# An Inverse Economic Lot-Sizing Approach to Eliciting Supplier Cost Parameters

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**Abstract**

This paper proposes an inverse lot sizing model for eliciting the cost parameters (the holding, backlogging, and the fixed production cost) of a supplier from known earlier one or more demand-optimal lot-size pairs. It is assumed that the buyer knows that the supplier solves a single-item, multi-period, uncapacitated lot-sizing problem with backlogs to compute its optimal delivery periods and quantities. The inverse optimization problem is reformulated to a mixed-integer linear program and solved using a commercial solver. The approach is illustrated on some sample problems.

**Keywords:** Economic lot-sizing, inverse combinatorial optimization, eliciting cost parameters.

## 1 Introduction

Inverse combinatorial optimization deals with reconstructing values of certain parameters of a combinatorial problem from one or more known optimal solutions. In this paper we introduce an inverse combinatorial approach to elicitating the cost parameters of a supplier who determines its delivery periods and quantities by solving a single-item, multi-period, uncapacitated lot-sizing problem with backlogs. The input of the approach is a historic record of demand vs. lot-size pairs. To the best of our knowledge, this is the first inverse lot-sizing model investigated in the literature.

Such a model may find applications in various scenarios involving a buyer-supplier relationship. Beyond the obvious and general benefit of knowing the partner's cost parameters, e.g., in price negotions, a specific application derives from the numerous recent cooperation mechanism for lot-sizing in supply chains, based on Stackelberg games: such models assume that the buyer knows all cost parameters of the supplier, but the coordination mechanisms themselves do not present any incentives for the supplier to reveal the true values of its cost parameters.

The inverse lot-sizing problem has been encoded as a linear program, based on a recent model for bilevel lot-sizing and a shortest path representation. The approach is illustrated on a few sample instances.

# 2 Literature review

## 2.1 Lot-sizing

Fundamental results on dynamic lot-sizing models were published by [35], and [37]. These papers consider uncapacitated lot-sizing models where the deterministic, time varying demand is known in advance over a finite planning horizon. Over the past decades the basic models have been extended by production capacities and various side constraints, for an overview see e.g., [3, 26, 29]. Albeit dynamic programming is still the most efficient method for solving the tractable cases [25, 34, 35, 37], they have been complemented by linear programming formulations for describing the convex hull of feasible solutions, see e.g., [4, 5, 27, 28, 30, 33]. Many times, it is easier to work with *extended formulations*, when new variables and constraints are introduced to obtain the linear formulation. The modeling of various features in lot-sizing by mixed-integer programs (MIP) are investigated in e.g., [6, 11]. As further extensions, different lot-sizing and scheduling models, including small-bucket and large-bucket, discrete and continuous time formulations, as well as single- and multi-level models are presented in [16, 26].

The need for studying the interacting lot-sizing decisions of multiple autonomous parties in a supply chain is widely recognized. One of the possible approaches is *integration*, when the different parties jointly solve the interrelated planning problems, see e.g., [23] for an overview, and [1, 19, 25] for applications. Important recent results in integrated planning include the work of [19], who investigated the case of serial chains, constrained capacities, and concave cost functions, and introduced a dynamic program whose running time is polynomial when the number of levels in the chain is fixed. A dynamic program that runs in $O(n^2 \log n)$ and a tight extended formulation is presented by [25] for uncapacitated two-level lot-sizing, and a formulation is derived for the multi-item, multi-client case. [20] developed efficient methods for solving the integrated production and transportation planning problem under various assumptions.

A drawback of integration is the mutual sharing of all the planning relevant information, which is sometimes unrealistic. A game theoretic approach alleviates this burden by using coordination mechanisms between the parties to drive the supply chain towards a system-wide optimal performance [2, 8]. The decentralized planning, integrated, coordinated, and bilevel approaches to the same lot-sizing problem in a two-player supply chain are compared in [22].

### 2.2 Inverse combinatorial optimization

Inverse combinatorial optimization is relatively new field of operations research. A survey of this field, including the studied problem models, algorithms, and the main results achieved as been given in [18]. Most of the previous work in the field focesed on graph theoretical problems, such as the inverse shortest path problem [7] or the inverse center location problem [9]. A generic optimization model for a class of inverse problems has been introduced in [38], together with a Newton-type algorithm that runs in strongly polynomial time under mild conditions.

A field of operations research, closely related to inverse optimization, is *bilevel programming*. It addresses decision and optimization problems whose outcome is determined by the interplay of two self-interested decision makers who decide sequentially. The first decision maker, the so-called *leader*, is assumed to have a complete knowledge of the second decision maker's, the *follower's* problem and parameters. Therefore, to optimize its own objective function, the leader must consider the response that it can expect from the follower. The basic modeling and solution techniques in bilevel programming are presented in [13]. A review of applicable solution methods for various classes of bilevel programs is given in [10], whereas reformulations of continuous bilevel optimization problems into single-level problems are discussed in [14]. A combinatorial perspective on bilevel problems is presented in [24]. Recent development in solution methods are presented in e.g., [15, 17, 32]. Results in bilevel inventory control include [12], [31], and [36].

The inverse lot sizing problem model investigated in this paper is tightly related to the bilevel lot sizing model presented in [21] in several ways. The current model corresponds to the inverse of the follower's problem in the bilevel model, and a possible application of the inverse approach is eliciting the cost parameters for the bilevel model. Furthermore, the MIP reformulation introduced in the two papers are based on a common idea, although type of decisions to be made are different.

## 3 Problem definition

This paper investigates cost elicitation in a two-level supply chain that consists of a single buyer and a single supplier. The supplier aims to meet the request made by the buyer at a minimum cost. These cost include a fixed production cost, $f$, a per period and per unit holding cost, $h$, and a per period and per unit backlog cost, $g$. Therefore, the supplier must solve a single-item uncapacitated lot-sizeing problem with backlogs (ULSB) each time it receives a request from the buyer. This fact is known to the buyer as well, but the values of the supplier's cost parameters are unknown to it. The buyer addresses to elicitate these cost parameters from historic records of requests over time and delivery lot-sizes received from the supplier. The historic records contain $M$ samples, each sample consisting of a vector of demand values over time, $d_i^m$, and a vector of delivery lot-sizes, $x_i^m$, $m = 1...M, i = 1...T$.

# 4 The solution approach

In this section, an inverse combinatorial optimization aproach is proposed for the above cost elicitation problem. A single optimization problem is considered that looks for possible values of $h$ and $g$ that are suitable for all samples at once. Note that different values of $h$ and $g$ might be feasible for the same problem. To characterize the range of feasible cost parameters, the four instances of the inverse problem are solved with four different objective functions: the minimum and maximum values of the holding cost and the backlog cost parameters, $h_{\min}$, $h_{\max}$, $g_{\min}$, and $g_{\max}$, respectively.

## 4.1 A mixed-integer linear programming reformulation

The reformulation of the inverse optimization problem to a MIP is based on the standard shortest path representation of the ULSB problem, see, e.g., [21]. In this graph-based representation, variable $c_{ijk}^m$ corresponds to the length of the edge indicating that in sample $m$, all demand in the interval $[i, k]$ is satisfied in period $j$, with $i \leq j \leq k$. The binary parameter $A_{ijk}^m$ indicates whether this edge is selected in the shortest path ($A_{ijk}^m = 1$) or not ($A_{ijk}^m = 0$). Note that this information can be simply read out of the input demand-lot size pairs.

Now, given values of cost variables $h$ and $g$, and the implied edge costs $c_{ijk}^m$ are a feasible solution of the inverse problem, i.e., result in a situation that the flow encoded in parameters $A_{ijk}^m$ is optimal, if and only if a potential value can be assigned to each variabe $\pi_k^m$ in such a way that the complementarity constraints are tight for the edges in the shortest path only, see (3, 4). The complete MIP reformulation therefore can be stated as follows:

Minimize or maximize

$$h \text{ or } g \tag{1}$$

subject to

$$\pi_0^m = 0 \qquad \forall m \tag{2}$$

$$c_{ijk}^m - \pi_k^m + \pi_{i-1}^m = 0 \qquad \forall m, i, j, k : A_{ijk}^m = 1 \tag{3}$$

$$c_{ijk}^m - \pi_k^m + \pi_{i-1}^m \geq 0 \qquad \forall m, i, j, k : A_{ijk}^m = 0 \tag{4}$$

$$c_{ijk}^m = f + \sum_{u=i}^{j-1} (j - u) g * d_u^m + \sum_{u=j+1}^{k} (u - j) h * d_u^m \qquad \forall m, i, j, k \tag{5}$$

$$c_{ijk}^m, \pi_k^m, h, g \geq 0, \qquad \forall m, i, j, k \tag{6}$$

Let us formulate two basic properties of the above approach. The proofs of the properties are omitted here.

**Lemma 1** *If two values of the holding cost variable $h = h_1$ and $h = h_2$ (or respectively, the backlog cost parameter, $g = g_1$ and $g = g_2$) with $h_1 < h_2$ are two feasible solutions of the above inverse problem with some, potentially different values of $g$, then any value in $[h_1, h_2]$ is a feasible solution of the above problem with some, potentially different value of $g$.*

**Lemma 2** *If $h = h_1$ is a feasible solution of the above inverse problem with some choice of $g$, and $g = g_1$ is a feasible solution of the above inverse problem with some choice of $h$, then $(\{h = h_1, g = g_1\}$ may or may not be a feasible solution of the problem.*

# 5 Experiments

Compuational experiments addressed the measurement of the precision of the cost parameter prediction, including the dependence of the prediction on various factors, such as the number of samples, the length of the time horizon, as well as the characteristics of the instance. Two types of problem instances have been generated: in the *random samples* instances, the samples were generated using independent random demand, $x_i^m \leftarrow U[1, 10]$, where $U[a, b]$ denotes the integer uniform random distribution over the interval $[a, b]$. In the *rolling horizon* instances, the demand in the first sample, $x_i^1$ was generaed in a similar fashion, whereas subsequent demand vectors were generated by shifting the previous demand earlier by one period, and perturbing the demand value by at most 10%, i.e., $x_i^{m+1} = x_{i+1}^m(U[90, 110]/100)$. A horizon of 10 time periods were considered, and 50 samples have been generated per instance.

All instances and all samples have been solved to optimality using a standard MIP formulation of the ULSB problem, and the instances were characterized by the following two measures: the *stock ratio*, $S$, denotes the ratio of periods over all samples where in the optimal solution, the supplier satisfied demand from stoick. Analoguously, the *backlog ratio*, $R$, denotes the ratio of periods where the supplier backlogs.

Figure 1 shows the results achieved for a random samples instance with $S = 59.2\%$ and $R = 3.2\%$, i.e., very rare backlogs. The diagram on the l.h.s. refers to the prediction of the holding ost parameter, while the diagram on the r.h.s to the backlog cost parameter. The horizontal axis shows the number of samples applied for the parameter elicitation. The maximum and minimum cost parameter curves indicate that the precision of the elicitatation improved gradually until up to 25 samples, but no improvement was experienced afterwards. The final precision was 4.5/

Figure 2 displays the results for different, frequent-backlog instance with $S = 24.2\%$ and $R = 41.4\%$ using a similar diagram design. The proposed method achieved a much better precision for this instance, 1.51% for $h$ and 0.81% for $g$, and the precision improved continuously until the 50th sample.

A natural conclusion from the above results is that for a fixed horizon length and number of samples, the higher the stock ratio, $S$ (resp., the backlog ratio, $R$), the higher precision can be achieved for the estimation of $h$ (resp., $g$).

The thorough evaluation of the approach on a larger set of data will consist the subject of future work.

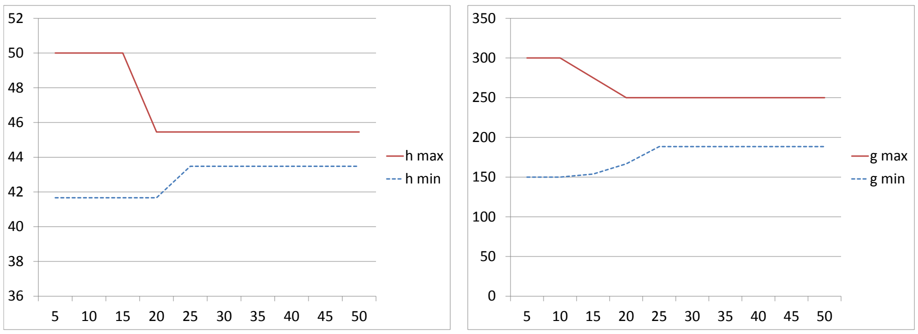


Figure 1: Estimation of the holding cost $h$ (left) and the backlog cost $g$ (right) for the first, rare-backlog sample instance.

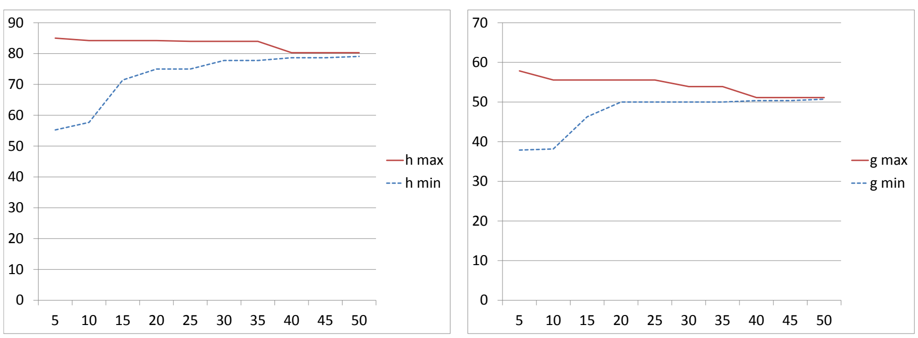


Figure 2: Estimation of the holding cost $h$ (left) and the backlog cost $g$ (right) for the second, frequent-backlog sample instance.

# 6 Conclusions and future work

This paper proposed a novel technique for eliciting supplier cost parameters from earlier records of demand-lot size pairs using an inverse optimization approach. An inverse lot sizing model, and its reformulation to a mixed-integer program was introduced.

Based on the prelimiary experimentation, we conclude that the approach is efficient enough to predict the future actions of the supplier, but it is not sufficient on its own for learning cost parameters for price discussions. Future work will focus on a detailed analysis of the proposed approach, including a thorough evaluation of the precision of the estimation under different condition. The extension of the model to more realistic assumptions, e.g., rolling horizon models and costs varying over time, is also an interesting direction of research.

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