Instructions for use

1. A-lists and B-lists are two different listings of the same sets of (equivalence classes of) complete n-arcs.

Explicit listings of complete n-arcs are prepared only for those cases where a given file of complete n-arcs contains no more than 70 records.

Larger files of complete n-arcs or files of all n-arcs can be generated by a freely downloadable computer program.

2. Coding of fields elements in A-lists. For a prime p, a possibility of ordering in GF(p) is the natural ordering of non-negative integers less than p. In GF(7), e.g., we have the ordering

$$0 < 1 < 2 < 3 < 4 < 5 < 6.$$

For a prime power $q = p^k, k > 1$, the k-tuples of non-negative integers less than p can be ordered according to the natural ordering. In GF(9), e.g.,

$$0 < 1 < 2 < \alpha < \alpha + 1 < \alpha + 2 < 2\alpha < 2\alpha + 1 < 2\alpha + 2$$

where α is a primitive root for GF(9).

Thus, for a prime p and for the alternative of using A-list, $0, 1, 2, \ldots, p-1$ are used just for $0, 1, 2, \ldots, p-1$ as residue classes.

For $q = 2^k$, $0, 1, 2, \ldots$ are used for $0, 1, \alpha, \alpha+1, \alpha^2, \alpha^2+1, \ldots$

For $q = 3^k$, $0, 1, 2, \ldots$ are used for $0, 1, 2, \alpha, \alpha+1, \alpha+2, 2\alpha, 2\alpha+1, 2\alpha+2, \ldots$

(Etc.)

3. Coding of fields elements in B-lists. The ordering is accomplished according to the powers of a primitive root. As 3 is a primitive root for GF(7), the latter can be ordered as

$$0 < 1 < 3 < 3^2 (= 2) < 3^3 (= 6) < 3^4 (= 4) < 3^5 (= 5).$$

Similarly, GF(9) can be ordered as

$$0 < 1 < \alpha < \alpha^2 (= 2\alpha + 1) < \alpha^3 (= 2\alpha + 2) < \alpha^4 (= 2) < \alpha^5 < \alpha^6 < \alpha^7.$$

As regards a B-list, $0, 1, 2, \ldots, q-1$ are always used for $0, 1, \alpha, \alpha^2, \alpha^3, \ldots, \alpha^{q-2}$.

4. Primitive roots. For GF(p), where $p \leq 31$ is a prime, it is convenient to choose a primitive root α which is the smallest possible field element so that $1, \alpha, \alpha^2, \alpha^3, \ldots \alpha^{p-2}$ are all different, that is, we use $\alpha = 2$ for $p = 5, 11, 13, 19, 29$, $\alpha = 3$ for $p = 7, 17, 31$, $\alpha = 5$ for $p = 23$. For GF(q), where $q \leq 32$ is a prime power, a primitive root α is a root of an irreducible polynomial.

5. Irreducible polynomials. For GF(q), where $q \leq 32$ and q is not a prime, we use the irreducible polynomials specified as follows:

q	**irreducible polynomial used**
4	$\alpha^2 + \alpha + 1$
8	$\alpha^3 + \alpha + 1$
9	$\alpha^3 + \alpha + 1$
16	$\alpha^4 + \alpha + 1$
25	$\alpha^2 + \alpha + 2$
27	$\alpha^3 + 2\alpha + 1$
32	$\alpha^5 + \alpha^2 + 1$

6. Compact notation of arcs. Any n-arc in PG(r, q) is shown in the listings (either for A-list or B-list) as an $r \times n - r - 2$ matrix, which should be extended to an $r + 1 \times n$ matrix by adding

– a row of all 1's,

– a column of all 1's,

– the $r + 1$ unit vectors.

The column vectors of this extended matrix constitute the points of the arc in full.

To avoid storing matrices which are the transpose of each other, non-sqare matrices are always stored and displayed in 'landscape' form. This means the transposition of matrices – introduced above for the compact notation of arcs – when $n < 2r + 2$.

7. Examples. To display, e.g., the list of complete arcs in PG(4,11), first select 'PG(4,11)' by a mouse click. When the table of PG(4,11) appears either in the same or in a new window, select 'complete 9-arcs'. The A-list of the result shows the 'raw material'

$$\begin{matrix} 2&3&4&5 \\ 3&6&5&7 \\ 6&2&3&9 \end{matrix}$$

defining the unique complete 9-arc in PG(4,11). As now $n < 2r + 2$ ($n = 9$ and $2r + 2 = 10$), the displayed matrix should be transposed (or rotated by 90). The extension of the transposed matrix is

$$\begin{matrix} 1&0&0&0&0&1&1&1&1 \\ 0&1&0&0&0&1&2&3&6 \\ 0&0&1&0&0&1&3&6&2 \\ 0&0&0&1&0&1&4&5&3 \\ 0&0&0&0&1&1&5&7&9 \end{matrix}$$

(This is the generator matrix of an MDS code.)

We conclude that the unique complete 9-arc in PG(4,11) consists of the points

$$(1\ 0\ 0\ 0\ 0),\ (0\ 1\ 0\ 0\ 0),\ (0\ 0\ 1\ 0\ 0),$$
$$(0\ 0\ 0\ 1\ 0),\ (0\ 0\ 0\ 0\ 1),\ (1\ 1\ 1\ 1\ 1),$$
$$(1\ 2\ 3\ 4\ 5),\ (1\ 3\ 6\ 5\ 7),\ (1\ 6\ 2\ 3\ 9).$$

For the unique complete 10-arc in the same space - considering now the B-list - we obtain the 'raw material'

$$\begin{matrix} 2&3&5&8 \\ 3&2&8&5 \\ 5&8&6&4 \\ 8&5&4&6 \end{matrix}$$

As now $n = 2r + 2$, transposition is not required, and the extended matrix is

$$\begin{matrix} 1&0&0&0&0&1&1&1&1&1 \\ 0&1&0&0&0&1&2&3&5&8 \\ 0&0&1&0&0&1&3&2&8&5 \\ 0&0&0&1&0&1&5&8&6&4 \\ 0&0&0&0&1&1&8&5&4&6 \end{matrix}$$

(Do not forget that, for B-list and for GF(11), an integer $a \neq 0$ means the residue class of 2^{a-1}.)

We conclude that the unique complete 10-arc in PG(4,11) consists of the points

$$(1\ 0\ 0\ 0\ 0),\ (0\ 1\ 0\ 0\ 0),\ (0\ 0\ 1\ 0\ 0),\ (0\ 0\ 0\ 1\ 0),\ (0\ 0\ 0\ 0\ 1),$$

$$(1\ 1\ 1\ 1\ 1),\ (1\ 2\ 3\ 5\ 8),\ (1\ 3\ 2\ 8\ 5),\ (1\ 5\ 8\ 6\ 4),\ (1\ 8\ 5\ 4\ 6).$$