Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - Eg: $A \rightarrow C$ is redundant in: $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g. on RHS: $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow CD\}$ can be simplified to $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow D\}$
    - E.g. on LHS: $\{A \rightarrow B,\ \ B \rightarrow C,\ \ AC \rightarrow D\}$ can be simplified to $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow D\}$
- Intuitively, a canonical cover of F is a “minimal” set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.

  a.) Attribute A is extraneous in α if $A \in \alpha$
  and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$

  b.) Attribute A is extraneous in β if $A \in \beta$
  and the set of functional dependencies
  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.

- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one

  a.) augmentation

  b.)decomposition

- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - ★ B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies

  $A \rightarrow C$ (I.e. the result of dropping B from $AB \rightarrow C$, *which is* $A \rightarrow C$, *given*).

  So the canonical cover is $\{A \rightarrow C\}$ by (left) augmentation we can get $AB \rightarrow C$

- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - ★ C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C (AB+)

Testing if an Attribute is Extraneous

- Consider a set *F* of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in *F*.
- To test if attribute A $\in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in *F*
  2. check that $(\{\alpha\} - A)^+$ contains all attributes of β; if it does, *A* is extraneous
- To test if attribute *A* $\in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in
     $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
  2. check that α^+ contains *A*; if it does, *A* is extraneous

Canonical Cover

- A *canonical cover* for F is a set of dependencies F_c such that
  - ★ F logically implies all dependencies in F_c, and
  - ★ F_c logically implies all dependencies in F, and
  - ★ No functional dependency in F_c contains an extraneous attribute, and
  - ★ Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for F:

  repeat

  Use the union rule to replace any dependencies in F

  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_1$ with $\alpha_1 \rightarrow \beta_1 \beta_2$

  Find a functional dependency $\alpha \rightarrow \beta$ with an

  extraneous attribute either in α or in β

  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$

  until F does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Example of Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - ✓ Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - ✓ Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
      - Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$