Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  
  ★ Eg: \( A \rightarrow C \) is redundant in: \( \{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow C\} \)
  
  ★ Parts of a functional dependency may be redundant
    
    ✔ E.g. on RHS: \( \{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow CD\} \) can be simplified to \( \{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow D\} \)
    
    ✔ E.g. on LHS: \( \{A \rightarrow B, \ B \rightarrow C, \ AC \rightarrow D\} \) can be simplified to \( \{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow D\} \)

- Intuitively, a canonical cover of \( F \) is a “minimal” set of functional dependencies equivalent to \( F \), having no redundant dependencies or redundant parts of dependencies
Consider a set $F$ of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in $F$.

a.) Attribute $A$ is extraneous in $\alpha$ if $A \in \alpha$ and $F$ logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$

b.) Attribute $A$ is extraneous in $\beta$ if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies $F$.

Note: implication in the opposite direction is trivial in each of the cases above, since a “stronger” functional dependency always implies a weaker one

a.) augmentation

b.) decomposition

Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$

- $B$ is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (i.e. the result of dropping $B$ from $AB \rightarrow C$, which is $A \rightarrow C$, given).

So the canonical cover is $\{A \rightarrow C\}$ by (left) augmentation we can get $AB \rightarrow C$

Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$

- $C$ is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting $C$ ($AB+$)
Testing if an Attribute is Extraneous

Consider a set $F$ of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in $F$.

To test if attribute $A \in \alpha$ is extraneous in $\alpha$
1. compute $({\alpha} - A)^+$ using the dependencies in $F$
2. check that $(\{\alpha\} - A)^+$ contains all attributes of $\beta$; if it does, $A$ is extraneous

To test if attribute $A \in \beta$ is extraneous in $\beta$
1. compute $\alpha^+$ using only the dependencies in $F' = (F - (\alpha \rightarrow \beta)) \cup \{\alpha \rightarrow (\beta - A)\}$,
2. check that $\alpha^+$ contains $A$; if it does, $A$ is extraneous
A **canonical cover** for $F$ is a set of dependencies $F_c$ such that

- $F$ logically implies all dependencies in $F_c$, and
- $F_c$ logically implies all dependencies in $F$, and
- No functional dependency in $F_c$ contains an extraneous attribute, and
- Each left side of functional dependency in $F_c$ is unique.

To compute a canonical cover for $F$:

1. **repeat**
   - Use the union rule to replace any dependencies in $F$
     - $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_1$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
     - Find a functional dependency $\alpha \rightarrow \beta$ with an extraneous attribute either in $\alpha$ or in $\beta$
     - If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
   - until $F$ does not change

2. **Note:** Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied.
Example of Computing a Canonical Cover

- \( R = (A, B, C) \)
- \( F = \{ A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C \} \)

- Combine \( A \rightarrow BC \) and \( A \rightarrow B \) into \( A \rightarrow BC \)
  - Set is now \( \{ A \rightarrow BC, B \rightarrow C, AB \rightarrow C \} \)

- \( A \) is extraneous in \( AB \rightarrow C \)
  - Check if the result of deleting \( A \) from \( AB \rightarrow C \) is implied by the other dependencies
  - Yes: in fact, \( B \rightarrow C \) is already present!
  - Set is now \( \{ A \rightarrow BC, B \rightarrow C \} \)

- \( C \) is extraneous in \( A \rightarrow BC \)
  - Check if \( A \rightarrow C \) is logically implied by \( A \rightarrow B \) and the other dependencies
  - Yes: using transitivity on \( A \rightarrow B \) \textit{and} \( B \rightarrow C \).
    - Can use attribute closure of \( A \) in more complex cases

- The canonical cover is: \( A \rightarrow B \)
  \( B \rightarrow C \)