies that this approach can lead to increased efficiency in practical situations. However, it did not consider any theoretical models of channel coordination, which I introduce in its framework model below.

The channel coordination in the rolling horizon case is also based on Axiom 4.1. In order to force truthful information sharing, I took similar approach as in coordinating the newsvendor model: the quality of the shared information should be measured and the supplier should be compensated proportional to the measurement. However, regarding rolling horizon planning this results in a fundamentally different problem. The basic assumptions of the model are as follows:

**Vertically integrated two echelon supply chain system.** I consider a customer and a supplier working together on the long run. This is a typical situation with the custom component suppliers in case of mass customisation. The supply network is reconfigured time and again, but I consider the stable periods of its operation when suppliers are contracted for producing particular components. There is no overlap between the supply channels, hence suppliers are not competitors. They compete at reconfiguration, but this strategic network design problem is out of the scope of the present dissertation.

**Service level.** The most important criteria is still that end-customer demand must be fulfilled at the highest service level as possible. I assume, that the forecast-demand difference follows certain rules (i.e., some random alteration on the long-term and demand shifting between periods on short-term horizon). Hence with the assumption of practically infinite capacity and the usage of appropriate safety stock calculations to cover the production lead-time (see Section 3.3.3) the shortages can be avoided with very high probability. Therefore I assume, 100% service level is achievable.

**Asymmetric information.** I consider similar information decentralisation as in the decentralised newsvendor model: the supplier knows the production related cost parameters, while the manufacturer is more familiar with the component demand forecast and uncertainty.

**Plan sharing.** As far as here, I suggest, that the supplier should decide about component production and inventories. For this reason, customer should signal the demand related information to him. Again, if the information sharing is truthful, the supplier will possess all available information for the appropriate planning. Since the manufacturer has clear incentives to distort the information (or to plan it sloppily), this phenomenon should be eliminated by a proper compensation payment.

**Long-term coordination contract.** Since the network configuration is considered permanent, the compensation should prevail on the long term.

**Commonly known realised demand.** VMI and the 100% service level assumption implies that the supplier knows the realised demand—just as in the newsvendor model—, because the customer can call-off neither more, nor less from the inventory than demanded.

Firstly, I focus on the possible ways of measuring forecast quality [27] and then I describe how these measurements can be used for compensation in order to coordinate the channel [111]. The parameters and variables used in this section are as follows:



$n$	length of the forecast horizon,
$n'$	length of the stability horizon ( $n' \leq n$ ),
$F_{i,j}$	forecast for period $i$ made in period $j$ ( $j + 1 \leq i \leq j + n$ ),
$\xi_i$	realised demand in period $i$ ,
$c_0$	unit price of a component,
$c_1$	unit compensation price,
$c_2$	compensation price for run-out possibility,
$\alpha_i$	discount factor of period $i$ ,
$\eta_j \in \{1, \dots\}$	period of the run-out estimated in period $j$ ,
$e_i$	error of period $i$ ,
$d_j$	deviation of the forecast generated in period $j$ and
$P_k$	payment for the period $k$ .

#### 4.2.1 Measuring Forecast Imprecision on Rolling Horizon

By *planning imprecision* I mean the difference between the forecasted and the realised demand. The difference between consecutive forecasts—which is the cause of the *nervousness syndrome*—is measured as *planning instability* [44]. Imprecision can be measured in two ways: either in absolute or in relative form. While relative imprecision is meaningful for the management, the former approach is more useful for compensational purposes, therefore I describe only these measurements, however they can be easily modified to the relative ones.

Measuring planning imprecision on single-horizon seems to be easy: one must compare the forecasted and the realised demand for the same period. This kind of measurement is widely utilised, the most frequently used variants are

**Forecast Error:**  $FE_i = F_i - \xi_i$ ,

**Absolute Forecast Error:**  $AFE_i = |F_i - \xi_i|$ ,

**Squared Forecast Error:**  $SFE_i = (F_i - \xi_i)^2$ ,

where  $F_i$  is the forecasted demand for period  $i$  (with single-horizon planning the second index would be constant). Difference in an interval is measured as the mean of the errors:

**BIAS:** the mean of FEs. It is useful to show the direction of the planning difference on the horizon: if it is positive (negative), then the plan is systematically overestimated

(underestimated). Its absolute value shows the rate of the over-/underestimation. Since positive and negative errors weaken each other, BIAS cannot truly measure the imprecision.

**Mean absolute deviation (MAD):** the mean of AFEs. It is a better planning imprecision measurement than BIAS, while it eliminates its weakening effect.

**Mean squared deviation (MSD):** the mean of SFEs. It is frequently used instead of MAD, when the larger penalty rates for larger errors are more adequate.

In the rolling horizon case, plans are generated period by period for a fixed horizon. Since these plans are overlapping, measuring imprecision is not as straightforward, as the example in Fig. 4.2 shows (let us disregard the “stability horizon” for the present).

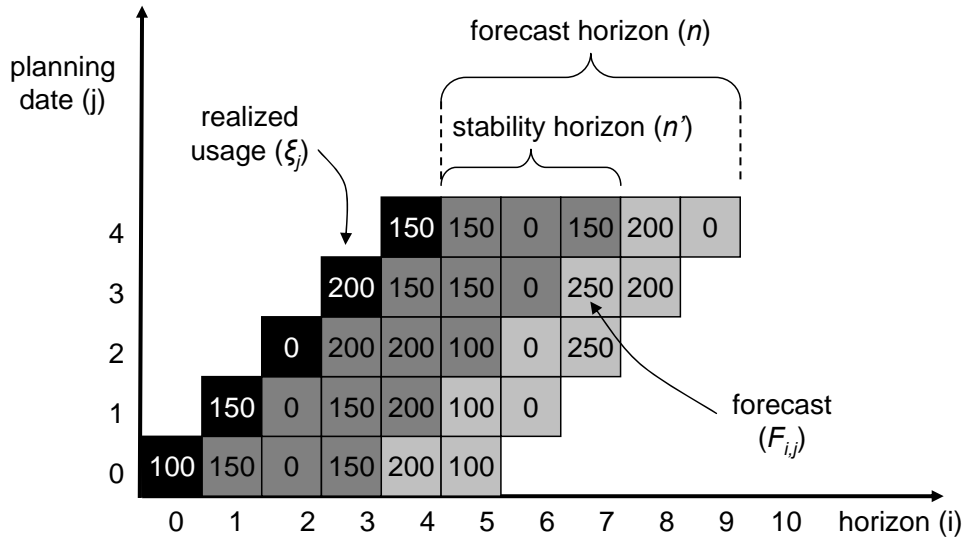


Figure 4.2: Rolling horizon planning.

In this example, the length of the forecast horizon is five periods. Therefore in period 0, the plan is made for periods 1..5 (see row 0). In addition, the realised demand for the period 0 is denoted with black background. For period 1, the plan is 150 just as the realised demand (row 1), therefore the plan proved to be right. This is true also for period 2, but for period 3, the forecast is 150 and the realised demand is 200 (row 3). There are other forecasts for period 3: the forecast made in period 1 is also wrong, but the forecast

generated in period 2 is right. (There are two other forecasts for period 3, they were made in periods -1 and -2, which are not included in the figure.)

The question is, how to measure the imprecision considering plans made in different periods? This is an important problem in the practice, because imprecise planning makes the behaviour of the system suboptimal even in the centralised case. An experimental study of this effect in multi-level rolling horizon planning can be found in [48].

Similarly to [44], I differentiate *forecast horizon* from *stability horizon*. Forecast horizon consists of periods for which forecast has been made. Stability horizon is a part of the forecast horizon, where imprecision of the forecast have serious effect (see again Fig. 4.2). Imprecision should be measured only on the stability horizon, therefore for this analysis the length of the forecast horizon is irrelevant, and only the stability horizon matters. This can be interpreted as  $n' = n$  an assumption made all along the theoretical part of this section. However, I discuss the  $n' < n$  case more informally in Section 4.4.2. Note that the name of “stability horizon” means, that one measures imprecision on this horizon and not that its demand is frozen.

In the next two subsections I present two of the several possible metrics that I found practically relevant. I suggest using the *error* in make-to-order purchasing, where small modifications in the forecast may completely alter the production plans, while the less rigorous *deviation* is more appropriate in a make-to-stock environment.

### Error of the Periods

The *error* of a period  $i$  measures the weighted absolute differences of a period’s realised demand and the previous forecasts for that period:

$$e_i = \sum_{j=i-n'}^{i-1} \alpha_{i-j} |F_{i,j} - \xi_i|. \quad (4.10)$$

The errors are weighted with discount factors  $\alpha_1 \geq \dots \geq \alpha_{n'} \geq 0$  on the stability horizon, such that  $\sum_{i=1}^{n'} \alpha_i = 1$ . This expresses that the forecast for far periods can be less important than the ones for near periods. The most widespread discount types are:

**Constant discount:** in this case every weight is equal, the periods are equally important thus the error becomes the mean of the differences, i.e.,  $\alpha_i = \frac{1}{n'}$ .

**Linearly decreasing discount:**  $\alpha_i = \frac{2-a}{n'} - (i-1)\frac{2-2a}{n'^2-n'}$ , where  $0 < a < 1$  is a parameter.

The weights approach the constant discount factor as  $a$  goes to 1.

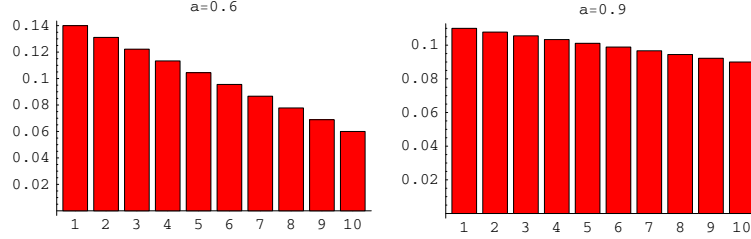


Figure 4.3: Sample linear discount factors ( $n' = 10$ ).

**Exponentially decreasing discount:**  $\alpha_i = \frac{1-a}{1-a^{n'}} a^{i-1}$ , where  $0 < a < 1$  is a parameter (see also [44]). The weights approach the constant discount factor as  $a$  goes to 1.

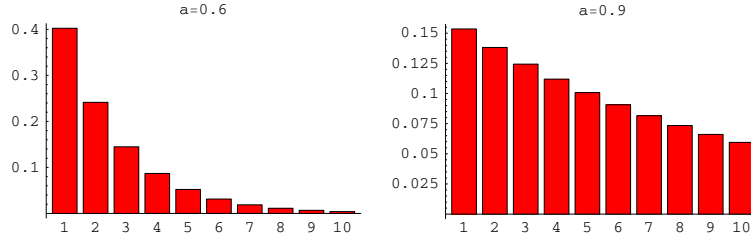


Figure 4.4: Sample exponential discount factors ( $n' = 10$ ).

### Average Deviation of the Plans

The *deviation* of the forecast generated in period  $j$  measures the absolute average difference between the total demand of a forecast and the total realised demand on the same horizon. This is a more appropriate measure when demand is fulfilled from the inventory and the demand shifts between periods are almost negligible. In this case, discounting is not desirable, because it would differentiate between the forward and backward direction of the demand shift, therefore I use  $\alpha_1 = \dots = \alpha_{n'} = \frac{1}{n'}$ :

$$d_j = \frac{1}{n'} \left| \sum_{i=j+1}^{j+n'} (F_{i,j} - \xi_i) \right|. \quad (4.11)$$

### 4.2.2 Compensation Schemes

In the proposed VMI setting the responsibility of the inventory related decision making is at the supplier's side, albeit some important inputs come from the customer. In order to inspire the customer towards truthful information sharing, she should pay compensation for the supplier in case of inaccurate forecasts. In this section I present two different payment schemes based on the imprecision measurements presented earlier in this section. The choice between these two types of evaluation should be based on the production and purchase characteristics: if the demand shifts can not cause shortage or necessary rescheduling, then plan deviation is appropriate, otherwise the forecast error should be used. Finally, I consider the case of short life-cycle products, where the customer should also share the run-out related information.

#### Compensation According to the Error

In this simple case the payment for the period  $k$  becomes the price of the called-off components plus the compensation for the forecast error:

$$P_k = c_0\xi_k + c_1e_k. \quad (4.12)$$

This payment can be paid immediately after the call-off, since the error can be determined using the past forecasts.

**Proposition 4.3** *Using the payment function 4.12, the customer will always share her true forecast, because if she either increases or decreases the forecasted quantity of a period, she increases the expected error and thus the payment.*

#### Compensation According to the Average Deviation

In this case a similar payment for the period  $k$  is the following:

$$P_k = c_0\xi_k + c_1d_k. \quad (4.13)$$

The complete payment can be fully paid only after  $n'$  periods: the price of the call-off can be paid immediately, but the amount of the deviation (and thus the compensation) only turns out at the end of the stability horizon.

**Proposition 4.4** *Using payment function 4.13, sharing the true forecast is optimal for the customer, but it is not unique. Any redistribution of the demand on the horizon is also optimal until the sum does not change, in special case, the customer can aggregate total forecast for the next period without any consequence to the payment.*

Thus the payment function 4.13 is inappropriate for channel coordination purposes. However, this problem can be resolved by introducing the idea of *rolling compensation*. Let

$$\tilde{P}_{k,0} = c_0 \xi_k \quad (4.14)$$

and

$$\tilde{P}_{k,l} = c_0 \xi_k + \frac{c_1}{l} \left| \sum_{i=k+1}^{k+l} (F_{i,k} - \xi_i) \right| \quad (l = 1, \dots, n') \quad (4.15)$$

be the estimated payments for period  $k$  computed in periods  $k+l$  ( $l = 0, \dots, n'$ ). According to the definitions  $P_k = \tilde{P}_{k,n'}$ . With rolling compensation, the customer should pay  $\tilde{P}_{k,0}$  in period  $k$  (i.e., only the price of the components called off) and  $\tilde{P}_{k,l} - \tilde{P}_{k,l-1}$  in periods  $k+l$  ( $l = 1, \dots, n'$ ) for the deviation of forecast generated in period  $k$ , respectively. This process can be interpreted as the partners estimate the expected payment for period  $k$  in periods between  $k$  and  $k+n'$ , and the customer revises the compensation that she has already paid. In the end, the total amount paid will be just  $P_k$ . Note that  $\tilde{P}_{k,l} - \tilde{P}_{k,l-1}$  can also be negative; this means that the customer paid too much compensation in the previous period and she gets back a part of it. The total payment in a period  $k$  using the rolling coordination method becomes:

$$P_k = \tilde{P}_{k,0} + \sum_{l=1}^{n'} \left( \tilde{P}_{k-l,l} - \tilde{P}_{k-l,l-1} \right). \quad (4.16)$$

**Proposition 4.5** *With rolling compensation, sharing the true forecast is the unique optimum for the customer, because any redistribution of the forecast means an expected early compensation, which will be paid back only later, i.e., loaning money without interest.*

For instance, if the customer aggregates the total forecast for the first period of the horizon, she can expect to pay a huge compensation on the next period, which will be amortised weekly by the supplier. If the payment is not made in every period but in larger time intervals, the payment function should be modified by including interest.

### Extending Compensation for Short Life-Cycle Products

In the WWr model I considered short life-cycle products by introducing the possibility of run-out. If one considers this setting, the customer must also share her knowledge about this probability. I assume that the distribution of run-out (or equivalently, the distribution of the length of the remaining product life) is a discrete distribution with one parameter (such as geometric, uniform or Poisson presented in Sect. 3.2.3). I also assume that *the type of distribution is common knowledge*, only the parameter is the private information of the customer.

Run-out mainly causes problem in the situation when the production (or purchasing) is made in large batches, so obsolete inventories may remain. Therefore the measurement based on the *forecast deviation* seems to be appropriate for such cases. I propose a payment function that has the following properties:

- i.) If no run-out happens, the customer pays the compensation for forecast deviation (payment 4.13), but besides, also pays compensation for the additional uncertainty. The smaller the communicated probability is, the smaller the compensation.
- ii.) If run-out happens, then the customer pays compensation for forecast deviation related before the run-out. She also pays compensation for uncertainty: the smaller the communicated probability is, the larger the compensation.
- iii.) The expected payment of the customer will be minimal, if she shares the forecast and probability parameter according to her best knowledge.

The proposed payment function is the following:

$$P_k = \begin{cases} \tilde{P}_{k,n'} - c_2 \ln(\Pr(\eta_k > n')) & , \text{ if no run-out happens,} \\ \tilde{P}_{k,l-1} - c_2 \ln(\Pr(\eta_k \leq n')) & , \text{ if product runs-out in period } l, \end{cases} \quad (4.17)$$

where  $\tilde{P}_{k,l}$  was defined by Eqs. 4.14–4.15 and  $\eta_k$  is the period of run-out estimated in period  $k$ . This payment function consists of three parts:

- i.) Payment for quantity called-off, which is independent from the decision variables.
- ii.) Compensation for forecast deviation, which is independent from the run-out probability. If no run-out happens, it is exactly the same as before, and in case of run-out

it is similar, but on a shorter horizon. Therefore the customer has incentives to share the true forecast to minimise this term of compensation. Note that because run-out may occur, the redistribution of the demand on the horizon ruins the optimality of the expected payment.

- iii.) The compensation for possibility of run-out is independent from the forecast, depends only on whether run-out actually happens or not. Note that the arguments of the logarithm functions are between 0 and 1, therefore the compensation terms are positive.

**Proposition 4.6** *Using payment function 4.17 the customer should communicate her best available parameter of the distribution of run-out in case of geometric, uniform or Poisson distributions.*

**Proof.** I prove the statement in case of Poisson distribution, the geometric and uniform cases are similar, but  $p_k$  and  $N_k$  should be used instead of  $\lambda_k$ .

Let us assume that the customer estimates the parameter  $\lambda_k$  (where the probability that the product does not run-out in the stability horizon is  $\Pr_{\lambda_k}(\eta_k > n')$ ), but she communicates  $\lambda'_k$  instead (wherewith the probability is  $\Pr_{\lambda'_k}(\eta_k > n')$ ). Then the expected compensation term will be:

$$\Pr_{\lambda_k}(\eta_k > n') \left( -c_2 \ln(\Pr_{\lambda'_k}(\eta_k > n')) \right) + (1 - \Pr_{\lambda_k}(\eta_k > n')) \left( -c_2 \ln(1 - \Pr_{\lambda'_k}(\eta_k > n')) \right). \quad (4.18)$$

She wants to minimise this term, therefore the derivative must equal to zero:

$$\frac{1 - \Pr_{\lambda_k}(\eta_k > n')}{1 - \Pr_{\lambda'_k}(\eta_k > n')} c_2 \frac{d \Pr_{\lambda'_k}(\eta_k > n')}{d \lambda'_k} - \frac{\Pr_{\lambda_k}(\eta_k > n')}{\Pr_{\lambda'_k}(\eta_k > n')} c_2 \frac{d \Pr_{\lambda'_k}(\eta_k > n')}{d \lambda'_k} = 0. \quad (4.19)$$

Simplifying this equation leads to the condition  $\Pr_{\lambda'_k}(\eta_k > n') = \Pr_{\lambda_k}(\eta_k > n')$ . The second derivative test shows, that this yields a minimum of the expected compensation. In case of geometric, uniform or Poisson distributions  $\Pr_{\lambda'_k}(\eta_k > n') = \Pr_{\lambda_k}(\eta_k > n')$  is fulfilled iff their parameters are equal, therefore the customer is inspired to share this parameter according to her best knowledge. Using this payment function, sharing the true forecast is optimal for the customer and it is unique.  $\square$

Note that in the proof we used the following properties of the distribution:



- i.) The probability distribution has only one parameter and the CDF is continuously differentiable in the parameter. This guarantees that the expected compensation has a unique stationary point; furthermore, if the expected payment is convex, then the stationary point is a global minimum.
- ii.) The CDF is an injective function of the distribution parameter. This means that two different parameters would lead to different probabilities, hence, the customer should share the real parameter in order to minimise the expected payment.

Thus the proposed compensation scheme is applicable with every distribution having the above properties.

The proposed compensations in case of geometric, uniform or Poisson distributions can be found in Table 4.1.

Table 4.1: Special cases of compensation.

	Geometric ( $p_k$ )	Uniform ( $N_k > n'$ )	Poisson ( $\lambda_k$ )
no run-out	$-c_2 \ln((1 - p_k)^{n'})$	$-c_2 \ln\left(1 - \frac{n'}{N_k}\right)$	$-c_2 \ln\left(1 - e^{-\lambda_k} \sum_{l=0}^{n'-1} \frac{\lambda_k^l}{l!}\right)$
run-out	$-c_2 \ln(1 - (1 - p_k)^{n'})$	$-c_2 \ln\left(\frac{n'}{N_k}\right)$	$-c_2 \ln\left(e^{-\lambda_k} \sum_{l=0}^{n'-1} \frac{\lambda_k^l}{l!}\right)$

This type of payment can be fully paid only after  $n'$  periods (if no run-out happens): the price of the call-off can be paid immediately, but the compensation terms only turn out at the end of the stability horizon. However, the rolling compensation approach can be used here, too.

Note that if the product runs out, the compensation for possibility of run-out is independent from the period in which the run-out happens. This may seem odd at first sight, but remember: a rolling horizon forecasting is considered. When a run-out happens, there will always be  $n'$  forecast with the run-out in different periods (from 1, ...,  $n'$ ), thus differentiating the period of run-out would be redundant.

### 4.3 A Game Theoretic Generalisation

In this section I study a two-player non-cooperative game where the utilities depend on a stochastic variable whose distribution is known by only one of the players. I formulate this

situation first as a principal–agent model that I call a *forecast sharing game*, which is a generalisation of the special models presented in the previous two sections. I show some preliminary results for such contracts that can guarantee efficiency. Then I generalise the model as a mechanism design problem and prove an impossibility theorem that excludes the fair cost and profit sharing in the general case. Finally, I enumerate some open questions and future research directions related to this model. General introduction to mechanism design can be found in [46, 80, 81]. The principal–agent model and the contracting theory are described in [57, 71, 87, 102].

### 4.3.1 Forecast Sharing Games in Principal–Agent Setting

Let us consider a market, where the stochastic demand should be entirely fulfilled. If a single enterprise serves the market, it firstly determines a production plan  $x \in \mathcal{K}$ , then the demand  $\xi \in D$  realises. Since the entire demand has to be fulfilled, if  $x$  underestimates the demand, new and costly productions are necessary—usually in overtime, with outsourcing or with additional setups—while overestimation leads to obsolete inventories. I consider the *utility* in the following form:

$$v(\xi) - c(x, \xi), \quad (4.20)$$

where  $v$  is the income function depending only on the demand and  $c$  is the cost function. I assume that the enterprise creates a demand forecast  $\theta \in \Theta$  based on its beliefs about the market and it also has an appropriate *choice* function  $f : \Theta \rightarrow \mathcal{K}$  that can determine an optimal production plan for a given forecast, i.e.,

$$f(\theta) \in \operatorname{argmax}_{x \in \mathcal{K}} \mathbb{E}_\theta[v(\xi) - c(x, \xi)] \quad (4.21)$$

or equivalently

$$f(\theta) \in \operatorname{argmin}_{x \in \mathcal{K}} \mathbb{E}_\theta[c(x, \xi)]. \quad (4.22)$$

Therefore if a  $\theta$  forecast is given, the utility of the enterprise becomes

$$u(\theta, \xi) = v(\xi) - c(f(\theta), \xi), \quad (4.23)$$

where  $f(\theta)$  is called the *first-best* solution.

Let us now consider that the market is served by a chain of a customer and a supplier applying the VMI business model. This means that the customer forecasts the demand

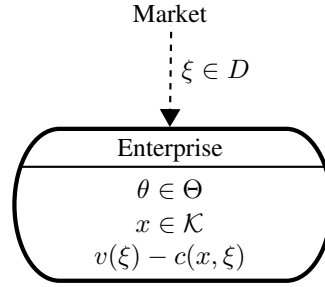


Figure 4.5: Centralised setting.

since she is more familiar with the market, while the supplier is responsible for determining the production plan and for supplying the products. This can be modelled as a *principal-agent* problem, where the customer is the agent who has the forecast as a private information (also called her *type*). Note that I do not assume that an a priori distribution about the type is known by the principal, i.e., I regard *strict incomplete information*.

According to VMI, the agent should share her forecast with the principal, this calls for a *direct-revelation mechanism*, i.e., the possible strategy of the customer is to report a forecast  $\hat{\theta} \in \Theta$ . This can also be interpreted as  $\theta$  is the “best achievable forecast”, but the agent may not make effort in forecasting and generates only  $\hat{\theta}$ . However, the value of the random variable (the demand) is independent from this effort, thus it is different from the standard *moral hazard* problem.

Since  $\theta$  cannot be observed by the principal, it is not *contractible*; the agent pays therefore depending only on  $\hat{\theta}$  and  $\xi$ . The utility of the agent is the difference between the valuation of the income and the payment:

$$v(\xi) - t(\hat{\theta}, \xi), \quad (4.24)$$

while the principal’s profit becomes the difference between the payment and the cost:

$$t(\hat{\theta}, \xi) - c(x, \xi). \quad (4.25)$$

Let us call the payment function  $t$  *strongly strategy-proof* if

$$\mathbb{E}_{\theta}[t(\hat{\theta}, \xi)] > \mathbb{E}_{\theta}[t(\theta, \xi)] \quad \forall \hat{\theta} \in \Theta \setminus \{\theta\}, \quad (4.26)$$

i.e., the agent with forecast  $\theta$  can minimise the payment—thus maximise her utility—by choosing  $\hat{\theta} = \theta$ . Applying such payment assures that truth-revelation is the only *dominant*

*strategy* of the agent. It is easy to see that if  $t$  is strongly strategy-proof then the principal chooses the first-best for maximising his expected utility (c.f., Axiom 4.1).

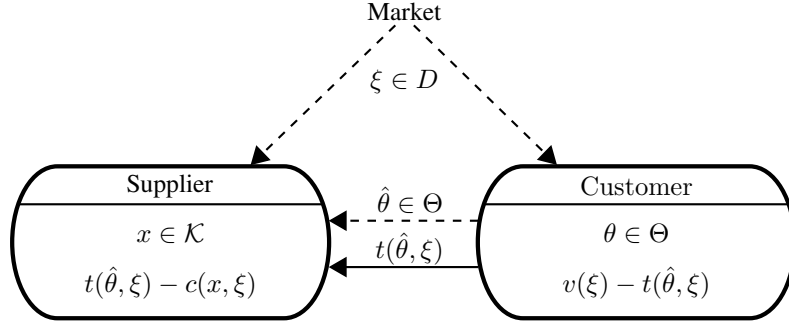


Figure 4.6: Decentralised setting with commonly known realised demand.

To sum up, the characteristics of the mechanism resulted by a strongly strategy-proof payment are as follows:

- The truth-revealing is dominant strategy for the agent.
- It is *efficient*, i.e., it results the first-best.
- It is *budget-balanced*, i.e., the payment is transferred only between the players.
- From efficiency and budget-balance properties follows the *Pareto-optimality*, i.e., any mechanism that results in a higher utility for one of the players, generates lower utility for the other.
- The mechanism does not guarantee *individual rationality*, i.e., the expected utilities of the players can be negative<sup>1</sup>.

If both the income and cost functions are common knowledge, then the payment

$$t(\hat{\theta}, \xi) = v(\xi) - \lambda \left( v(\xi) - c(f(\hat{\theta}), \xi) \right) \quad (4.27)$$

with an arbitrary  $\lambda \in (0, 1]$  is strongly strategy-proof and provides arbitrary cost and profit allocation between the players, specifically  $\lambda = \frac{1}{2}$  results in equal profits for them. If the

<sup>1</sup>Although a player with negative expected utility may decide not to participate in the game, it is not always the best alternative. Getting a new customer is always much more expensive than keeping an existing one, thus allowing temporary negative utility is common in the global competition.

expected profit of the centralised problem is non-negative, i.e.,  $\mathbb{E}_\theta[v(\xi) - c(f(\theta), \xi)] \geq 0$ , then the mechanism defined by Eq. 4.27 is (interim) individual rational for the players, i.e., their expected utilities are also non-negative.

Unfortunately, when the cost function is unobservable for the customer, then such a result does not hold as I will prove in Section 4.3.2. Before this, I present examples of strongly strategy-proof mechanisms when the cost function is not contractible.

Firstly, let us consider a trivial example, where the forecast is simply the expected value of the demand. In this case it is easy to see that for example the payment function in the form

$$t(\hat{\theta}, \xi) = \alpha|\hat{\theta} - \xi| + \beta(\xi) \quad (4.28)$$

is strongly strategy-proof, where  $\alpha > 0$  is a constant and  $\beta$  is an arbitrary function. However, if one refines the model assuming the forecast is given by the expected value and the standard deviation, finding an appropriate payment is not so straightforward. In the following, I present a strongly strategy-proof payment which is a generalisation of Theorem 4.2, without assuming any particular distribution.

**Theorem 4.7** *Let us consider a one-period problem where the forecast is given by an expected value and a standard deviation, i.e.,  $\theta = (m, \sigma)$ . Then the payment function in the form of*

$$t(\hat{m}, \hat{\sigma}, \xi) = \alpha \left( \frac{(\hat{m} - \xi)^2}{\hat{\sigma}} + \hat{\sigma} \right) + \beta(\xi) \quad (4.29)$$

*is strongly strategy-proof, where  $\alpha > 0$  is a constant and  $\beta$  is an arbitrary function.*

**Proof.** The proof is similar to the proof of Theorem 4.2.  $\square$

Practically,  $\beta(\xi)$  can be considered as the payment for the supplied products, while  $\alpha$  is the price of the VMI service. Furthermore, there is a simple intuition behind the term  $(\hat{m} - \xi)^2/\hat{\sigma} + \hat{\sigma}$ : if the customer states that the forecast is fairly precise (i.e.,  $\sigma$  is small), she is ready to pay larger compensation for the difference between the expected and the realised demand. This could be avoided by signalling higher uncertainty, but then this increases the second part of the term.

### 4.3.2 A Mechanism Design Formulation

In this section I consider a more general form of the forecast sharing game with strict incomplete information. I assume that the cost function  $c \in C = \{c : \mathcal{K} \times D \rightarrow \mathbb{R}_0^+\}$

is a private information of the supplier. Due to the *revelation principle*, we can focus on direct-revelation mechanisms, thus in the form  $\mathcal{M} = (f, t_1, t_2)$  where  $f : \Theta \times C \rightarrow \mathcal{K}$  is the choice function and  $t_i : \Theta \times C \times D \rightarrow \mathbb{R}$  are the payment functions ( $i = 1, 2$ ). The utilities of the customer and the supplier are

$$u_r(\theta, \hat{\theta}, \hat{c}, \xi) = v(\xi) - t_1(\hat{\theta}, \hat{c}, \xi) \quad (4.30)$$

and

$$u_s(c, \hat{\theta}, \hat{c}, \xi) = t_2(\hat{\theta}, \hat{c}, \xi) - c(f(\hat{\theta}, \hat{c}), \xi) \quad (4.31)$$

respectively, if their real types are  $\theta$  and  $c$  but they claim  $\hat{\theta}$  and  $\hat{c}$  instead.

In this formulation the mechanism can be considered as an *independent third party*, whom the players share their information, who decides about the production plan, which then has to be executed by the supplier. For this service, the mechanism keeps the difference between  $t_1$  and  $t_2$ , see Fig. 4.7.

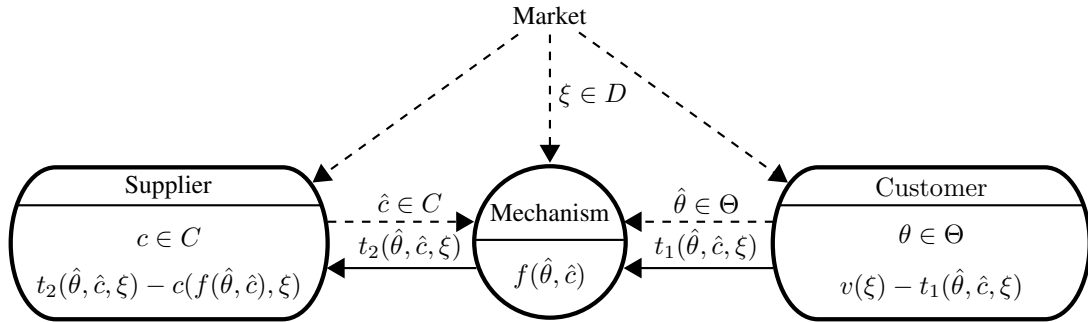


Figure 4.7: Mechanism design setting.

**Definition 4.8** The choice function  $f$  is efficient if it maximises social welfare<sup>2</sup>, i.e.,

$$f(\theta, c) \in \operatorname{argmax}_{x \in \mathcal{K}} \mathbb{E}_\theta[v(\xi) - c(x, \xi)] = \operatorname{argmin}_{x \in \mathcal{K}} \mathbb{E}_\theta[c(x, \xi)] \quad \forall \theta \in \Theta, \forall c \in C. \quad (4.32)$$

**Definition 4.9** A mechanism  $\mathcal{M}$  is (weakly) strategy-proof, if  $\forall \theta, \hat{\theta} \in \Theta, \forall c, \hat{c} \in C$ :

$$\mathbb{E}_\theta[u_r(\theta, \theta, c, \xi)] \geq \mathbb{E}_\theta[u_r(\theta, \hat{\theta}, c, \xi)] \quad (4.33)$$

<sup>2</sup>The sum of the utilities without the payments.

and

$$\mathbb{E}_\theta[u_s(c, \theta, c, \xi)] \geq \mathbb{E}_\theta[u_s(c, \theta, \hat{c}, \xi)], \quad (4.34)$$

i.e., truth telling is a dominant strategy for both players. The definition can be modified for strong strategy-proofness, which assures that truth telling is the only dominant strategy.

Firstly, I show that if a strategy-proof mechanism gives the same output for different cost functions, then it gives the same payment for the supplier.

**Proposition 4.10** *If  $\mathcal{M}$  is a strategy-proof mechanism,  $c, \hat{c} \in C, \theta \in \Theta$  such that  $f(\theta, c) = f(\theta, \hat{c})$ , then  $\mathbb{E}_\theta[t_2(\theta, c, \xi)] = \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)]$ .*

**Proof.** Let us assume that  $\mathbb{E}_\theta[t_2(\theta, c, \xi)] < \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)]$ . But then

$$\mathbb{E}_\theta[t_2(\theta, c, \xi)] - \mathbb{E}_\theta[c(f(\theta, c), \xi)] < \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)] - \mathbb{E}_\theta[c(f(\theta, \hat{c}), \xi)], \quad (4.35)$$

i.e., the mechanism is not strategy-proof, since the supplier with type  $c$  would state that his type is  $\hat{c}$ .  $\square$

I now prove that if we are looking for an efficient and strategy-proof mechanism, then  $t_2$  should be independent from the cost function of the supplier, therefore it excludes the cost and profit sharing between the players.

**Theorem 4.11** *Let  $\mathcal{M}$  be an efficient, weakly strategy-proof mechanism. Then*

$$\forall \theta \in \Theta, \forall c, \hat{c} \in C : \quad \mathbb{E}_\theta[t_2(\theta, c, \xi)] = \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)]. \quad (4.36)$$

**Proof.** The proof is similar to the proof of the uniqueness of Groves mechanism among efficient and strategy-proof mechanisms, thus I exploit that the cost function can be arbitrary function.

Let us consider a fixed  $\theta$  and indirectly assume that  $\exists c, \hat{c} \in C : \mathbb{E}_\theta[t_2(\theta, c, \xi)] > \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)]$  and define

$$\epsilon = \mathbb{E}_\theta[t_2(\theta, c, \xi)] - \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)] > 0. \quad (4.37)$$

Due to the modus tollens of Proposition 4.10,  $f(\theta, c) \neq f(\theta, \hat{c})$ . Because the cost function can be arbitrary,  $\exists \tilde{c} \in C, \exists k \in \mathbb{R} :$

$$\mathbb{E}_\theta[\tilde{c}(f(\theta, \hat{c}), \xi)] = k \quad (4.38)$$

$$\mathbb{E}_\theta[\tilde{c}(x, \xi)] > k \quad \forall x \neq f(\theta, \hat{c}) \quad (4.39)$$

$$\mathbb{E}_\theta[\tilde{c}(f(\theta, c), \xi)] < k + \epsilon \quad (4.40)$$

From the efficiency of the mechanism follows that  $f(\theta, \tilde{c}) = f(\theta, \hat{c})$  and then from Proposition 4.10,  $\mathbb{E}_\theta[t_2(\theta, \tilde{c}, \xi)] = \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)]$ . But then

$$\mathbb{E}_\theta[u_s(\tilde{c}, \theta, \tilde{c}, \xi)] = \mathbb{E}_\theta[t_2(\theta, \tilde{c}, \xi)] - \mathbb{E}_\theta[\tilde{c}(f(\theta, \tilde{c}), \xi)] = \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)] - k \quad (4.41)$$

and

$$\mathbb{E}_\theta[u_s(\tilde{c}, \theta, c, \xi)] = \mathbb{E}_\theta[t_2(\theta, c, \xi)] - \mathbb{E}_\theta[\tilde{c}(f(\theta, c), \xi)] > \mathbb{E}_\theta[t_2(\theta, c, \xi)] - k - \epsilon = \mathbb{E}_\theta[t_2(\theta, \hat{c}, \xi)] - k, \quad (4.42)$$

thus the mechanism is not strategy-proof.  $\square$

The theorem proves the reasonable conjecture that the supplier can claim higher costs in such a way, that the optimal production plan does not change, thus he may try to obtain more payment without increasing his costs.

Although theorem 4.11 is rather negative if we are aimed at fair cost and profit sharing, it has some positive consequences as well. If  $t_1 = t_2$  is independent from the cost function, the supplier will chose the efficient outcome without any central decision maker (Axiom 4.1). The outcome may be even unobservable for the customer, since her utility is independent from the outcome.

### 4.3.3 Open Questions and Future Work

Since the VMI business model has become widespread recently, I believe that it is important to study it and the presented forecast sharing game points towards this direction. Despite the negative consequences of theorem 4.11, there are several possible solutions to guarantee individual rationality, some of them are as follows

- In the proof of theorem 4.11, I exploited that the cost function can be arbitrary. If one considers a certain form of the cost function, maybe even the customer has a belief about its parameters—which assumption can be conceivable in practice—then we may get different results.
- In this model I assumed that the entire demand should be fulfilled. One can study the situation, when lost sales are allowed. In this case theorem 4.11 holds no more.
- If lost sales are allowed, one may also study such mechanisms that are not effective, only approximating.



## 4.4 Practical Considerations

### 4.4.1 Length of the Stability Horizon

Throughout the section I have assumed that the stability horizon equals to the forecast horizon, i.e.,  $n' = n$ . This assumption is explicable with the fact that if imprecision is not measured in the end of the planning horizon, the planners do not have incentives to improve forecast quality in those periods. In the practice however, this restriction would lead either to an unacceptably long stability horizon or an insufficiently short planning horizon.

We implemented these measurements in an industrial supply chain information sharing application (see Sect. 5.1). Depending on the production characteristics of the supplier, the stability horizon is set to 4-16 weeks, while the planning horizon is generally 36-48 weeks long. As extensive data analysis showed, the deviations in the stability horizon are relatively small, but usually large on the whole planning horizon. However, the long planning horizon is necessary: due to seasonality, a part of the high demand of autumn and winter must be produced in springtime. Without this *production smoothing*, idle capacities and capacity shortages would cause serious problems both in the service level and in the production efficiency. This means that improving planning on the post-stability horizon implies better precision on the stability horizon, too. All in all, the customer has no incentives to share the true forecast beyond the stability horizon.

### 4.4.2 Compensation Parameters

The models presented in this chapter assume that  $c_0$  unit price,  $c_1$  and  $c_2$  compensation prices are preliminary fixed parameters. In a theoretical situation, when the production costs and end-market prices were common knowledge, these parameters could be used for cost and profit sharing between the partners. However, in real supply chains these are also private information, like the demand forecasts. For a fair allocation of costs and benefits a complete coordination mechanism resulting in truthful sharing of all private information—if such exists at all—should be developed, but this problem is beyond the scope of this dissertation; it is an open issue of contemporary research of supply chain coordination.

As an acceptable practical trade-off, I propose that the negotiation process about payment parameters between the partners should be supported by a simulation environment

based on both historical and random data. Such a tool can help estimating the long-term results of different parameters. Fig. 4.8 shows an example of the payments in each period where the compensation terms are usually much less than the price paid for components—as long as no run-out happens. When the product runs-out however, the compensation increases (cf. Eq. 4.17) to redeem for the obsolete inventory at the supplier.

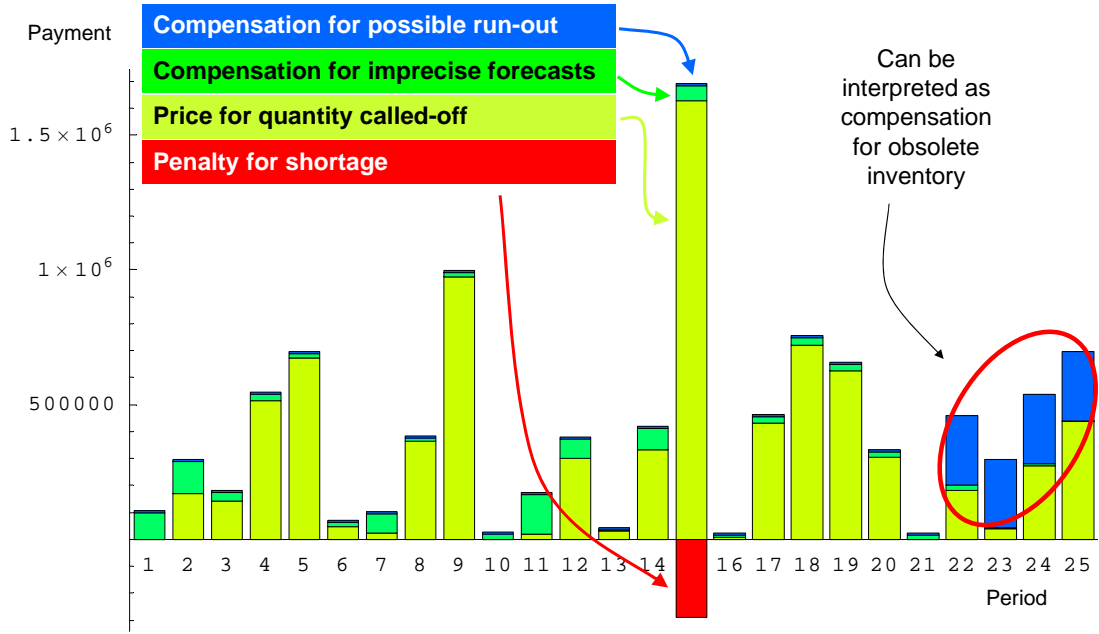


Figure 4.8: Components of the payment.

## 4.5 Experiments

In this section I analyse the examples of Sect. 3.4 in a decentralised setting. In this case the profit of the customer is the income for the end-products minus the payment for the required components minus further production, inventory holding and logistic costs. Since I do not have not only the necessary parameters, but also the structure of this profit function for the real cases, I rather focus on the supplier's profit, which can be modelled with the payment minus the cost. According to the main results of this chapter, I also assume that the optimal lot-sizes of the decentralised problems equal to the solutions of the centralised systems.

### 4.5.1 Experiments with the Decentralised Newsvendor Model

I present here the result of the simulations using the parameters from the first row of Table 3.3, i.e.,  $c_s = 55269.5$ ,  $c_p = 3.15$  and  $m = 7152$ . I used  $b = 393.30$  (which means 10% relative deviation),  $c_0 = 10$  and simulated the arisen costs and payments for different  $c_1$  compensation parameters by averaging 1000 simulation runs in each scenario. The results can be found in Table 4.2.

Table 4.2: Compensation's effect on supplier's profit.

$c_1$	$q^*$	$p$	$C_d$	$C_u$	$P$	$Z$	$\pi_S$
1	8640	71410	1375	1297	74082	83846	-9765
2	8640	71385	2504	2594	76483	83778	-7295
3	8640	71533	3814	3892	79239	83849	-4611
4	8640	71458	6043	5189	82690	83849	-1159
5	8640	72052	7295	6486	85833	84179	1654
6	8640	71544	8416	7783	87743	83723	4019
7	8640	71292	9286	9081	89658	83899	5759
8	8640	71614	10962	10378	92954	83682	9272
9	8640	72102	12017	11675	95794	83674	12120
10	8640	71498	11704	12972	96173	83787	12386

Here  $q^*$  denotes the optimal lot size, which does not change, since the solution of the newsvendor model is independent from the compensation parameter. The  $p$ ,  $C_d$  and  $C_u$  mean the three parts of the payment function: the price paid for the components, the compensation for deviation and compensation for forecast uncertainty, respectively. The price is independent from  $c_1$ , but fluctuates slightly due to the random demand. The two parts of compensation evidently increase with  $c_1$ . The total payment is the sum of the three parts, denoted by  $P$  and can be compared with the total arisen cost ( $Z$ ). The profit of the supplier is  $\pi_S = P - Z$ .

I also performed *ceteris paribus* sensitivity analysis of each parameter, i.e., with all other parameters left fixed. An interesting example of changing the expected demand can be seen in Table 4.3, where I used the same  $c_s$ ,  $c_p$  and  $c_0$  parameters as in the previous example and set  $c_1 = 1$  and the relative deviation 10%.

Table 4.3: Expected demand's effect on supplier's profit.

$m$	$b$	$q^*$	$p$	$C_d$	$C_u$	$P$	$Z$	$\pi_S$
6000	331	7307	59933	982	1088	62003	79017	-17014
7000	386	8464	70189	1339	1270	72797	83467	-10670
8000	441	9613	79927	1516	1451	82894	86972	-4077
9000	496	10755	90087	1646	1632	93366	90630	2736
10000	551	11890	100351	1797	1814	103962	94761	9201
11000	606	13019	110264	2028	1995	114286	97998	16289
12000	662	14143	119937	1999	2177	124113	101424	22689
13000	717	15262	130013	2319	2358	134690	105415	29275
14000	772	16376	140218	2519	2539	145277	110068	35209
15000	827	17486	149878	2839	2721	155438	113415	42023
16000	882	18592	160751	3105	2902	166758	117680	49079

In this case, both the payment and the cost increase with the expected demand, but due to their different slopes, the larger the demand, the larger the profit of the supplier. One can notice that  $C_d \approx C_u$  in every simulation. This is not accidental: if  $m' = m$  and  $b' = b$ , then  $\mathbb{E}[C_d] = C_u$ . The comparison of costs (narrow bars) and payments (wide bars) is graphically illustrated in Fig. 4.9.

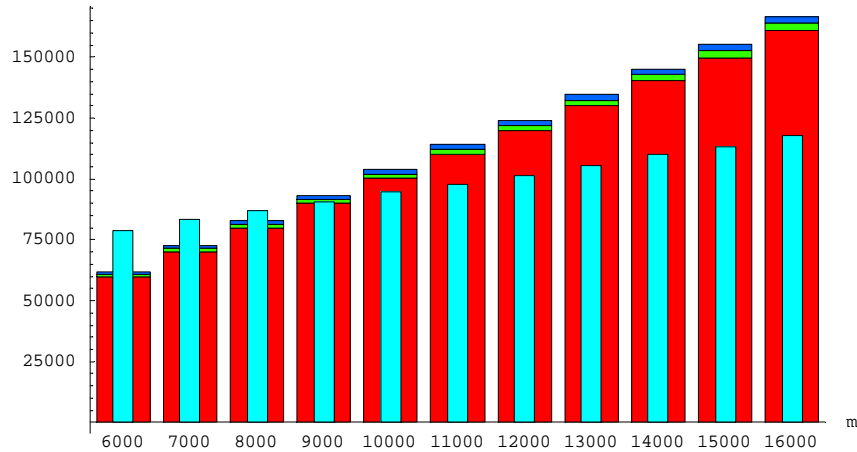


Figure 4.9: Payment versus cost.

### 4.5.2 Experiments with the Rolling Horizon Coordination Model

I illustrate the imaginary payment in Fig. 4.10 for the same component that was presented in Fig. 3.15. I applied the same parameters as in Sect. 3.4 plus the following ones:  $n' = 4$ ,  $c_0 = 100$ ,  $c_1 = 2$  and  $c_2 = 10000$ .

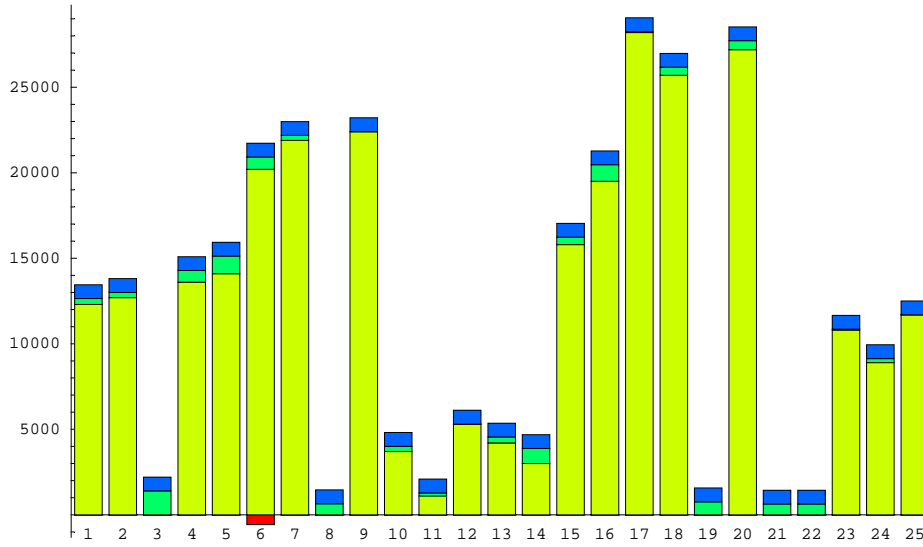


Figure 4.10: Example payments based on real forecast and consumption data.

In this case, the simulation approach can also be used to compare the cost and the payment of the supplier facing stochastic demand. Table 4.4 contains the results of the same simulation as in Table 3.4, but this time I included the payment—in total and in parts—to be able to estimate the profit of the supplier.

Simulation can also illustrate how the costs and payments change when the customer shares inappropriate estimations for the run-out probability. Table 4.5 presents the results of the experiments with the 1153 materials introduced in Sect. 3.4.

I generated the forecasts assuming geometric run-out distribution with  $p = 0.02$ . As we have already seen in the last chapter, communicating the parameter increased or decreased by 0.01 results in approximately 2% larger costs for the supplier. But due to the compensation, the payment also increases; in this case almost with 4%. The effect of the parameter distortion to the supplier's profit depends on other parameters as well, thus it

Table 4.4: Summary of 1000 simulation runs.

Statistics	AVG	STD	MAX	MIN
Cost	114579.04	1301.70	118793.79	110262.21
Total payment	303129.42	31910.67	403088.70	195304.70
Payment for call-offs	282825.2	31910.46	382800	175000
Compensation for deviation	101.52	18.49	180	56
Compensation for possible run-out	20202.70	0	20202.70	20202.70

Table 4.5: The effect of the parameter estimation.

	$p' = 0.01$	$p' = 0.03$
Cost	102.1%	102.27%
Total payment	103.93%	103.85%
Profit of the supplier	105.51%	108.94%

might increase—as in this example—or decrease, while customer always loses on being not truthful.

The same data source and parameters was used to study the effects of the inappropriate forecasting, but in this case, the forecasted demand was manipulated instead of the run-out parameter. Fig. 4.6 shows the relative differences when the demand is systematically over/underestimated by 5/10%. While the costs and payments both increase with the imprecision, the profit of the customer—as in the previous experiment—behave unpredictable. These two empirical studies confirm the proven property: the customer can expect to pay more when the shared information is not appropriate.

Table 4.6: The effect of the forecast quality.

	−10%	−5%	+5%	+10%
Cost	102.66%	101.601%	101.226%	101.274%
Total payment	104.08%	103.272%	103.535%	103.537%
Profit of the supplier	102.3%	100.503%	105.484%	102.293%

## 4.6 Summary

In this chapter I studied the two previously presented models in a decentralised setting with two autonomous, rational enterprises along a supply chain with asymmetric information. In order to achieve the optimal centralised solution, I proposed that the supplier should provide a service of managing the channel inventory (the practically widespread VMI approach) and the customer should pay also for this service. I presented the appropriate payment functions for achieving channel coordination, and finally, I illustrated their properties with some simulation results.

Although the newsvendor model is widespread in the channel coordination literature, my approach differs in several aspects from the existing results. I consider asymmetric information, quantitative demand forecast, VMI, 100% service level requirement and I presented a protocol with compensational payment that coordinates the channel. Hence, my formulation and solution for this special case is a novel research result.

As I already mentioned in the literature review, the channel coordination problem on a longer horizon, considering rolling horizon planning is neglected in the literature; therefore my results study a new direction of the channel coordination. The presented approach is based on the similar idea of compensation that I applied in the newsvendor case, but it is able to consider practical planning problems better than the one-period models.

In addition, I started to generalise the problem using the apparatus of the mechanism design theory. I presented some preliminary results and mentioned some interesting future research direction in this field.

# Chapter 5

## Applications in Supply Chain Planning

This last chapter briefly overviews two implemented systems aimed at increasing supply chain performance. The first one is a complex information sharing and monitoring platform deployed at the focal manufacturer participating in our research project. The system contains different performance measurements including the *error* and the *deviation* of the forecasts, presented in Section 4.2.1. The other application is a pilot simulation environment for analysing the behaviour of a VMI supply relation. The system applies a combination of the two lot-sizing models presented in Chapter 3, and evaluates the performance and the payment considering the rolling horizon coordination contract. The data interface of the simulation can work either with the database of the previously mentioned information sharing platform, or with a specialised random forecast generator.

### 5.1 Designing an Information Sharing Platform

In this section, I present a real-life development case study of a distributed planning system. Since this work was done in a team, I use first person plural instead of singular here.

#### 5.1.1 Problem Statement

Our work in the production network presented in Section 1.3 was aimed in general at the improvement of the overall logistic and production performance, but at the beginning the



actual details of the means to this end were unclear even for the industrial partners. We started the work by creating a specialised network model based on a multiagent organisational reference model [33]. In particular, we identified, analysed and described every planning role in each enterprise of the network—including the used decision support systems, algorithms, human heuristics, planning granularity, cycle times, etc.—, the interaction protocols between enterprises as well as the existing information resources. As a result, we could point out several issues that were critical to some key performance indicators (KPI), and also some special circumstances, which could be exploited in the algorithms proposed in Chapter 3. Some important characteristics of the suppliers are as follows:

**Setup.** Some suppliers have huge setup costs, therefore they produce in large batches based on medium-term forecasts. Others have negligible setups, thus—whenever lead-times allow—they produce only to order.

**Capacity constraint.** For some components, the production time is very small, therefore they can be produced in arbitrary large batches—as long as there is enough raw material. Hence, there is practically no capacity constraint. In other cases such constraints should be considered.

**Raw material.** For almost every supplier there are some raw materials that can be procured with long lead-time (months), therefore their supply should be based on medium-term forecasts.

**Customisation.** The components can be either standardised or customised. The demand for the former ones can be considered stable, but in the latter case, run-out can occur and sometimes obsolete inventories remain.

**Supply lead-time.** The production of the components usually takes five days—as long as raw material is available—, while the transportation time is one day—due to the regional type of the network. Therefore the supply lead-time—the possible minimal time between component order and consumption—is considered to be one week.

In our case, we could classify suppliers into two groups: (i) the suppliers of standard components work with small setup times and costs, but strict capacity constraints applying make-to-order approach, while (ii) the suppliers of mostly customised packaging materials

have high setup costs, produce to stock based on medium-term forecasts, but the production capacities are large enough to consider them infinite. In the latter case the relatively high risk of producing obsolete inventories may involve huge financial losses, hence we decided to concentrate on this problem that offered the best possibility to decrease inefficiency. For this purpose I developed the algorithms presented in Chapter 3 and proposed the channel coordination approach of Chapter 4. Note that although we concentrated on packaging material suppliers, the precision and stability of the medium-term forecasts are important also for other suppliers so as to manage their long lead-time procurements.

### 5.1.2 System Design

Based on the detailed network model, we concluded that information transparency is essential in order to provide basis for the MTS production and for the procurement at the suppliers. In accordance with the industrial partners, we decided to develop an information sharing system called *Logistics Platform* (LP) and prepare it to support the VMI approach and JIT deliveries, too. Therefore the designed system covers two planning levels: medium and short term.

On medium term the goal is to achieve more efficient component production and raw material purchase, thus the customer enterprise should share her component forecasts derived from his production plan. On the short term, however, the supply service level and the cost efficient delivery are the main objectives. For this reason, the short-term component consumption plan derived from the customer's production schedule, the inventory levels and the suppliers' transportation schedule ought to be shared in the system. The production schedule of the customer and its dependent daily material demand in our case were generated by a custom-tailored scheduler system that was developed in the same project [26].

Since the studied network was focal, the system could be deployed at the customer. This way, it can easily access the required information from other systems, and in addition, it can use the existing corporate security and single sign-on (SSO) authentication technologies applied by the customer. The users of every enterprise can access only the permitted information via a controlled web interface. The architecture of the system can be seen in Fig. 5.1.

Beyond information sharing, the platform has two further functions. On one hand, it

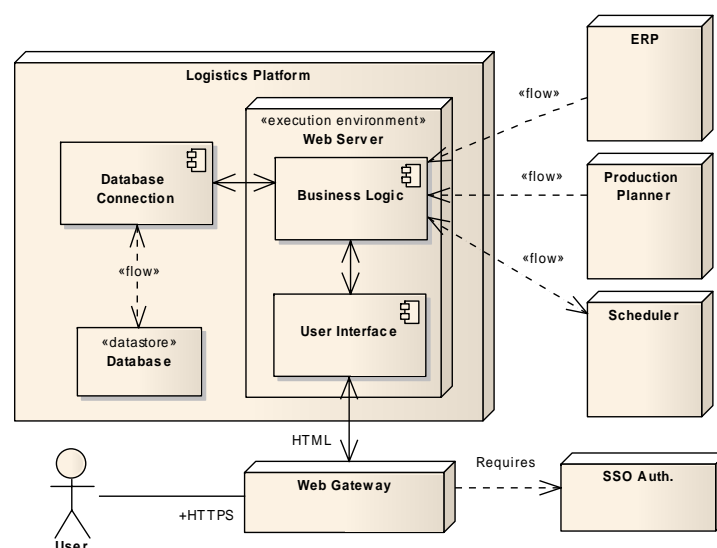


Figure 5.1: Architecture of the information sharing platform.

monitors the supply process by comparing planned component consumption and expected delivery, i.e., helps detecting and avoiding possible future shortages. On the other hand, it evaluates past performance in terms of the forecast imprecision presented in Section 4.2.1 as well as the service level of the suppliers. Note that decision making function is not included in the system.

The LP is in a daily use for more than a year now, and it is constantly improved based on the experiences and new requirements of its users. Currently, more than 40 plants—1 focal customer, 5 internal and several external suppliers—, more than 60 users and 10000 different components are defined in the system.

Automated information sharing between enterprises along supply chains is sorely needed for coordinating supply with demand and even for enhancing efficiency by cooperation. As it turned out, using the LP also helped human experts detecting serious glitches and inconsistencies in the existing planning processes and data administration. For some further details and lessons of the usage I refer to publications [28, 110].

Forecast week	Manufacturing week								PD
	2008-01-21 (-8)	2008-01-28 (-7)	2008-02-04 (-6)	2008-02-11 (-5)	2008-02-18 (-4)	2008-02-25 (-3)	2008-03-03 (-2)	2008-03-10 (-1)	
2008-01-14 (-9)	4,623.6	5,201.8	2,244.4	2,209.6					-10.7 %
2008-01-21 (-8)	3,218.4	5,688.5	9,835.7	3,359.1	2,719.2				19.7 %
2008-01-28 (-7)		5,597.6	4,756.9	9,386.7	3,028.2	3,432.0			24.8 %
2008-02-04 (-6)			3,657.3	6,163.8	6,643.5	3,460.8	2,370.3		-3.0 %
2008-02-11 (-5)				3,519.1	8,318.3	8,304.7	3,876.9	3,758.7	9.0 %
2008-02-18 (-4)					5,279.3	7,278.9	8,516.6	3,315.8	N/A
2008-02-25 (-3)						4,054.4	N/A	N/A	N/A
2008-03-03 (-2)							6,368.5	9,440.1	N/A
2008-03-10 (-1)								6,554.5	N/A
MND	0.0 %	-7.1 %	-38.6 %	-37.2 %	-48.5 %	-15.4 %	-62.8 %	-49.4 %	
MPD	43.7 %	1.6 %	168.9 %	166.7 %	57.6 %	104.8 %	33.7 %	44.0 %	
DFE	16.4 %	-1.5 %	52.5 %	71.2 %	14.2 %	55.4 %	-6.2 %	0.9 %	
DAFE	16.4 %	2.7 %	68.6 %	82.4 %	44.1 %	65.4 %	25.8 %	32.1 %	

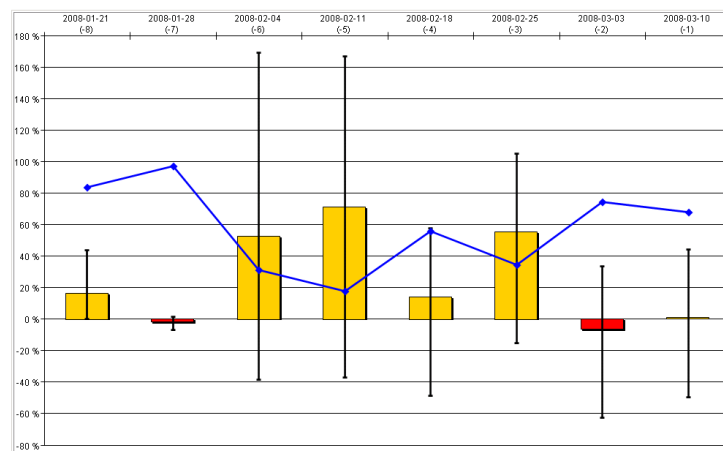


Figure 5.2: Forecast evaluation in the LP.

## 5.2 Simulation Environment

For testing the algorithms and protocols, I developed a simulation system called *InventoSim*, whose architecture can be seen in Fig. 5.3. The programme was written in Mathematica 5.2, however, the WWr algorithm was implemented in Java and called through the JLink API. As the figure shows, it can operate in two modes: it either reads real forecasts from the database of the LP, or uses a random number generator for this purpose. While the former method is useful for evaluating the algorithms with real problem instances, the latter facilitates carrying out systematic tests and gives more insights to the average performance.

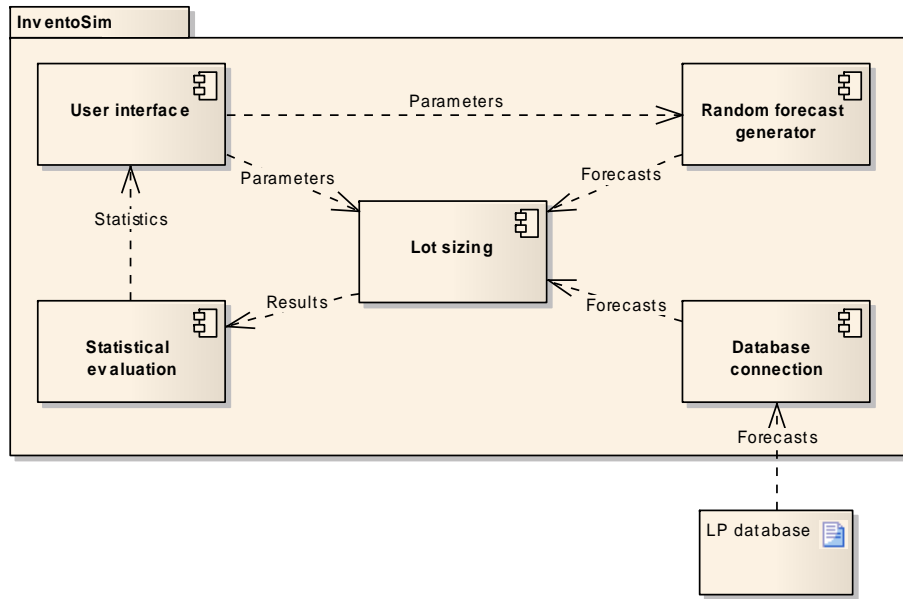


Figure 5.3: Architecture of the simulator.

### 5.2.1 Parameters

On the user interface several parameters can be set (the parameters denoted with \* can be set automatically or are not necessary when working from the LP database):

- Horizon related data.

**Horizon\***. The number of generated forecast in a rolling horizon manner.

**Forecast horizon\***. Length of the forecasts ( $n$ ).

**Stability horizon**. Length of the measurement time window ( $n'$ ).

- Cost parameters.

**Setup cost**. The  $c_s$  parameter of the lot-sizing models.

**Production cost**. The  $c_p$  parameter of the lot-sizing models.

**Inventory holding cost**. The  $h$  parameter of the lot-sizing models.

**Shortage cost**. Although in the models the shortages are excluded, practically they can happen, therefore I introduced a penalty for them, proportional to the absent quantity.

- Price parameters.

**Unit price.** The  $c_0$  parameter of the coordination models.

**Compensation for deviation.** The  $c_1$  parameter of the coordination models.

**Compensation for possible run-out.** The  $c_2$  parameter of the coordination models.

- Demand parameters.

**Average demand\*.** Used for forecast generation (see below).

**Relative deviation\*.** Used for forecast generation.

**Shift probability\*.** Used for forecast generation.

**Run-out distribution.** Used both for forecast generation and in the WWr algorithm. It can be either geometric, uniform or Poisson distribution (see Sect. 3.2.3).

**Real run-out parameter\*.** Used for forecast generation.

**Estimated run-out parameter.** Used in the WWr algorithm. It can differ from the real run-out parameter, thus the effect of an inappropriate estimation can be analysed with simulations.

**Shortage effect.** When shortage occurs, the absent quantity can be either (i) backlogged and satisfied later, (ii) lost or (iii) lost with the whole order cancelled, i.e., the order can be fulfilled only completely or not at all.

- Miscellaneous parameters.

**Initial inventory\*.** The forecast will be decreased with the available quantity.

**Safety stock policy.** The simple policies presented in Sect. 3.3.3 can be chosen.

**Safety stock parameter.** The parameter of the previous policy.

**Number of simulation runs\*.** The simulations can be run several times with the same parameters for statistical evaluation.

Note that the LP does not contain any information about prices or costs of production, most of these data are known approximately, though. Another difficulty is that the system does not register whether a component runs out or not. The run-out components have

constant zero forecasts, but this does not mean necessarily a run-out: temporary long pauses in the demand can also happen.

### 5.2.2 Random Rolling Horizon Forecast Generation

Demand forecast is a fundamental input for most inventory and production planning methods. Forecast is usually generated for a given horizon of the future and as the time goes by and new information becomes available, the forecast is repetitively updated. This phenomenon is expressed in the second law of forecasting: “forecasts always change” [45].

The most widespread quantitative methods of forecasting are the *time series models* (e.g., moving average, exponential smoothing, etc.) which predict the future demand based on the past demand values. In practice however, these quantitative forecasts are always revised by human experts based on market information.

Thus for modelling an existing forecasting process, applying time series model is inappropriate, for it completely disregards the changes caused by qualitative methods [43]. In several situations the *Martingale Model of Forecast Evolution* (MMFE) is considered instead, that will be further studied hereinafter.

It is important emphasising that MMFE is not a forecasting method. It can be used in stochastic inventory and production planning models ([39, 106]) and in simulations ([48]). For a short review of models applying MMFE, I refer to [13]. The basic definitions related to martingales can be found e.g., in [40].

#### The Standard MMFE Model

The first use of MMFE was presented in [38], which considered the forecast evolution of a single-item with uncorrelated demand. The model was generalised for the multi-product case by Heath and Jackson in [43], where the correlation across products and time periods were considered and where the model was named as MMFE. For the sake of simplicity, I overview only the single-item version of the model below, that I use as a basic in my forecast generation.

Let  $F_{t,t+i}$  denote the forecast made in period  $t$  for demand in period  $t+i$ ,  $i \in \{1, \dots, n\}$ , where  $n$  is the length of the forecast horizon. It is assumed that beyond the horizon, the forecast is implicitly given by a long-run average<sup>1</sup>  $d$ :  $F_{t,t+i} = d$ ,  $i > n$ . It is also assumed

<sup>1</sup>The model can be modified considering varying  $d_{t+i}$  for modelling life-cycle phases (e.g., ramp-up),

that the realised demand in period  $t$  is known in that period and it is denoted by  $F_{t,t}$ . Past “forecast” are considered to be known:  $F_{t,t-i} = F_{t-i,t-i}$ ,  $i \in \{1, \dots, t\}$ .

Let us define the *forecast update*<sup>2</sup>  $\varepsilon_{t,t+i} = F_{t,t+i} - F_{t-1,t+i}$  and the *forecast update vector*  $\varepsilon_t = (\varepsilon_{t,t}, \dots, \varepsilon_{t,t+n})$ . The MMFE model has four assumptions:

**A1** If we consider  $F_t = (F_{t,t}, F_{t,t+1}, \dots)$  then it is assumed that  $F_0, \dots, F_t$  are known in period  $t$ . Let us define  $\mathcal{F}_t = \sigma(F_0, \dots, F_t)$  as the smallest  $\sigma$ -field with respect to which  $F_0, \dots, F_t$  are each measurable, then the sequence  $\{\mathcal{F}_t\}$  is a *filtration*. This assumption simply states that the knowledge grows with time.

**A2** It is assumed that  $\{(F_{t,\tau}, \mathcal{F}_t), t \geq 0\}$  is a *martingale*  $\forall \tau \geq 0$ , i.e.,  $\mathbb{E}[F_{t+1,\tau} \mid \mathcal{F}_t] = F_{t,\tau}$ . This assumption states that if we are in period  $t$ , then the expected value of the demand in the period  $t+i$  is  $F_{t,t+i}$ . From this assumption  $\mathbb{E}[\varepsilon_{t,t+i}] = 0$  follows.

**A3** The  $\varepsilon_t$  vectors are independent and identically distributed (they form a static stochastic process).

**A4** The distribution of the  $\varepsilon_t$  vector is multivariate normal (with mean 0 from **A2**).

It follows from the assumptions that the model can be described with two parameters: the initial  $F_0$  state and the  $\Sigma$  covariance matrix of the normal distribution, which can be determined from past forecasts and demand data.

A weakness of the additive model pointed out by the authors is that the variance of the forecast update is independent from the forecasts. They found however that the deviations of the forecasts are usually proportional to the sizes of the forecasts, thus they introduced the multiplicative model which can grasp this property, but still fits in the model described with martingales.

In the following I show and analyse two further phenomena of the forecast generation which cannot be expressed in the original MMFE.

### Products with Uncertain Life-Cycle

An important characteristic of the today’s markets is that the demand for customised products (or components) can suddenly cease. This means that if the product runs out in

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although this possibility was disregarded in the previous papers.

<sup>2</sup>In fact, this is the *additive* model. For the *multiplicative* model I refer to [43].



period  $\tau$ , then  $F_{\tau,\tau} = F_{\tau+1,\tau+1} = \dots = 0$ . This phenomenon cannot be described with the above defined MMFE, thus it is necessary to study this situation and extend the model.

Let  $r_t$  denote the event of the run-out, i.e.,  $r_t = 0$  if the product did not run out until period  $t$ , and 1 otherwise. Furthermore, let  $p_{t,t+i} < 1$  denote the estimation of the probability that the product will run out exactly in period  $t+i$ ,  $i > 0$ , made in period  $t$ . I assume that for all  $t$ :  $r_t + \sum_{i=1}^{\infty} p_{t,t+i} = 1$  and I define  $p_t = (p_{t,t+1}, p_{t,t+2}, \dots)$ . Let us define the probability that the product runs out in the next  $i$  periods:  $c_{t,t+i} = \sum_{\tau=t+1}^{t+i} p_{t,\tau}$ .

Take as filtration the sequence  $\mathcal{F}_t = \sigma(F_0, \dots, F_t, r_0, \dots, r_t, p_0, \dots, p_t)$ . I assume that

$$\mathbb{E}[r_{t+i} \mid \mathcal{F}_t] = r_t + c_{t,t+i} \quad (i > 0), \quad (5.1)$$

thus  $\{(r_t, \mathcal{F}_t), t \geq 0\}$  is a *submartingale*.

It is a basic question how should we define the  $F_{t,t+i}$  forecast in this setting. I suppose the following interpretation: as far as there is no run-out, the expected demand in period  $t+i$  is  $F_{t,t+i}$ , otherwise it is 0. More precisely let us assume that  $\mathbb{E}[F_{t+1,\tau} \mid \mathcal{F}_t, r_{t+1} = r] = (1-r)F_{t,\tau}$ . Using Eq. 5.1:  $\mathbb{E}[F_{t+1,\tau} \mid \mathcal{F}_t] = (1-r_t - p_{t,t+1})F_{t,\tau}$ , i.e.,  $\{(F_{t,\tau}, \mathcal{F}_t), t \geq 0\}$  is a *supermartingale* for all  $\tau \geq 0$ .

This contradicts with the second assumption of MMFE, whose authors argue that “if the second assumption is not satisfied it will be possible to construct improved predictions (in the mean squared error sense)”. Although it is true, I still stick to the presented interpretation, due to the following reason: the probability of the run-out is usually small<sup>3</sup>,  $\mathbb{E}[r_{t+i} \mid \mathcal{F}_t] \approx r_t$ , and the most probable value of  $F_{t+i,t+i}$  in period  $t$  is usually  $F_{t,t+i}$ . In probability theory this value (or set of values) is called the *mode*, i.e., where the probability function attains its maximum.

For an illustration, let us consider the following simple example. Let  $\xi$  be a random variable with the following distribution:  $\Pr(\xi = 0) = 1/2$ ,  $\Pr(\xi = 1) = \Pr(\xi = 2) = \Pr(\xi = 3) = 1/6$ . In this case  $\mathbb{E}[\xi] = 1$ , but its mode is 0. Which would be the more rational forecast for  $\xi$ ?

## Demand Shifts

The multiplicative MMFE can characterise the phenomenon when the forecast change of a period depends on the forecasted quantity, i.e.,  $F_{t,t+i} = (1 + \varepsilon_{t,t+i})F_{t-1,t+i}$ . I have found

---

<sup>3</sup>If the probability is high, it is better to use one-period models instead of the rolling horizon ones, see Section 3.

however that the change also depends on the neighbouring quantities: the planners tend to hasten urgent task, postpone less important ones or simply redistribute tasks to fill the capacities<sup>4</sup>. This kind of demand shifts highly affects the size of the optimal safety stocks, and although the main goal of Heath and Jackson was to determine safety stock levels by simulation, they disregarded this kind of variations.

I consider the following model of demand shifts: let the random variable  $s_{t,t+i} \in \{-1, 0, 1\}$  denote the direction of the demand shift of period  $t+i$  in period  $t$  (left, none and right, respectively). The proportion of the shifted demand is the random variable  $r_{t,t+i} \in [0, 1]$ . Thus the new forecast is the previous forecast, minus the shifted out quantity, plus the shifted in quantity:

$$\begin{aligned} F_{t,t+i} = & (1 - s_{t,t+i}^2 r_{t,t+i}) F_{t-1,t+i} \\ & + \frac{s_{t,t+i-1}^2 + s_{t,t+i-1}}{2} r_{t,t+i-1} F_{t-1,t+i-1} \\ & + \frac{s_{t,t+i+1}^2 - s_{t,t+i+1}}{2} r_{t,t+i+1} F_{t-1,t+i+1}, \end{aligned} \quad (5.2)$$

where I assumed that  $s_{t,t+i} = 0$  if  $i > n$  or  $i < 0$ , furthermore  $s_{t,t} \geq 0$  and  $s_{t,t+n} \leq 0$ . This means that shifts can occur only in the horizon.

It is clear that in general the martingale assumption does not hold again. The forecast could be modified to satisfy the assumption, but I argue that this would again distort the representation of the process. However, the following equality still holds for the cumulative forecast:  $\mathbb{E}[\sum_{i=0}^n F_{t,t+i} \mid \mathcal{F}_t] = \sum_{i=0}^n F_{t-1,t+i}$ , since the demand shift only redistributes the demand on the horizon, but does not affect the total quantity.

## Summary

The MMFE or its extensions can be used for various purposes, like in production/inventory planning models and simulations for determining the optimal/acceptable base-stock level, planning horizon, safety stock; comparing different rolling-horizon MRP techniques; estimating stockouts, costs, etc. My simulation system includes a combined forecast generator: MMFE with demand shifts and run-outs and used the simulation not only for the above enumerated purposes, but also for estimating the cost-and-profit-sharing in a VMI situation.

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<sup>4</sup>Some medium-horizon master planners cannot optimise for filling the capacities, thus it is adjusted later manually.

The random forecast generator module determines the component forecast on a rolling horizon. The first forecast contains uniformly distributed demand on the planning horizon between zero and twice of the average demand. The other forecasts and the realised demands are computed from the previous ones with basically the following procedure. Firstly, the forecast is rolled, i.e., the first period is left out, while a new random forecast is added to the end of the horizon. Then a normally distributed random noise is added to the demand with zero expected value and increasing standard deviation along the horizon. Thirdly, certain quantities can be shifted back and forth between the periods—this captures the phenomenon when the planners urge or postpone certain works. Finally, the possible negative forecasts are eliminated and the quantities are rounded to the nearest integer number.

The specific properties of the implemented forecast generation and some lessons from the simulation runs are the following:

- The  $\Sigma$  covariance matrix of the MMFE is diagonal (i.e., the forecast updates are independent across time periods). I also relaxed the normality assumption: while the  $\varepsilon_{t,t}, \dots, \varepsilon_{t,t+n-1}$  are still considered to have normal distribution with 0 mean and increasing variance, the  $\varepsilon_{t,t+n}$  has uniform distribution on  $[-d, d]$  instead. The variance is linearly increasing, but I am planning to allow also logarithmically increasing in the future.
- The distribution of run-out can be geometric, uniform or Poisson with arbitrary parameters. The distribution is considered to be static, which does not seem to be reasonable (except from the geometric distribution case due to its “lack of memory” property), thus I intend to change this in the next version of the simulation system.
- For the demand shift model  $\Pr(s_{t,t+i} = -1) = \Pr(s_{t,t+i} = 1)$  (except for  $i = 0$  and  $i = n$ ) is an arbitrary constant which is independent both from  $t$  and  $i$ . I plan to modify it to be inversely proportional to  $i$ , which would be a more realistic assumption.
- The proportion of the shifted demand is assumed to be uniform (on  $[0, 1]$ ).

### 5.2.3 Lot-Sizing Logic

The lot-sizing module of the supplier operates according to the approach presented in Fig. 3.11: it basically uses the WWr method, but when it results in one aggregated lot, then it switches to the newsvendor model. The pseudo-code of one simulation run is as follows:

---

```

variable inventory  $\leftarrow$  initialInventory
variable wip  $\leftarrow$  0 ▷ Work-in-progress

for i  $\leftarrow$  1 to length_of_horizon + 1 do
  inventory  $\leftarrow$  inventory + wip -  $\xi_i$  ▷ Updating inventory level
  if inventory < 0 then ▷ Shortage
    inventory  $\leftarrow$   $\begin{cases} 0 & \text{in case of partial lost sales} \\ \text{inventory} + \xi_i & \text{in case of full lost sales} \\ \text{inventory} & \text{in case of backorders} \end{cases}$ 
  end if
  if run - out then
    inventory  $\leftarrow$  wip  $\leftarrow$  0
  else
    wip  $\leftarrow$   $\begin{cases} \text{WWr solution} & \text{if } \text{inventory} < F_{i,1} + \text{safetystock} \\ 0 & \text{otherwise} \end{cases}$ 
    if wip =  $\sum_{k=1}^n F_{i,k} - \text{inventory}$  then ▷ Producing in one lot
      wip  $\leftarrow$  Newsvendor solution ▷ With estimated standard deviation
    end if
  end if
end for

```

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Finally, the statistical evaluation module computes the most important aspects of the simulation in terms of average, standard deviation, minimal and maximal observed value of the simulation runs.

This simulation system is useful not only for validating my algorithms, but also for decision support purposes. Industrial planners could use it for estimating the performance of the system with certain parameters, e.g., determining whether a particular safety stock policy is appropriate in order to achieve some service level goal; or studying the consequences of applying the proposed channel coordination protocols. Furthermore, the simulation results could be used for calculating the internal rate of return (IRR) or return on investment (ROI) indicators when deciding whether to switch to a VMI business model.

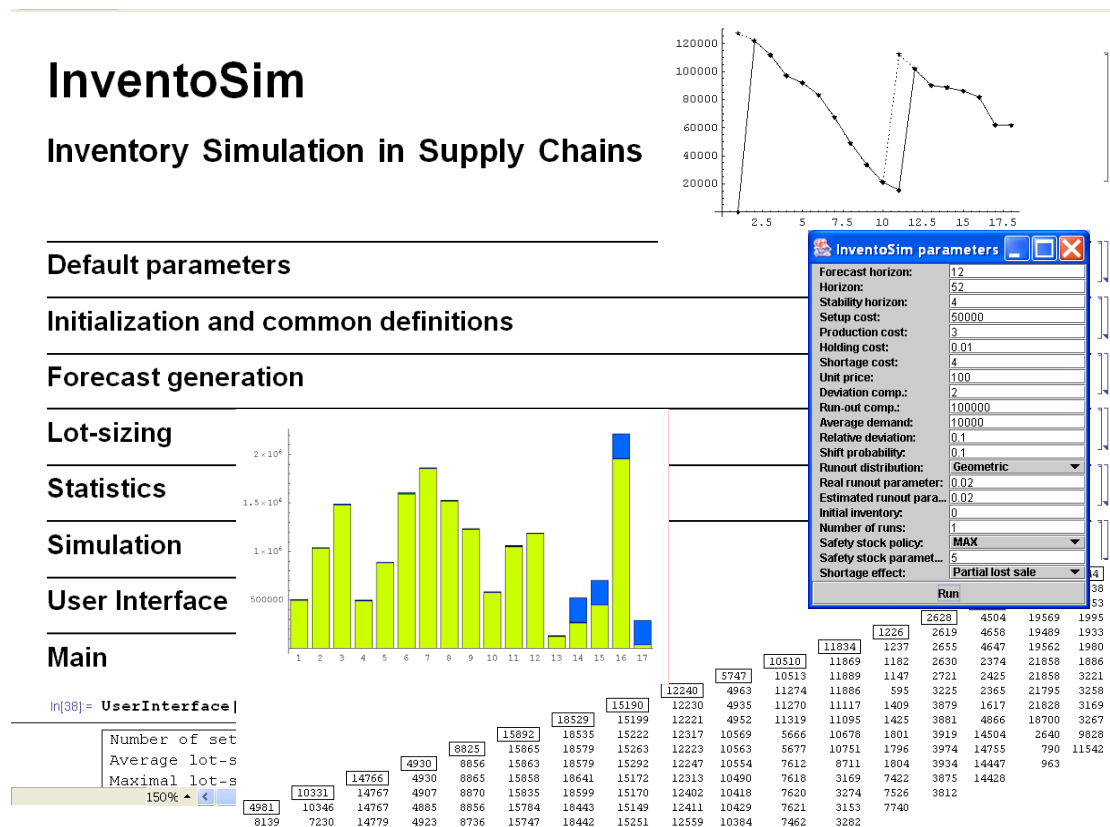


Figure 5.4: A collage of the InventoSim user interface.

# Chapter 6

## Summary and Conclusion

This dissertation investigated the recent trends and issues in cooperative production networks, and tried to answer the challenges in some special, practically relevant cases. In the introductory chapters, I reviewed the current situation in manufacturing of mass customised products, described an industrial case study that gave a specific motivation to my work, and presented the three main research topics related to this dissertation, based on several recent scientific publications.

In Chapter 3, I presented novel extensions of two classical lot-sizing models considering the recent market trends, such as the short product life-cycles and the increasing customer expectations towards high service levels. My first model in this chapter solves the one-period newsvendor problem aiming at satisfying the demand with minimal cost, even if it necessitates setting up an additional production. My second model extends the dynamic Wagner – Whitin problem with a stochastic variable describing the length of the product life-cycle.

In Chapter 4, I studied the two previously presented models in a decentralised setting. The goal in these cases is to achieve the optimal centralised performance with two autonomous enterprises along a supply chain with asymmetric information. For this purpose, I presented supply protocols for both models based on the VMI approach and such payment functions that achieve channel coordination, i.e., provide optimal channel behaviour assuming rational enterprises.

Finally, in Chapter 5, I presented two software systems that partially use the models I had developed: an industrial information sharing system and a pilot simulation environment supporting decision making.

The results of this dissertation consider the problems of a special configuration and neither address nor answer every general issues in production networks. There are several open questions in this intensively studied field; I enumerate some of them as possible directions in my future work.

- Firstly, the simplifying assumptions of the centralised model should be omitted, and the resulting complex problem with multiple items, capacity constraints and *sequence-dependent setups* may be studied.
- Secondly, the channel coordination models can be further analysed by exploring the possibilities of *fair cost and profit allocation* between the partners. The coordination can be studied also in *make-to-order* or *engineer-to-order* manufacturing environments (e.g., machine and ship building industries) applying *robust local production planning* methods.
- Finally, the simulation system may be extended to the network level with several supplier tiers in order to analyse the ramification of the *events*—e.g., machine breakdowns, shortages, urgent orders—in *complex adaptive networks* and their effects to the overall network behaviour.

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