Evolving Decision Principles

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Outline of talk

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Motivation

- Multicriteria decision problems – many subjective decisions
- Subjective decisions – not very exact
- Sensitivity analysis for small changes in the subjective values
- The need for stable solutions

Introduction

- Buying a TV set – criteria: cost, quality of image, physical aspect, trade-name, etc.
- Ranking the alternatives – by using some decision principle (a function of the values of an alternative with respect to the different criteria)
- Sensitivity analysis – the analysis of the results, when we make small perturbations to the subjective values
- Stability index – takes values between 0 and 1 and measures the stability of a solution

Multicriteria decision problem

- Criteria C_i with weights w_i
- Alternatives A_j with aggregate values x_j

$$
\begin{array}{ccc}
 & & \begin{array}{cccc} x_1 & . & . & x_n \\ \mathbf{A}_1 & . & . & \mathbf{A}_n \end{array} \\
\begin{array}{c} w_1 \\ . \\ . \\ w_m \end{array} & \begin{array}{c} \mathbf{C}_1 \\ . \\ . \\ \mathbf{C}_m \end{array} & \left[\begin{array}{cccc} a_{11} & . & . & a_{1n} \\ . & . & . & . \\ . & . & . & . \\ a_{m1} & . & . & a_{mn} \end{array}\right]
\end{array}
$$

Decision principle

$$f : \mathbf{R}_+^m \times \mathbf{R}_+^m \to \mathbf{R}_+$$

A1. $f(\lambda \mathbf{v}, \mathbf{z}) = f(\mathbf{v}, \mathbf{z})$ and $f(\mathbf{v}, \lambda \mathbf{z}) = \lambda f(\mathbf{v}, \mathbf{z})$ (Homogeneity)

A2. $f\left(v_{\sigma(1)}, \ldots, v_{\sigma(m)}; z_{\sigma(1)}, \ldots, z_{\sigma(m)}\right) = f\left(v_1, \ldots, v_m; z_1, \ldots, z_m\right)$

A3. f is strictly increasing in variable z_i, $i = 1, \ldots, n$

A4. $\min\limits_{i=1,\ldots,m} z_i \leq f\left(v_1, \ldots, v_m; z_1, \ldots, z_m\right) \leq \max\limits_{i=1,\ldots,m} z_i$

The aggregated value

$$x_j = f\,(w_1, \ldots, w_m; a_{1j}, \ldots, a_{mj})$$

α power mean as decision principle:

$$m_\alpha(w_1, \ldots, w_m; y_1, \ldots, y_m) = \left(\frac{\sum_{i=1}^{m} w_i\, y_i^\alpha}{\sum_{i=1}^{m} w_i} \right)^{1/\alpha}$$

Sensitivity analysis

- The analysis of the results, when we make small perturbations to the weights w_i and/or values a_{ij} of a decision table
- A solution $\{x_1, \ldots, x_n\}$ is stable with respect to these perturbations if the decreasing order of the new solution $\{x'_1, \ldots, x'_n\}$ coincides with the decreasing order of $\{x_1, \ldots, x_n\}$

Sensitivity with respect to alternatives

Minimum and maximum aggregate value

$$x_j^-(\varepsilon) = f(w_1, \ldots, w_m; a_{1j}^-(\varepsilon), \ldots, a_{mj}^-(\varepsilon))$$
$$x_j^+(\varepsilon) = f(w_1, \ldots, w_m; a_{1j}^+(\varepsilon), \ldots, a_{mj}^+(\varepsilon))$$

$x_j^-(\varepsilon)$ is attained for $a_{ij}^-(\varepsilon) = a_{ij} - \varepsilon\, a_{ij}$

$x_j^+(\varepsilon)$ is attained for $a_{ij}^+(\varepsilon) = a_{ij} + \varepsilon\, a_{ij}$

Stability with respect to alternatives

Let σ, $x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}$

- Stability of the order $x_{\sigma(j)} \geq x_{\sigma(j+1)}$
  the greatest $\delta_j \in [0,1]$ for which
  $x^{-}_{\sigma(j)}(\delta_j\,\varepsilon) \geq x^{+}_{\sigma(j+1)}(\delta_j\,\varepsilon)$
- Stability index of decision principle f

$$S(\varepsilon) = \left(\prod_{j=1}^{n-1} \delta_j \right)^{\frac{1}{n-1}}$$

Genetic programming

- Evolutionary search for the program that produces the output corresponding to the input given in the fitness cases
- Algorithm
  1. Generate an initial population of individuals
  2. Execute each program of the population on the fitness cases and assign it a fitness value
  3. Create a new population of individuals by reproduction, crossover and mutation
  4. Iterate through steps 2-3

Genetic programming setup

- Representation (only feasible solutions)

  Terminals - α power means applied to an alternative's values

  Functions - α power means with equal weights

- Evaluation method

  1. For each $\mathbf{A}_j$ compute x_j, $x_j^-(\varepsilon)$, $x_j^+(\varepsilon)$
  2. Order the alternatives: $x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}$
  3. For each pair $\mathbf{A}_{\sigma(j)},\ \mathbf{A}_{\sigma(j+1)}$ compute δ_j
  4. Compute $Fitness = S(\varepsilon)$

Sensitivity with respect to weights

Minimum and maximum aggregate value

$$x_j^-(\varepsilon) = f(w_{1j}^-(\varepsilon), \ldots, w_{mj}^-(\varepsilon); a_{1j}, \ldots, a_{mj})$$
$$x_j^+(\varepsilon) = f(w_{1j}^+(\varepsilon), \ldots, w_{mj}^+(\varepsilon); a_{1j}, \ldots, a_{mj})$$

- Maximum (minimum) of a monotone function on a subset of the simplex $S = \sum_{i=1}^{m} v_i = 1$ $(W_\varepsilon = [\mathbf{w} - \varepsilon\mathbf{w}, \mathbf{w} + \varepsilon\mathbf{w}])$
- Branch and bound technique
- Starting from the initial simplex, we maintain a list of simplices
- We always divide the simplex with the largest upper bound and keep those of the resulting two simplices, which have intersection with the set of interest

Stability with respect to weights

Let σ, $x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}$

- Stability of the order $x_{\sigma(j)} \geq x_{\sigma(j+1)}$
  the greatest $\delta_j \in [0,1]$ for which
  $x^-_{\sigma(j)}(\delta_j\,\varepsilon) \geq x^+_{\sigma(j+1)}(\delta_j\,\varepsilon)$
- Stability index of decision principle f

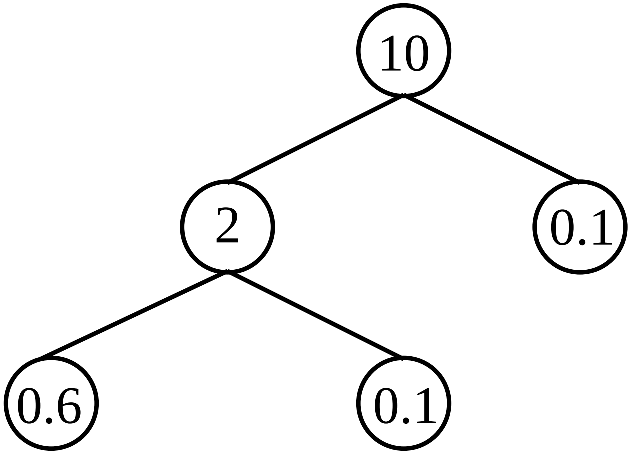
$$S(\varepsilon) = \left(\prod_{j=1}^{n-1} \delta_j\right)^{\frac{1}{n-1}}$$

Example 1

		$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{A}_3$	$\mathbf{A}_4$	$\mathbf{A}_5$	$\mathbf{A}_6$	$\mathbf{A}_7$	$\mathbf{A}_8$	$\mathbf{A}_9$	$\mathbf{A}_{10}$
$w_1 = 0.1$	$\mathbf{C}_1 =$ aesthetics	1	2	9	3	1	4	8	9	5	6
$w_2 = 0.2$	$\mathbf{C}_2 =$ warranty	3	9	1	6	9	7	8	2	3	1
$w_3 = 0.3$	$\mathbf{C}_3 =$ trade-name	9	1	7	1	8	8	3	6	2	9
$w_4 = 0.4$	$\mathbf{C}_4 =$ image	5	6	2	9	1	2	4	3	7	4

best	order	$\mathbf{A}_1$	$\mathbf{A}_7$	$\mathbf{A}_6$	$\mathbf{A}_{10}$	$\mathbf{A}_4$	$\mathbf{A}_9$	$\mathbf{A}_8$	$\mathbf{A}_2$	$\mathbf{A}_5$	$\mathbf{A}_3$
GP	x_j	4.80	4.60	4.38	4.31	4.28	4.06	3.91	3.75	3.39	3.20
	δ_j	0.21	0.25	0.09	0.06	0.29	0.20	0.24	0.68	0.36	
arith-	order	$\mathbf{A}_1$	$\mathbf{A}_4$	$\mathbf{A}_{10}$	$\mathbf{A}_6$	$\mathbf{A}_7$	$\mathbf{A}_2$	$\mathbf{A}_5$	$\mathbf{A}_9$	$\mathbf{A}_8$	$\mathbf{A}_3$
metic	x_j	5.4	5.4	5.1	5	4.9	4.7	4.7	4.5	4.3	4
mean	δ_j	0	0.39	0.1	0.1	0.26	0	0.23	0.25	0.40	

Best solution



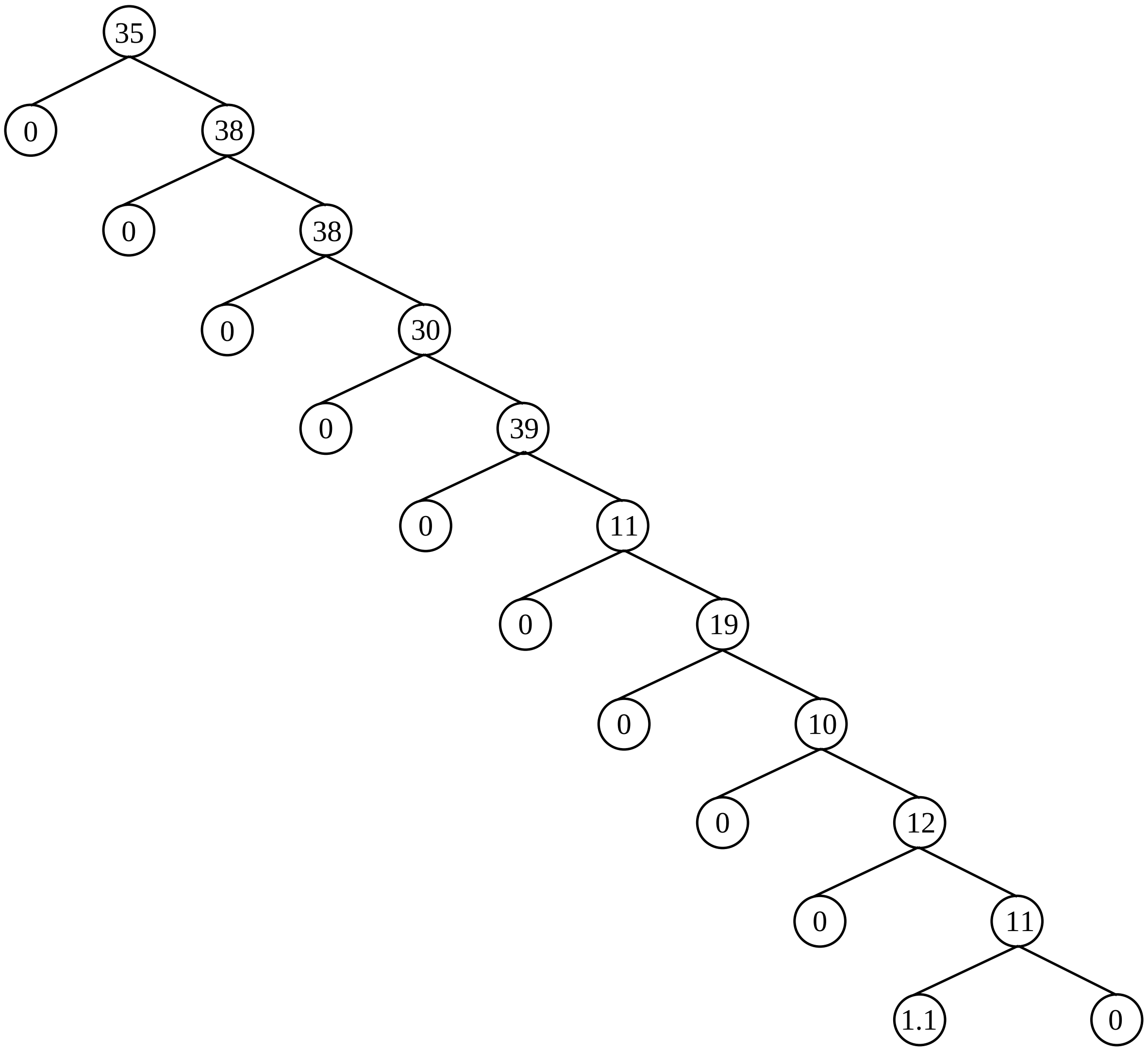
$$f = \left(\frac{\left(\left(\frac{m_{0.6}^2 + m_{0.1}^2}{2} \right)^{\frac{1}{2}} \right)^{10} + m_{0.1}^{10}}{2} \right)^{\frac{1}{10}}$$

Example 2

$$\begin{array}{lc} & \mathbf{A}_1 \ \mathbf{A}_2 \ \mathbf{A}_3 \ \mathbf{A}_4 \ \mathbf{A}_5 \ \mathbf{A}_6 \\ \begin{array}{ll} w_1 = 0.3 & \mathbf{C}_1 \\ w_2 = 0.2 & \mathbf{C}_2 \\ w_3 = 0.5 & \mathbf{C}_3 \end{array} & \begin{bmatrix} 1 & 8 & 9 & 9 & 5 & 9 \\ 7 & 9 & 1 & 6 & 9 & 7 \\ 9 & 1 & 7 & 1 & 8 & 8 \end{bmatrix} \end{array}$$

best GP	order	$\mathbf{A}_6$	$\mathbf{A}_5$	$\mathbf{A}_3$	$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{A}_4$
	x_j	8.07	7.11	5.13	4.52	3.33	3.12
	δ_j	0.7	1	1	1	0.5	
arithmetic mean	order	$\mathbf{A}_6$	$\mathbf{A}_5$	$\mathbf{A}_3$	$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{A}_4$
	x_j	8.1	7.3	6.4	6.2	4.7	4.4
	δ_j	0.59	0.82	0.25	1	0.51	

Best solution



Conclusions

- A novel method for sensitivity and stability analysis of multicriteria decision problems with respect to alternative values and weights
- Alternatives – genetic programming for generating more stable decision principles
- Weights – special algorithm for computing stability

Future work

- Evolutionary stability analysis for weights
- Joint stability analysis for alternative values and weights
- More general stability analysis where only a number of alternatives are of interest