

Bounds on $K(t, b, R)$
(bounds on the size of mixed ternary/binary optimal covering codes)

t	b	$R = 1$	$R = 2$	$R = 3$	$R = 4$
1	1	$\mathbf{2}^2$	$\mathbf{1}^1$		
1	2	$\mathbf{3}^1$	$\mathbf{2}^4$	$\mathbf{1}^1$	
1	3	j $\mathbf{6}^{24}$ a	$\mathbf{2}^2$	$\mathbf{2}^6$	$\mathbf{1}^1$
1	4	b $\mathbf{8}^2$ c	$\mathbf{3}^1$	$\mathbf{2}^4$	$\mathbf{2}^8$
1	5	j $\mathbf{16}^{120}$ a	j $\mathbf{6}^{155}$ a	$\mathbf{2}^2$	$\mathbf{2}^6$
1	6	b $\mathbf{24}^2$ b	c $\mathbf{8}^6$ b	$\mathbf{3}^1$	$\mathbf{2}^4$
1	7	s 42–48 a	d $\mathbf{12}^1$ b	j $\mathbf{6}^{573}$ a	$\mathbf{2}^2$
1	8	b 76–84 b	w 20 b	d $\mathbf{8}^{21}$ b	$\mathbf{3}^1$
1	9	s 134–160 c	a 24–35 c	d $\mathbf{12}^5$ b	j $\mathbf{6}^{1102}$ a
1	10	b 253–284 c	b 44–60 b	d 15–20 b	z 8 z
1	11	s 448–548 b	b 74–96 b	b 19–32 c	h 11–12 x
1	12	b 869–992 e	s 121–179 c	b 30–50 e	w 12–20 x
1	13	a 1536–1920 e	a 205–320 e	a 44–84 t	h 16–32 x
2	1	b $\mathbf{4}^2$ b	$\mathbf{2}^3$	$\mathbf{1}^1$	
2	2	a $\mathbf{6}^1$ b	$\mathbf{3}^5$	$\mathbf{2}^6$	$\mathbf{1}^1$
2	3	i $\mathbf{12}^{23}$ a	b $\mathbf{4}^6$ b	$\mathbf{2}^3$	$\mathbf{2}^9$
2	4	j $\mathbf{20}^1$ b	j $\mathbf{6}^1$ b	$\mathbf{3}^5$	$\mathbf{2}^6$
2	5	s 32–36 b	d $\mathbf{11}^{91}$ b	j $\mathbf{4}^{12}$ b	$\mathbf{2}^3$
2	6	s 57–64 b	d 16 b	j $\mathbf{6}^2$ b	$\mathbf{3}^5$
2	7	s 101–122 d	d 22–28 b	d 11 b	j $\mathbf{4}^{20}$ j
2	8	b 187–232 c	s 33–48 b	d 14–16 b	j $\mathbf{6}^2$ j
2	9	d 338–408 c	b 56–74 c	d 18–24 c	h 8–11 x
2	10	b 646–768 b	s 93–144 b	b 23–40 e	h 11–16 x
2	11	d 1173–1472 e	b 165–256 b	s 36–64 e	w 13–24 x
2	12	a 2169–2560 e	a 274–475 e	a 56–117 v	a 16–36 s
3	1	c $\mathbf{9}^4$ b	$\mathbf{3}^1$	$\mathbf{2}^4$	$\mathbf{1}^1$
3	2	i $\mathbf{16}^3$ b	j $\mathbf{5}^1$ b	$\mathbf{3}^{16}$	$\mathbf{2}^8$
3	3	b $\mathbf{24}^2$ b	c $\mathbf{8}^{15}$ b	$\mathbf{3}^1$	$\mathbf{2}^4$
3	4	s 44–48 a	d $\mathbf{13}^1$ b	j $\mathbf{5}^2$ b	$\mathbf{3}^{19}$
3	5	s 78–92 c	d 17–23 b	d $\mathbf{8}^{131}$ b	$\mathbf{3}^1$
3	6	s 140–171 c	s 25–36 b	w 12 b	j $\mathbf{5}^6$ j
3	7	b 256–312 c	b 43–56 c	d 15–20 b	z $\mathbf{8}^{583}$ z
3	8	b 480–576 c	s 72–96 b	b 18–32 c	h 8–12 x
3	9	d 888–1056 d	b 123–188 c	s 29–54 e	w 12–18 s
3	10	b 1689–2016 c	b 223–356 c	s 46–86 e	a 13–30 s
3	11	a 3072–3456 p	a 367–576 e	a 71–140 e	a 20–48 s

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4	1	b 18 ⁵ a	j 6 ¹¹ a	3 ⁵	2 ⁵
4	2	i 36 a	d 10 ⁴ b	c 4 ¹¹ b	3 ⁴⁰
4	3	s 60–72 a	w 15–18 b	c 6 ⁴⁵ b	3 ⁶
4	4	s 107–128 l	s 20–24 b	d 10 b	z 4 ²² z
4	5	d 195–238 l	b 32–48 a	d 12–16 b	z 6 ¹³² z
4	6	b 356–432 b	b 55–72 b	d 16–24 b	h 8–10 x
4	7	d 672–852 d	s 93–144 a	b 22–40 c	w 10–15 s
4	8	b 1257–1296 c	b 168–252 b	s 36–60 s	w 12–22 s
4	9	d 2370–2592 a	b 290–480 c	b 58–107 s	a 16–36 s
4	10	a 4366–5184 a	a 494–852 e	a 91–183 e	a 24–60 s
5	1	d 45–54 a	d 12 ¹ b	b 4 ¹ b	3 ¹⁴
5	2	s 80–96 b	d 16–21 c	d 7 ¹ c	3 ²
5	3	d 148–168 b	b 24–36 b	d 11–12 b	z 4 ² z
5	4	s 268–324 c	b 42–64 j	d 13–21 c	z 7 ⁴ z
5	5	d 509–624 d	s 71–108 b	s 18–32 b	h 8–12 x
5	6	b 936–1184 d	b 126–192 b	s 28–54 c	w 11–18 s
5	7	d 1791–1944 b	b 222–348 c	b 44–86 e	a 12–29 y
5	8	b 3353–3888 a	b 385–664 m	b 76–144 b	a 19–48 s
5	9	a 6221–7776 a	a 669–1224 e	a 117–245 e	a 30–79 y
6	1	d 113–132 b	d 19–27 b	d 9 ⁴ c	3 ¹
6	2	s 204–252 b	b 33–48 b	d 11–16 c	z 5 ¹ z
6	3	d 386–468 c	s 54–72 b	d 15–24 n	z 8 ¹¹ z
6	4	s 700–864 c	b 94–144 a	s 22–42 e	w 10–14 s
6	5	d 1353–1620 k	b 170–276 c	s 35–71 c	w 12–24 m
6	6	b 2500–2916 b	b 290–513 c	b 58–96 c	a 15–36 s
6	7	d 4827–5832 a	b 532–936 c	b 96–180 c	a 23–60 s
6	8	a 8887–10944 e	a 911–1728 e	a 151–288 e	a 37–96 x
7	1	d 292–333 c	s 42–54 b	s 12–18 b	z 6 ³⁷ z
7	2	s 525–648 c	s 72–108 a	s 18–33 c	w 9–11 w
7	3	d 1022–1296 a	b 130–216 a	s 27–54 b	w 11–18 x
7	4	b 1864–2304 c	b 220–374 u	b 44–80 e	w 13–24 s
7	5	d 3641–4374 b	b 397–690 e	b 74–144 b	a 18–48 s
7	6	b 6762–8532 d	b 729–1280 e	b 121–232 s	a 28–78 e
7	7	a 12725–15552 q	a 1245–2302 e	a 197–432 w	a 45–136 e

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t	b	$R = 1$	$R = 2$	$R = 3$	$R = 4$
8	1	d 772–948 d	b 99–162 a	s 21–42 c	h 9–14 s
8	2	s 1395–1728 c	b 167–288 b	s 34–70 e	w 11–24 s
8	3	d 2747–3374 d	b 295–504 b	b 57–108 c	a 14–36 s
8	4	b 5047–6408 d	b 555–876 e	b 93–198 e	a 22–63 y
8	5	d 9904–11664 d	b 964–1566 d	b 155–324 b	a 35–100 e
8	6	a 18257–23328 a	a 1707–2817 e	a 259–576 e	a 57–168 e
9	1	d 2072–2520 v	b 219–396 b	b 45–91 s	w 12–28 s
9	2	s 3770–4752 e	b 422–729 b	b 72–136 m	a 17–50 s
9	3	d 7463–9450 e	b 731–1184 e	s 119–243 e	a 27–76 e
9	4	b 13802–17496 c	b 1321–1782 e	b 207–432 b	a 44–136 m
9	5	a 26244–34992 a	a 2351–3078 v	a 341–837 e	a 72–240 e
10	1	d 5624–6804 d	b 555–729 b	b 91–184 y	a 21–54 s
10	2	b 10311–13122 c	b 983–1458 a	b 154–324 b	a 34–99 e
10	3	d 20458–25272 c	b 1894–1944 b	b 283–648 a	a 55–168 e
10	4	a 37792–49086 e	a 3247–3888 a	a 451–1138 v	a 91–270 e
11	1	d 15405–18954 a	b 1436–1458 a	b 217–486 a	a 43–136 r
11	2	b 28439–34992 e	b 2528–2916 a	b 365–729 b	a 70–216 e
11	3	a 54507–69984 a	a 4499–5832 a	a 600–1458 a	a 116–360 e
12	1	d 42516–52488 c	b 3405–4374 a	b 470–972 b	a 89–303 e
12	2	a 78732–102060 e	a 6253–8748 a	a 802–1458 v	a 149–486 e
13	1	d 118098 a	a 8713–13122 a	a 1076–2187 j	a 192–618 e

Key to Tables for $K(t, b, R)$

Lower bounds	
unmarked	value 1, 2, or 3
a	sphere-covering bound
b	van Lint–van Wee (1991)
c	Östergård–Hämäläinen (1997)
d	Bertolo–Östergård–Weakly (2004)
h	$K(t, b, R) \geq K(t - 1, b + 1, R)$
i	Kolev (1993, 1995)
j	Kolev–Landgev (1994)
s	Lang–Quistorff–Schneider (2007)
w	Kéri (2003–2008)
z	Kéri–Östergård (2003)

Key to Tables for $K(t, b, R)$

Upper bounds	
unmarked	value 1, 2, or 3
a	$K(t, b, R) \leq 2K(t, b - 1, R)$ or $K(t, b, R) \leq 3K(t - 1, b, R)$
b	Hämäläinen–Rankinen (1991)
c	Östergård–Hämäläinen (1997)
d	Bertolo–Östergård–Weakly (2004)
e	Bertolo–(Di Pasquale)–(Rugin)–Santisi (2003–2009)
j	Östergård (1991–1994) or Kolev–Landgev (1994)
k	Davies–Royle (1997)
l	Hämäläinen–Madsen (1995), Davies–Royle (1997)
m	Madsen (2006,2010)
n	Madsen (1987)
p	Di Nasso (1952)
q	Basile (1970)
r	Bertolo–Di Pasquale–Santisi–Rivas Soriano (2009)
s	Rivas Soriano (2004–2006,2009)
t	Lozano (2005)
u	Hämäläinen and Rivas Soriano (2009)
v	Hämäläinen (1987–2009)
w	Kéri (2003–2008)
x	amalgamated direct sum with a repetition code
y	Rivas Soriano (2011)
z	Kéri–Östergård (2003)
