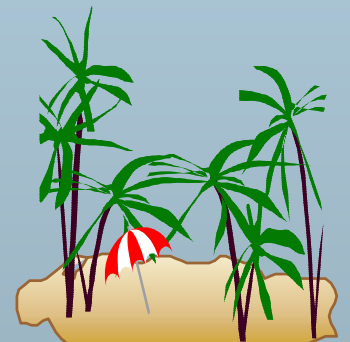




# Canonical Cover

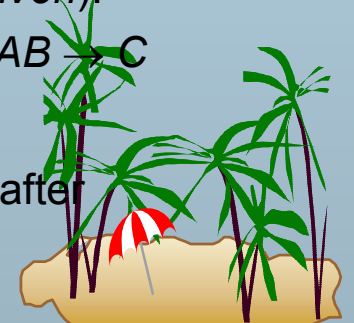
- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - ★ Eg:  $A \rightarrow C$  is redundant in:  $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
  - ★ Parts of a functional dependency may be redundant
    - ✓ E.g. on RHS:  $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$  can be simplified to  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
    - ✓ E.g. on LHS:  $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$  can be simplified to  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
- Intuitively, a canonical cover of  $F$  is a “minimal” set of functional dependencies equivalent to  $F$ , having no redundant dependencies or redundant parts of dependencies





# Extraneous Attributes

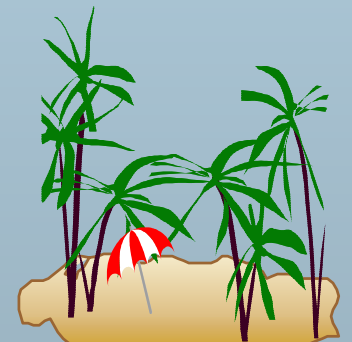
- Consider a set  $F$  of functional dependencies and the functional dependency  $\alpha \rightarrow \beta$  in  $F$ .
  - a.) Attribute  $A$  is **extraneous** in  $\alpha$  if  $A \in \alpha$  and  $F$  logically implies  $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$
  - b.) Attribute  $A$  is **extraneous** in  $\beta$  if  $A \in \beta$  and the set of functional dependencies  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  logically implies  $F$ .
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a “stronger” functional dependency always implies a weaker one
  - a.) augmentation
  - b.) decomposition
- Example: Given  $F = \{A \rightarrow C, AB \rightarrow C\}$ 
  - ★  $B$  is extraneous in  $AB \rightarrow C$  because  $\{A \rightarrow C, AB \rightarrow C\}$  logically implies  $A \rightarrow C$  (i.e. the result of dropping  $B$  from  $AB \rightarrow C$ , which is  $A \rightarrow C$ , given). So the canonical cover is  $\{A \rightarrow C\}$  by (left) augmentation we can get  $AB \rightarrow C$
- Example: Given  $F = \{A \rightarrow C, AB \rightarrow CD\}$ 
  - ★  $C$  is extraneous in  $AB \rightarrow CD$  since  $AB \rightarrow C$  can be inferred even after deleting  $C$  ( $AB \rightarrow D$ )





# Testing if an Attribute is Extraneous

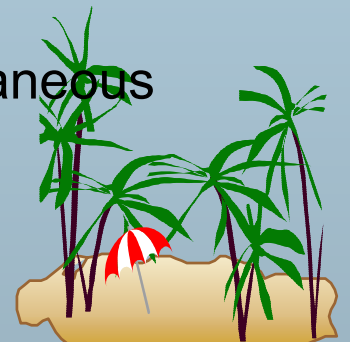
- Consider a set  $F$  of functional dependencies and the functional dependency  $\alpha \rightarrow \beta$  in  $F$ .
- To test if attribute  $A \in \alpha$  is extraneous in  $\alpha$ 
  1. compute  $(\{\alpha\} - A)^+$  using the dependencies in  $F$
  2. check that  $(\{\alpha\} - A)^+$  contains all attributes of  $\beta$ ; if it does,  $A$  is extraneous
- To test if attribute  $A \in \beta$  is extraneous in  $\beta$ 
  1. compute  $\alpha^+$  using only the dependencies in  $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ ,
  2. check that  $\alpha^+$  contains  $A$ ; if it does,  $A$  is extraneous

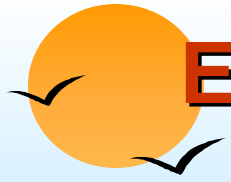




# Canonical Cover

- A *canonical cover* for  $F$  is a set of dependencies  $F_c$  such that
  - ★  $F$  logically implies all dependencies in  $F_c$ , and
  - ★  $F_c$  logically implies all dependencies in  $F$ , and
  - ★ No functional dependency in  $F_c$  contains an extraneous attribute, and
  - ★ Each left side of functional dependency in  $F_c$  is unique.
- To compute a canonical cover for  $F$ :  
**repeat**
  - Use the union rule to replace any dependencies in  $F$   
 $\alpha_1 \rightarrow \beta_1$  and  $\alpha_1 \rightarrow \beta_2$  with  $\alpha_1 \rightarrow \beta_1 \beta_2$
  - Find a functional dependency  $\alpha \rightarrow \beta$  with an  
extraneous attribute either in  $\alpha$  or in  $\beta$
  - If an extraneous attribute is found, delete it from  $\alpha \rightarrow \beta$**until**  $F$  does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied





# Example of Computing a Canonical Cover

- $R = (A, B, C)$   
 $F = \{A \rightarrow BC$   
     $B \rightarrow C$   
     $A \rightarrow B$   
     $AB \rightarrow C\}$
- Combine  $A \rightarrow BC$  and  $A \rightarrow B$  into  $A \rightarrow BC$ 
  - ★ Set is now  $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- $A$  is extraneous in  $AB \rightarrow C$ 
  - ★ Check if the result of deleting  $A$  from  $AB \rightarrow C$  is implied by the other dependencies
    - ✓ Yes: in fact,  $B \rightarrow C$  is already present!
  - ★ Set is now  $\{A \rightarrow BC, B \rightarrow C\}$
- $C$  is extraneous in  $A \rightarrow BC$ 
  - ★ Check if  $A \rightarrow C$  is logically implied by  $A \rightarrow B$  and the other dependencies
    - ✓ Yes: using transitivity on  $A \rightarrow B$  and  $B \rightarrow C$ .
      - Can use attribute closure of  $A$  in more complex cases
- The canonical cover is:  $A \rightarrow B$   
     $B \rightarrow C$

