Inventory Control in Supply Chains: A Comparative Analysis of Fundamental Approaches

András Kovács[1], Péter Egri[1], Tamás Kis[1], József Váncza[1,2]

[1] Fraunhofer Project Center for Production Management and Informatics,
Computer and Automation Research Institute, Hungarian Academy of Sciences

[2] Department of Manufacturing Science and Technology,
Budapest University of Technology and Economics, Hungary
{andras.kovacs,egri,tamas.kis,vancza}@sztaki.hu

September 10, 2010

Abstract

The principal challenge of inventory control in supply chains consists of planning for a set of autonomous enterprises under information asymmetry and disparate, potentially conflicting objectives. In this paper, we investigate four different computational approaches to cope with this challenge: the decomposition, the integrated, the coordinated, and the bilevel programming approaches. Beyond their analytical characterization, we illustrate every approach on the same inventory control problem. For this purpose, we also developed a new coordination mechanism as well as a new model and solution algorithm for the bilevel optimization problem. Comparative computational experiments are carried out on a set of randomly generated problem instances.

Keywords: Supply chain, inventory, integration, coordination, bilevel programming.

1 Introduction

The key question in modeling and solving inventory management problems in supply chains is the handling of the fact that supply chains are comprised of autonomous enterprises, having different, often conflicting objectives. Moreover, the individual enterprises typically make decisions that effect the entire supply chain based in part on private information inaccessible to the other parties.

The goal of this study is to provide a clear-cut comparison of the fundamental approaches to this challenge, comparing their main modeling, computational and managerial implications.

In particular, we investigate four different computational approaches to inventory control in supply chains. According to the classical *decomposition approach*, each party optimizes its own production and logistic decisions without explicitly considering the consequences on, and the future actions of, the partners in the supply chain. The *integrated approach* optimizes the overall performance of the supply chain by centralized planning, however, this requires a tight integration of the parties. To lift the latter requirement, the *coordinated approach* seeks for mechanisms that motivate the autonomous enterprises to cooperate in finding mutually beneficial plans by using means like standardized communication and benefit sharing. Finally, the *bilevel approach* enables an individual party, in possession of sufficient information about the relevant partners in the supply chain, to optimize its production taking into account the actions that it can expect from its partners.

The comparative analysis of the above approaches is made on the same *inventory control* problem, which is essentially a multi-period lot-sizing problem in a two-echelon supply chain. In a dyadic situation where a *buyer-supplier* chain has to meet external demand, this problem involves both production related decisions of the supplier, as well as logistic decisions from the side of the buyer. Although for the sake of analytical clarity we had to take some simplifying assumptions, the basic problem has direct application relevance. Primarily, a retailer may assume the role of the buyer, connecting exogenous market demand and the service of the supplier. Further on, planning functions at an enterprise are typically decomposed both along the horizon (strategic, tactical, operational) and the type of activity (procurement, production, distribution and sales) [12]. A department specialized in supply planning (and interested in minimal logistic cost) may again assume the role of the buyer. In this case, it transmits dependent and aggregated component demand generated by production planning towards an external supplier.

A common treatment of the above inventory control problem required the development of novel models and solution methods, too. Beyond results of the generic comparative study, the paper presents also new contributions, specifically a coordination mechanism (Section 5) and a bilevel formulation and solution algorithm (Section 6).

To the best of our knowledge, this is the first study that provides a self-contained comparison of the above fundamental computational approaches, and shows how they can be applied to solving the same inventory control problem in different settings. For a review of inventory control problems, both as faced by a single decision maker and in a supply chain, the reader is referred to [3]. The potential gain by integrated versus decentralized decision making in supply chains has been investigated in [17], where the difference of the induced costs is defined as the *price of anarchy.* The coordination of supply chains consisting of autonomous enterprises is studied in detail in [2], while Sarmah et al. [21] provide a comprehensive taxonomic survey of coordinated buyer-vendor models in a deterministic, time invariant setting. The fundamental ideas of bilevel programming are presented in [7], and the application of this approach to the management of multi-divisional organizations has been studied in [4].

The rest of the paper is organized as follows. In Section 2 we introduce the sample problem that we used for demonstrating the computational approaches. Each of Sections 3-6 investigates one of the approaches in detail, providing a review of the related literature, the analysis of the approach in the context of supply chains, as well as an illustration on the sample problem. The main characteristics of the approaches are contrasted, and their performance in computational experiments is compared in Section 7. Finally, conclusions are drawn in Section 8.

2 Problem definition

2.1 A two-stage lot sizing problem

We study the different modeling approaches on a two-stage single-item uncapacitated lot-sizing problem as follows. Let us consider a supply chain that provides a single item to its customers. The supply chain consists of two independent companies, a *buyer* and a *supplier*. The buyer (and hence, the supply chain) faces dynamic, deterministic external demand d_t, $t = 1, ..., T$, over a discrete time horizon of T time periods.

Departing from the known demand, the buyer generates a *supply plan*, which specifies that at the beginning of each period t, the amount x_t^1 of the item should be delivered from the supplier to the buyer. This incurs a fixed cost of f^1 to be paid in each period where a positive amount is delivered, independently of the amount. The buyer may use the delivered amount partly to satisfy the demand in the same period t, partly to keep it on stock to cover future demand in periods $t' > t$, and partly to satisfy backlogged demand from previous periods $t'' < t$. Holding inventory and backlogging at the buyer take h^1 and g^1 per unit and per period cost, respectively. These delivery, holding, and backlogging costs are paid by the buyer.

To cover the demand set by the buyer's supply plan, the supplier generates a *production plan* that specifies the x_t^2 amount of the item to be produced in period t over the planning horizon. In each period t where a positive amount $x_t^2 > 0$ is produced, a fixed f^2 setup cost is incurred. Just as the buyer, the supplier can hold stock or backlog demand, for a cost h^2 and g^2 per unit and per period. Moreover, we assume that the production and holding costs that occur at the supplier are paid by the supplier to an external party, whereas the backlogging cost is paid by the supplier to the buyer as a penalty for the delay caused.

It is assumed that all demand must be satisfied by the end of the horizon, i.e., $\sum_{t=1}^T d_t = \sum_{t=1}^T x_t^1 = \sum_{t=1}^T x_t^2$. As usual (see e.g. [3]), we assume that both the inventory holding and the backlogging costs are higher at the buyer than at the supplier, $h^1 > h^2$ and $g^1 > g^2$. The production and delivery lead times are zero.

In all models studied in the sequel the decision variables of the buyer are the x_t^1 supply, s_t^1 inventory and r_t^1 backlog quantities for each time period $t = 1, ..., T$ of the planning horizon. The supplier has a decision problem of identical structure, with x_t^2 production, s_t^2 inventory and r_t^2 backlog quantities. Whenever appropriate, we distinguish the two parties with an upper index k, where $k = 1$ stands for the buyer's and $k = 2$ for the supplier's decision variables and parameters. Auxiliary binary variables y_t^1 and y_t^2 are introduced to capture events of delivery and production, respectively.

2.2 Plan execution

The above model allows the supplier to apply backlog, and according to some of the approaches investigated later, the buyer may not be able to anticipate this kind of deviation from the supply plan. Therefore, the executed scenario may differ from the plan, and the rules of the execution must be established. We applied the following rules.

If the supplier produces the goods on time, then the buyer must call off the amount indicated in the supply plan. Otherwise, i.e., if the supplier produces backlog, then the buyer calls off the ordered goods as soon as they are available. Formally, in each period t, the buyer must call off the amount that has been ordered and actually produced, which is calculated as:

$$x_t^{1R} = \min\left(\sum_{t'=1}^{t} x_{t'}^1, \sum_{t'=1}^{t} x_{t'}^2\right) - \sum_{t'=1}^{t-1} x_{t'}^{1R}. \tag{1}$$

Likewise, external demand is served as soon as possible:

$$d_t^R = \min\left(\sum_{t'=1}^{t} d_{t'}, \sum_{t'=1}^{t} x_{t'}^{1R}\right) - \sum_{t'=1}^{t-1} d_{t'}^R. \tag{2}$$

Similarly, we differentiate the realized setup, inventory, and backlog from the planned values by using an index R.

In all cases, the cost of the realization, and not the cost of the original plan will be investigated. Hence, the total cost of the buyer can be computed by the following formula (with the last component standing for the backlog compensation received from the supplier):

$$C^1 = \sum_{t=1}^{T} \left(f^1 y_t^{1R} + h^1 s_t^{1R} + g^1 r_t^{1R} - g^2 r_t^{2R}\right). \tag{3}$$

The supplier's cost is also computed based on the realized values, though, in the approaches investigated, the supplier is always able to execute its plan. Note that late delivery by the supplier will often imply backlog at the buyer as well, which is disadvantageous for the buyer, due to the assumption that $g^1 > g^2$.

3 Decomposition approach

In industrial practice, where business is run by legally separated enterprises, the parties in a supply chain have very limited access to private data of the others (and especially to the sensitive cost factors), and hardly participate in each others' planning processes. A natural consequence of this information asymmetry is that each party focuses on optimizing its own production and logistics based on information locally accessible.

From the computational aspect, this leads to a *decomposition approach*, where the overall planning problem of the supply chain is divided into as many sub-problems as the number of parties, and the sub-problems are solved one by one. Of course, so as to satisfy external demand, decentralized decisions have to be coordinated in some way. Typically, local planning problems are solved in a sequence, where the solution of one problem sets *target* for the next one. The most common procedure is *upstream planning* [8, 18], a hierarchical sequential decision scheme starting at the downstream party (e.g., retailer) who, after solving its own planning problem, generates demand to its supplier. In a longer chain, this pattern is repeated upstream.

The decentralized approach has three main characteristics:

- The parties necessarily make, often implicitly, assumptions on the actions of the other related parties. For example, the buyer may assume that its supplier always delivers on time.
- Each party optimizes its own actions without considering the consequences to the performance of the others.
- The assumptions that provide grounds for the decomposition may fail (e.g., the supplier may deliver late), in which case the realization deviates from the plans.

Disparate objectives and the decentralization of decisions may easily lead to suboptimal overall system performance–a phenomenon known as double marginalization in microeconomics [23]. Information asymmetry and local autonomy cause together time and again inefficiencies like acute shortage situations or excess inventories. Recently, Albrecht analyzed and classified a number of drivers that lead to sub-optimality in decentralized planning [2]. In any case, satisfying the target set by one partner incurs some extra costs (by, e.g., too large quantities, or too frequent deliveries required) at another one, increasing thus the system-wide costs, too.

Even though the decomposition approach cannot provide any guarantee of the system-wide solution quality, it has advantages in real applications: it complies with the usual business environment, has moderate information requirements and does not require special contracts. From the computation aspect, models and algorithms for the problems of individual parties have been studied widely [3], and the approach can naturally be applied in a supply chain of arbitrary size.

3.1 Computational model

In the decomposed model of our sample problem, although the decisions of the buyer and the supplier are made separately, the supply chain as a whole should satisfy the external demand (see the basic problem definition in Section 2). The buyer makes the assumption that the supplier will deliver on time (and therefore it does not hold any buffer stock), which assumption may or may not be satisfied in the realized scenario. Since the external demand is known by the buyer, the upstream planning approach is taken, in the following steps:

1. The buyer decides about its supply, inventory and backlog quantities.
2. The generated x_t^1 supply plan is passed to the supplier as target.
3. The supplier regards these quantities as incoming demand, and computes the corresponding production plan for minimizing its own costs.
4. Knowing the x_t^2 production quantities, the delivery from the supplier to the buyer, as well as from the buyer to the external customer, are realized according to the rules of the plan execution.

The decomposition method involves solving two identically structured single-stage lot-sizing problems, as defined by the following mixed-integer linear program (MIP). The buyer's (supplier's) model can be received by substituting $k = 1$ ($k = 2$).

Minimize

$$\sum_{t=1}^{T} \left(f^k y_t^k + h^k s_t^k + g^k r_t^k \right) \tag{4}$$

subject to

$$x_t^k + (r_t^k - r_{t-1}^k) = d_t^k + (s_t^k - s_{t-1}^k) \quad t = 1, ..., T \tag{5}$$

$$x_t^k \leq D y_t^k \quad t = 1, ..., T \tag{6}$$

$$s_0^k = s_T^k = r_0^k = r_T^k = 0 \tag{7}$$

$$x_t^k, r_t^k, s_t^k \geq 0 \quad t = 1, ..., T-1 \tag{8}$$

$$y_t^k \in \{0, 1\} \quad t = 1, ..., T \tag{9}$$

The MIP model minimizes the sum of the fixed, inventory holding, and backlog cost at a partner (4). Line (5) describes the inventory balance constraint, while inequality (6) states that a positive amount can be delivered/produced in a given time period only if a setup is performed in that period. Constant D is the total demand, i.e., $D = \sum_{t=1}^{T} d_t$. Line (7) sets the initial and final stock and backlog to zero, which also implies that the total demand will be satisfied throughout the planning horizon.

The buyer directly faces the external demand, i.e., $d_t^1 = d_t$ ($t = 1, ..., T$). Given that decisions on the supply plan at the buyer have already been made, the two serial decision problems are coupled: $d_t^2 = x_t^1$ for each period over the horizon.

After solving also the supplier's problem, from values of the computed vector of x_t^2 the realized delivery x_t^{1R} and the served demand d_t^R can be determined according to the rules of plan execution (see Subsection 2.2). The realization will incur costs C_{Dec}^1 and C_{Dec}^2 at the buyer and the supplier, respectively.

The above single-stage lot-sizing problem (with slightly more general assumptions) was investigated first in the seminal paper of Wagner and Whitin [25], where a backward induction algorithm with $\mathcal{O}(T^2)$ time complexity was proposed. Later it was shown that this problem can be solved in $\mathcal{O}(T)$ (even a generalization of the model in $\mathcal{O}(T \log T)$) time [24].

3.2 Sample problem

Each of the approaches will be illustrated on the same sample problem instance, whose parameters are given below:

$$f^1 = 100 \qquad h^1 = 6 \qquad g^1 = 18 \qquad T = 10$$
$$f^2 = 492 \qquad h^2 = 5 \qquad g^2 = 6$$
$$d = [71, 84, 43, 21, 4, 81, 59, 44, 32, 46]$$

The solution computed according to the decomposition approach is displayed in Table 1. The buyer plans to satisfy demand from just-in-time supply in all periods except for period 5, where it wishes to use the quantity on stock from period 4. However, the supplier, who has much higher setup cost, produces only in periods 2, 6, and 9, which causes backlogs in periods 1 and 8 in both the supplier-buyer and the buyer-external customer relations. This causes excess cost for the buyer compared to its plan.

	t	1	2	3	4	5	6	7	8	9	10
Demand	d_t	71	84	43	21	4	81	59	44	32	46
Supply plan	x_t^1	71	84	43	25		81	59	44	32	46
Production plan	x_t^2		223				140			122	
Realized delivery	x_t^{1R}		155	43	25		81	59		76	46
Served demand	d_t^R		155	43	21	4	81	59		76	46

$C_{Dec}^1 = 2204 \quad C_{Dec}^2 = 3156 \quad C_{Dec}^{\Sigma} = 5360$

Table 1: Solution of the sample problem according to the decomposition approach.

4 Integrated approach

The inevitable sub-optimality of the decomposition approach motivated researchers to investigate *integrated approaches* to planning in the supply chain (see [16] for a recent overview, or [1] for an application to inventory control). Consequently, integrated (or centralized) models are of great theoretical relevance, but they may only be applied if the parties are strongly tied together, e.g., they are different divisions of the same enterprise. In any case, the integrated approach presumes a central agency that knows all the parameters and whose decisions are adopted by all partners.

The integrated approach minimizes the total cost on the supply chain level, while, in itself, it may increase or decrease the costs of the individual parties depending on the actual parameters. Often, integration incurs more savings realized at the supplier than at the buyer, because the aspects of the supplier were neglected in the first round of decision making in the baseline decomposition approach. To guarantee that integrated planning is beneficial for both parties, its practical implementation often involves some settlement on the sharing of benefits, which may range from the reduced unit prices to complex pricing schemes.

In theory, the integrated approach can be extended to supply chains with more than two parties, but the increasing computational complexity, the practical difficulties of sharing information, as well as of getting plans executed accurately even in face of uncertainties render this extension unpractical.

4.1 Computational model

Integrated planning implies that the demand set by the buyer equals the demand and the output of the supplier, i.e., $x_t^1 = d_t^2$, and the supplier's backlog r_t^2 is zero. Consequently, the supply and production plans can be realized without any modification.

Minimize

$$\sum_{t=1}^{T} \left(f^1 y_t^1 + h^1 s_t^1 + g^1 r_t^1 \;+\; f^2 y_t^2 + h^2 s_t^2 \right) \tag{10}$$

subject to

$$x_t^1 + (r_t^1 - r_{t-1}^1) = d_t^1 + (s_t^1 - s_{t-1}^1) \quad t = 1, ..., T \tag{11}$$

$$x_t^2 = x_t^1 + (s_t^2 - s_{t-1}^2) \quad t = 1, ..., T \tag{12}$$

$$x_t^1 \le D y_t^1 \quad t = 1, ..., T \tag{13}$$

$$x_t^2 \le D y_t^2 \quad t = 1, ..., T \tag{14}$$

$$s_0^1 = s_T^1 = s_0^2 = s_T^2 = r_0^1 = r_T^1 = 0 \tag{15}$$

$$x_t^1, r_t^1, s_t^1, x_t^2, s_t^2 \ge 0 \quad t = 1, ..., T-1 \tag{16}$$

$$y_t^1, y_t^2 \in \{0, 1\} \quad t = 1, ..., T \tag{17}$$

The MIP model of the integrated approach essentially corresponds to the duplication of the single-level MIP model. The objective function contains the sum of the total costs of the two parties (10). Lines (11) and (12) encode the inventory balance constraint. Inequalities (13) and (14) express that production requires a setup at each of the parties, with constant $D = \sum_{t=1}^{T} d_t$ as above.

The first polynomial algorithm for solving the integrated problem (with an arbitrary number of stages) has been proposed by Zangwill [27], using the concept of concave cost networks.

In this paper, we investigate the integrated approach with two different benefit sharing mechanisms. In the first case, each party bears its own costs, i.e.,

$$C^1_{Int} = \sum_{t=1}^{T} \left(f^1 y^1_t + h^1 s^1_t + g^1 r^1_t \right)$$

$$C^2_{Int} = \sum_{t=1}^{T} \left(f^2 y^2_t + h^2 s^2_t \right),$$

whereas in the second case they share the gain over the decomposition approach (denoted by G below) on a 50%–50% basis. Hence, first they have to compute the solution according to the decomposition approach; note that the assumptions allow them to do this in a truthful way:

$$G = C^1_{Dec} + C^2_{Dec} - C^1_{Int} - C^2_{Int}$$

$${C^1_{Int}}^* = C^1_{Dec} - \frac{G}{2}$$

$${C^2_{Int}}^* = C^2_{Dec} - \frac{G}{2}$$

4.2 Sample problem

The integrated solution for the sample problem is structurally different from the decomposed solution. Since the difference between the holding costs h^1 and h^2 is marginal, the items produced at the supplier (in periods 1, 2, 6, and 8) are immediately delivered to the buyer. According to this plan, the external demand will be satisfied on time, except for period 5, where it will be backlogged. The costs incurred at the individual parties decrease by 5.4% (buyer) and 37.6% (supplier), which means a 24.5% saving for the overall supply chain. These costs are displayed in Table 2 as they directly incurred at the parties, as well as after sharing the benefits.

5 Coordinated approach

Is it possible to circumvent the deficiencies of the decomposition method when there is no opportunity for integrated planning? This is the key question of *coordinated planning* that is aimed at improving the overall performance of the supply chain while maintaining the information asymmetry and local decision authority of the partners.

	t	1	2	3	4	5	6	7	8	9	10
Demand	d_t	71	84	43	21	4	81	59	44	32	46
Supply plan	x_t^1	71	148				144		122		
Production plan	x_t^2	71	148				144		122		
Realized delivery	x_t^{1R}	71	148				144		122		
Served demand	d_t^R	71	84	43	21		85	59	44	32	46

$$C_{Int}^1 = 2080 \qquad C_{Int}^2 = 1968 \qquad C_{Int}^{\sum} = 4048$$

$$C_{Int}^1{}^* = 1664.51 \qquad C_{Int}^2{}^* = 2383.49 \qquad C_{Int}^{\sum}{}^* = 4048$$

Table 2: Solution of the sample problem according to the integrated approach.

Even though the literature provides quite a number of alternative definitions for coordinated planning, it is generally accepted that coordination complements the division of labor by re-adjusting some actions of the partners so as to achieve certain common, system-wide goals [2, 16, 22]. (Hence, often the term collaborative planning is used). According to the strong notion of coordination, a supply chain is coordinated if and only if the partners' optimal local decisions are implemented and lead to optimal system-wide performance. This problem can be captured in a game theoretic setting: how to find a set of optimal supply chain actions (i.e., production and delivery) that result in such an equilibrium from which no partner has an interest to deviate? The game theoretic perspective leads to theoretical contract models that coordinate a supply channel under rigorous simplifying assumptions (e.g., typically, one-period models are handled) [5, 15].

In this paper, we take a weaker, albeit widely accepted concept: the supply chain is coordinated if the local, selfish production and delivery actions result in a *better (at least as good) overall performance than the decomposed solution.* This definition allows for a wide spectrum of coordination mechanisms that have though some generic features in common:

- While keeping the privacy of sensitive cost factors, there is a need for sharing information on the intentions–specifically, plans–of the partners.
- So as to arrive at a coordinated solution acceptable for both parties, alternative planning scenarios have to be generated and evaluated mutually by all concerned parties.
- An incentive scheme is required that–against their local interests–drives the partners towards coordinated solutions. I.e., potential benefits of coordination should be shared.

Although the need for coordinated planning in supply chains is generally recognized, there is still a gap between theoretical proposals and practical requirements. Recently, motivated by the requirements of the automotive industry where the supplier–at least in the phase of planning–must fully comply with the demand of the buyer, we elaborated a coordination scheme where the buyer generates alternative scenarios, which are evaluated and priced by the supplier. In the end, the buyer selects a scenario which incurs the lowest total cost to it [9]. As a form of benefit balancing, the buyer receives appropriate compensation whenever one of its locally sub-optimal scenarios is executed. In this scheme called Dynamic Supply Loops, the same pattern of coordinated planning is repeated between partners of subsequent tiers of the supply chain. The method outperforms traditional upstream planning in a multi-echelon model, facilitates coordination and is easy to implement by using the services of contemporary planner systems and communication channels [10].

According to our actual problem definition, backlogging is allowed at the supplier even at the time of planning. Hence, in the computational model below, it will be the supplier who offers alternative scenarios with appropriate pricing.

5.1 Computational model

Our model for coordinated planning builds on the elements of the decomposed model. The basic idea is that instead of a single production plan, the supplier responds to the demand of the buyer with a set of *alternative scenarios* from among which the buyer will finally select a single one to be executed. However, compared to its optimal production plan, the supplier can only have a *loss* on each of the alternative scenarios, hence, it assigns a compensation request to each of the alternatives. The buyer's final decision is based on the total cost of the scenarios: this $C^{1,n}$ cost is calculated as given in (3) plus the compensation to be paid for a suboptimal scenario.

The alternative planning scenarios are distinguished by index $n = 0, ..., N$, and the costs of a particular scenario from the perspective of the buyer and the supplier are expressed by $C^{1,n}$ and $C^{2,n}$, respectively. Locally optimal solutions of the decomposed approach are indexed by $n = 0$.

The coordinated planning protocol proceeds in the following steps:

1. The buyer solves its problem as formulated in equations (4)-(9), facing directly the external demand, i.e., $d_t^1 = d_t$, $(t = 1, ..., T)$.

2. The supply plan x_t^1 is communicated to the supplier who in return generates a baseline production plan $x_t^{2,0}$, incurring $C^{2,0}$ cost.

3. With some policy (discussed later) the supplier generates also a series of alternative production plans, $x_t^{2,n}$. The supplier's potential *loss* on each scenario is calculated as $L^{2,n} = C^{2,n} - C^{2,0}$.

4. The alternative scenarios are offered to the buyer, together with a compensation requirement $Z^{2,n}$. Obviously, the self-interested supplier will ask for a compensation that covers its loss, i.e., $Z^{2,n} \geq L^{2,n}$.

5. The buyer simulates the execution of each $x_t^{2,n}$ scenario received, and calculates the $C^{1,n}$ cost as a sum of the realized cost given in (3) and the $Z^{2,n}$ compensation.

6. Finally, the buyer selects the scenario with the minimal $C^{1,n}$ cost and the chain as a whole will be operated accordingly.

Thanks to the assumption that $Z^{2,n} \geq L^{2,n}$, the partners deviate from the baseline solution of the decomposition approach only if there is a scenario n where $C^{1,n} + C^{2,n} < C^{1,0} + C^{2,0}$. In the last resort, the baseline solution is executed. Hence, the above protocol coordinates the chain in the weak sense.

The policy for scenario generation is based on the idea that not only the buyer, but also the chain as a whole could be better off if the buyer did not have to face backlog caused by the mismatch of the supply and production plans. Hence, the supplier generates a series of alternative scenarios with less and less backlog, ending up with a production plan scenario N without any backlog (i.e., in each period t, $r_t^{2,N} = 0$). Such a series can be generated by incrementally increasing the backlog cost $g^{2,n}$. (In our experiments reported later this increment was 10%, i.e., $g^{2,n+1} = 1.1g^{2,n}$.) However, varying the backlog cost is only a simple technical means for generating alternative production plans; the $C^{2,n}$ costs are calculated with the original g^2 value. Note that since the supply plan is generated a priori, the proposed alternatives may not contain the optimal solution of the integrated approach.

In our model the *benefit* of eventual cooperation can be shared through the compensation required by the supplier for a sub-optimal scenario. A supplier with a *fully cooperative* but rational attitude does not require more than its eventual loss, i.e., $Z^{2,n} = L^{2,n}$. Though, in addition to the compensation the supplier may want to realize some gain, too. A *gain ratio* can express this gain in the percentage of the supplier's cost $C^{2,0}$ with the baseline solution. If the supplier has a greedy attitude and requires more compensation than its potential loss, then the chances of arriving at a coordinated solution are getting worse.

The above planning protocol can be generalized to multi-level chains, but so as to keep communication and decision complexity at bay, the feedback loop should be confined to immediate partners in a chain. Note that in this case there is no guarantee that the method coordinates the channel as a whole.

A shortfall of this coordination mechanism is that the supplier, if it has information about the buyer's parameters, can abuse the mechanism: it can deliberately generate a default plan that is unacceptable to the buyer, and assign massive compensation costs to any other alternatives. Finally, we note that, in contrast to the previous approaches, the solution computed by the coordination approach is characteristic to the defined coordination mechanism; different mechanisms may result in different solutions.

5.2 Sample problem

For the sample problem, the supplier generated a series of production plan alternatives, which contained the baseline decomposed solution and additional four alternative plans. The last alternative, with modified backlog cost $g^{2,4} = 9.663$, resulted in no backlog in either relation. This alternative incurs a higher cost of 3348 for the supplier (cf. the default $C^2_{Dec} = 3156$). However, it is worth for the buyer to compensate the supplier for eliminating the costly backlog, and hence, this plan alternative is selected. This solution decreases the total cost for the overall supply chain by 20.30% compared to the decomposition approach. Note, however, that this solution lags behind the integrated solution, because the supply plan of the buyer, prepared a priori, did not enable the partners to find a more economical solution.

	t	1	2	3	4	5	6	7	8	9	10
Demand	d_t	71	84	43	21	4	81	59	44	32	46
Supply plan	x^1_t	71	84	43	25		81	59	44	32	46
Production plan	x^2_t	71	152				140		122		
Realized delivery	x^{1R}_t	71	84	43	25		81	59	44	32	46
Served demand	d^R_t	71	84	43	21	4	81	59	44	32	46

$C^1_{Crd} = 1116 \quad C^2_{Crd} = 3156 \quad C^{\sum}_{Crd} = 4272$

Table 3: Solution of the sample problem according to the coordinated approach.

6 Bilevel approach

The *bilevel optimization approach* captures the decision situation of a well-informed buyer (*leader* in the terminology of bilevel optimization), who knows the decision problem of the supplier (*follower*), i.e., the parameters f^2, h^2 and g^2, and wants to take into account the optimal decision of the supplier when preparing its supply plan. We adopt the *optimistic* assumption, i.e., consider that in case of multiple optimal solutions for the supplier, it chooses the optimal solution which is the most favorable for the buyer. Notice that in the *pessimistic* case the supplier would always choose an optimal solution which yields the least favorable outcome for the buyer. The basic modeling and solution techniques in bilevel programming are presented in [7].

Up to now, the literature of bilevel approaches to inventory problems is rather scarce. One barrier to the wider application of bilevel techniques is that these problems are notoriously hard to solve. The few works in bilevel inventory problems include the paper of de Kok and Muratore [6], who investigate a planning problem in an extended supply chain with an arbitrary number of parties. A production and transshipment plan is sough for multiple items over a finite horizon. They present a bilevel approach where the optimality condition states that the resulting plan must be locally optimal for each of the parties. A heuristic solution approach based on iteratively solving a MIP is presented.

Ryu et al. [20] introduced a bilevel programming model to a production and distribution planning problem in a supply chain. The upper level corresponds to the distribution problem in the network, whereas the lower level captures the local planning problems of the multiple production plants. The local planning problems have a simple linear structure, which enabled the authors to extract parametric solutions, and hence, convert the bilevel problem into a tractable single level global optimization problem. A similar production and distribution problem subject to uncertainties is formulated as a probabilistic bilevel problem in [19]. Yang et al. [26] investigate the problem of coordinated planning in a supply chain under hard service time requirements, where a central coordinating agency allocates desired response times to the production and transportation companies in the supply chain. Finally, bilevel problems in production scheduling have been investigated in [13, 14].

6.1 Computational model

In the following mathematical program we model the decision problem of the leader. The decision variables and parameters are like in the previous approaches.

Minimize

$$\sum_{t=1}^{T} \left(f^1 y_t^1 + h^1 s_t^1 + g^1 r_t^1 - g^2 r_t^2 \right) \tag{18}$$

subject to

$$x_t^1 + r_{t-1}^2 - r_t^2 + (r_t^1 - r_{t-1}^1) = d_t^1 + (s_t^1 - s_{t-1}^1) \qquad t = 1, ..., T \tag{19}$$

$$x_t^1 + r_{t-1}^2 - r_t^2 \leq D y_t^1 \qquad t = 1, ..., T \tag{20}$$

$$s_0^1 = s_T^1 = r_0^1 = r_T^1 = 0 \tag{21}$$

$$x_t^1, r_t^1, s_t^1, \geq 0 \qquad t = 1, ..., T \tag{22}$$

$$y_t^1 \in \{0, 1\} \qquad t = 1, ..., T \tag{23}$$

$$\begin{pmatrix} y^2 \\ x^2 \\ s^2 \\ r^2 \end{pmatrix} \in \arg\min \{ \sum_{t=1}^{T} \left(f^2 y_t^2 + h^2 s_t^2 + g^2 r_t^2 \right) \mid (25) - (29) \} \tag{24}$$

where

$$x_t^2 + (r_t^2 - r_{t-1}^2) = x_t^1 + (s_t^2 - s_{t-1}^2) \quad t = 1, ..., T \tag{25}$$
$$x_t^2 \leq D y_t^2 \quad t = 1, ..., T \tag{26}$$
$$s_0^2 = s_T^2 = r_0^2 = r_T^2 = 0 \tag{27}$$
$$x_t^2, s_t^2, r_t^2 \geq 0 \quad t = 1, ..., T \tag{28}$$
$$y_t^2 \in \{0, 1\} \quad t = 1, ..., T \tag{29}$$

The objective (18) is minimizing the leader's total cost, minus the compensation received from the supplier for late deliveries. Variables x_t^1 represent the supply plan computed by the buyer, which is sent to the supplier and constitutes the demand it has to satisfy. Then, the realized supply is $x_t^{1R} = x_t^1 + r_{t-1}^2 - r_t^2$, which equals $x_t^2 - s_t^2 + s_{t-1}^2$ by the balance equation (25) of the supplier. Otherwise, the constraints (19)-(23) and (25)-(29) are identical to those in the decomposed model of the buyer and supplier, respectively. The supplier's optimality condition (24) expresses that the supplier chooses its optimal production plan for the given supply plan received from the buyer.

6.2 Solution algorithm

Albeit there exists a number of standard solution techniques for specially structured bilevel problems, e.g., linear bilevel programs, we were not able to apply these techniques to the problem at hand. Instead, we developed a customized solution algorithm, motivated by the the dynamic program (DP) of Zangwill [27] for the single-stage uncapacitated lot-sizing problem with backlogs.

Recall that this single-stage problem always admits an optimal solution of the following structure [27]: there exists an integer $K \geq 1$ and a sequence of $2K$ integers $1 = \ell_1 \leq i_1 < \ell_2 \leq i_2 < \cdots < \ell_K \leq i_K \leq T$ such that $x_j \neq 0$ only if $j \in \{i_1, i_2, \ldots, i_K\}$ from which periods $\ell_j, \ldots, i_j - 1$ are satisfied by backlogging, while periods $i_j + 1, \ldots, \ell_j - 1$ from stock, and $s_{\ell_j - 1} = r_{\ell_j - 1} = 0$. We call these $2K$ integers a *configuration*.

In order to solve the bilevel optimization problem, we will search over all possible configurations that may be implemented by the supplier. For each configuration, we derive the conditions under which the configuration may be optimal for the supplier for the demand x_t^1, based on the DP of Zangwill. The conditions will take the form of linear inequalities in x_t^1 and some extra variables. We will add them to the buyer's constraints (19)-(23), and solve the resulting MIP. Repeating this for each configuration, and taking the minimum value of the optimal solutions of the MIPs, we obtain the optimal solution of the bilevel optimization problem.

It remains to derive the conditions for a demand vector $d_t^2 = x_t^1$ such that a configuration is optimal for the supplier. The dynamic program of Zangwill, formulated with the demand and costs of the supplier, is as follows.

Let $\phi(u, v)$ denote the optimal solution value of the supplier provided the demand in period v is satisfied from period u. Furthermore, let $G(v)$ denote the optimal solution value of the problem restricted to the periods $v, \ldots, T$, i.e., $G(v) = \min_{u \geq v} \phi(u, v)$. Now we can define $\phi(u, v)$ formally:

$$\phi(u,v) = \begin{cases} (v-u)h^2 d_v^2 + \min\{G(v+1), \phi(u,v+1)\} & \text{when } u < v, \\ (u-v)g^2 d_v^2 + \phi(u,v+1) & \text{when } u > v, \\ f^2 + \min\{G(u+1), \phi(u,u+1)\} & \text{when } u = v. \end{cases}$$

The optimal supplier solution value is $G(1)$ which can be computed by decreasing u from T down to 1 and for each u in turn, iterating v from T down to 1. Now, all we need is to use this dynamic program to express conditions under which a configuration of the supplier is optimal. The following inequalities describe a relaxation of the dynamic program:

$$G(v) \leq \phi(u,v) \quad \text{when } u \geq v \tag{30}$$
$$\phi(u,v) \leq (v-u)h^2 d_v^2 + G(v+1) \quad \text{when } u < v \tag{31}$$
$$\phi(u,v) \leq (v-u)h^2 d_v^2 + \phi(u,v+1) \quad \text{when } u < v \tag{32}$$
$$\phi(u,v) = (u-v)g^2 d_v^2 + \phi(u,v+1) \quad \text{when } u > v \tag{33}$$
$$\phi(u,v) \leq f^2 + G(u+1) \quad \text{when } u = v \tag{34}$$
$$\phi(u,v) \leq f^2 + \phi(u,u+1) \quad \text{when } u = v \tag{35}$$
$$G(T+1) = \phi(u,T+1) = 0 \quad \text{when } u \leq T \tag{36}$$
$$\phi(u,v), G(v) \geq 0 \quad \text{for all } u,v \tag{37}$$

Lemma 1 *The configuration $1 = \ell_1 \leq i_1 < \ell_2 \leq i_2 < \cdots < \ell_K \leq i_K \leq T$ is optimal for demand d_t^2 if and only if there exists (ϕ, G) that satisfies (30)-(37), and also for each $j = 1, \ldots, K$:*

i) $G(\ell_j) = \phi(i_j, \ell_j)$,

ii) $\phi(i_j, v) = (v - i_j)h^2 d_v^2 + \phi(i_j, v+1)$ *for* $i_j < v < \ell_{j+1} - 1$,

iii) $\phi(i_j, v) = (v - i_j)h^2 d_v^2 + G(v+1)$ *for* $v = \ell_{j+1} - 1$,

iv) $\phi(i_j, i_j) = f^2 + \phi(i_j, i_j + 1)$ *if* $i_j < \ell_{j+1} - 1$,

v) $\phi(i_j, i_j) = f^2 + G(i_j + 1)$ *if* $i_j = \ell_{j+1} - 1$.

Proof The crux of the proof is that we observe how the quantities $G(v)$ and $\phi(u, v)$ relate in an optimal solution with the given configuration.
Necessity Suppose the given configuration is optimal. Since $G(v) = \min_{u \geq v} \phi(u, v)$, condition (i) is satisfied. Moreover, for $j = 1, \ldots, K$, the demand $d_{\ell_j}, \ldots, d_{\ell_{j+1}-1}$ is satisfied from the time period i_j, which implies conditions (ii)-(v).
Sufficiency We have to prove that the configuration is optimal, provided there exist $\phi(u, v)$ and $G(v)$ satisfying the conditions (i)-(v) along with (30)-(37). It suffices to show that (ϕ, G) can be chosen such that among (31) and (32) at least one holds at equality for each $u < v$, and among (34) and (35) at least one holds with equality for each u. Such a solution is the output of the above dynamic

program, and therefore, $G(1)$ is the optimal solution value, and the configuration is optimal. Notice that the conditions (i)-(v) ensure that the configuration is indeed a feasible solution for the lot-sizing problem.

It remains to show how to choose (ϕ, G). Let (ϕ, G) be arbitrary satisfying (i)-(v) and (30)-(37) with maximum $\sum_{u \neq v} \phi(u, v) + \sum_v G(v)$ value. Notice that this sum is finite, since (30)-(37) ensure that all the $\phi(u, v)$ and $G(v)$ values are bounded. Now suppose $\phi(u, v) < \min\{(v-u)h^2 d_v^2 + G(v+1), (v-u)h^2 d_v^2 + \phi(u, v+1)\}$ for some $u < v$. Clearly, (u, v) is not among the values for which $\phi(u, v)$ is fixed by (i)-(v), since those values cannot be changed due to (36). Notice also that $\phi(u, v)$ is involved only into two inequalities on the left hand side, i.e., there is one inequality in (31) and one in (32). Then we increase $\phi(u, v)$ until equality holds. Clearly, we obtain a solution with a larger sum of the values of ϕ and G, a contradiction. One similarly observes that none of the $G(v)$ values could be increased. Finally, if some of the $\phi(u, u)$ is strictly smaller than $f^2 + \min\{G(u+1), \phi(u, u+1)\}$, then we may increase $\phi(u, u)$ along with all the $\phi(u, k)$ with $k < u$, to keep (33) satisfied, a contradiction again. Hence, this choice of (ϕ, G) is as desired.

6.3 Sample problem

The solution of the sample problem according to the bilevel approach is shown in Table 4. It demonstrates the various ways how the buyer can manipulate the supply plan submitted to the supplier so as to minimize its own cost. Namely, in period 1, the buyer asks for a larger lot than its actual needs (82 instead of 71). This is necessary in order to prevent the supplier from backlogging this lot to period 2 (cf. the decomposed solution in Table 1), which would cause an extensive backlog cost for the buyer as well. On the other hand, the buyer anticipates some demand from period 6 to period 5. The supply plan for period 5 is then the maximum amount that does not trigger production at the supplier. While this kind of demand anticipation does not affect the material flow, it incurs a backlog compensation paid by the supplier to the buyer for the late satisfaction of the anticipated demand. Certainly, this can be regarded as an abuse of the contract between parties, but this is a rational action from a cost minimizing buyer. We even encountered problem instances where, with the extensive usage of this technique, the buyer realized a negative total cost, i.e., the backlog compensation received from the supplier dominated all costs of the buyer.

Regarding the costs incurred at the parties in the sample problem, the buyer could reduce its costs by 60% compared to the decomposition approach, whereas the costs of the supplier increased by 8%. The overall cost in the supply chain decreased by 20.2%. Note that in general there is no guarantee that the bilevel approach decreases the total cost.

	t	1	2	3	4	5	6	7	8	9	10
Demand	d_t	71	84	43	21	4	81	59	44	32	46
Supply plan	x_t^1	82	73	68		42.72	39.77	57.51	55.46	21.93	44.61
Production plan	x_t^2	82	141				140		122		
Realized delivery	x_t^{1R}	82	73	68			82.49	57.51	55.46	21.93	44.61
Served demand	d_t^R	71	84	43	21	4	81	59	44	32	46

$C_{Bl}^1 = 869.64 \quad C_{Bl}^2 = 3407.69 \quad C_{Bl}^{\sum} = 4277.33$

Table 4: Solution of the sample problem according to the bilevel approach.

7 Comparison of the approaches

7.1 Computational evaluation

The investigated approaches have been compared quantitatively in computational experiments on a set of randomly generated problem instances. For that purpose, we implemented the presented models and algorithms for all the four approaches in FICO XPress-MP [11], using the Mosel programming language. In the numerical study, 100 problem instances were generated with the following parameters. The number of time periods was fixed to $T = 10$, the buyer's setup cost to $f^1 = 100$, while the other cost parameters were randomized. In the sequel, $U[a, b]$ stands for the uniform distribution over the integers in interval $[a, b]$. Hence, we let $h^1 \leftarrow U[2, 10]$, $g^1 \leftarrow U[4, 20]$, $f^2 \leftarrow U[250, 500]$, $h^2 \leftarrow U[1, h^1]$, and $g^2 \leftarrow U[2, g^1]$.

We recall that for the decomposed, integrated, and bilevel approaches, optimal solutions of formal mathematical models were computed (in case of the decomposed approach, the typical upstream planning method was considered). In contrast, the solution found by the coordinated approach was characteristic to the developed coordination mechanism; different mechanisms may lead to different solutions. Finding the optimal solutions required a few minutes per instance for the bilevel solver (although there are several possibilities to speed up the presented algorithm), while running times were negligible for the other three solvers.

The results are displayed in Figures 1-3, which compare the results of the integrated, coordination, and bilevel approaches to those of the baseline decomposition approach. Each spot in the diagram corresponds to one problem instance, and its horizontal (vertical) position shows the difference of the buyer's (supplier's) cost in the given approach compared to the cost in the decomposed solution. The difference is measured in *percent of the overall total cost of the decomposed solution.* Hence, a spot with coordinates $(-25, -15)$ denotes that the total cost was decreased by 40%. The lower left quarter of the diagram corresponds to solutions that are beneficial for both parties, whereas the upper left quarter contains solutions advantageous for the buyer, but disadvantageous for the supplier, etc. A solution below the diagonal improves the cost of the overall supply chain. Note that the savings are higher (by a factor of 2 on average) compared to the cost of an individual party.

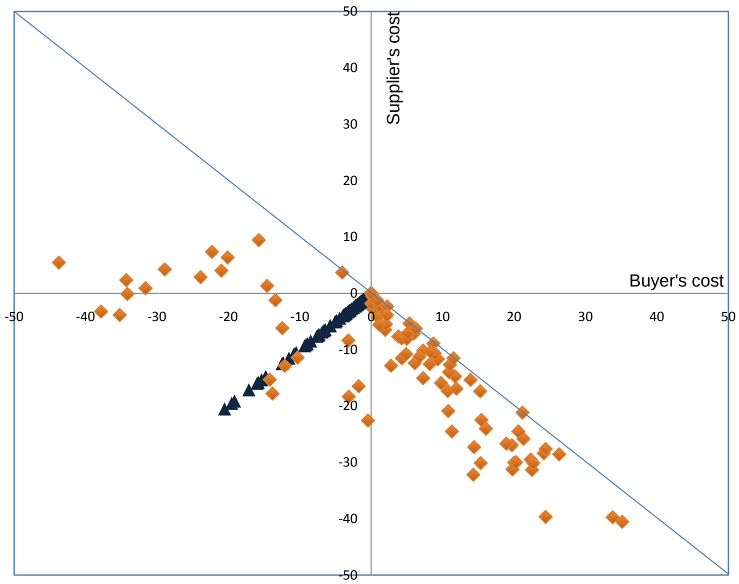

Figure 1: Results of the integrated approach compared to the decomposed solutions.

Figure 1 shows the results obtained by the integrated approach. Light (orange) diamonds correspond to solutions without benefit balancing, $[C^1_{Int}, C^2_{Int}]$. Dark (blue) triangles stand for solutions with benefit balancing, $[{C^1_{Int}}^*, {C^2_{Int}}^*]$. As expected, the integrated approach decreases the cost of the overall supply chain for all instances, by 8.67% on average. Without benefit balancing, this approach is advantageous especially for the supplier (its cost decreased in 80 problem instances), because its objectives are disregarded in the first round of decision making in the decomposition approach. Integration can be beneficial for the buyer when centralized planning eliminates its backlog originating from the difference of the planned and realized supply (25 instances).

The results of the coordinated approach are displayed in Figure 2, where light (orange) diamonds represent the case with a fully cooperative supplier. In this case, the supplier does not reduce its costs, but the overall cost is decreased for 36 instances, by 5.01% on average. This relative benefit is smaller than the benefit of the integrated approach. Dark (blue) triangles correspond to the case when the supplier works with a 30% gain ratio in the compensation required. In this case, only 14 solutions could improve the overall cost compared to the decomposition approach, by 3.62% on average, but this gain is shared between the parties.

Finally, Figure 3 contains the results of the bilevel approach. The informed

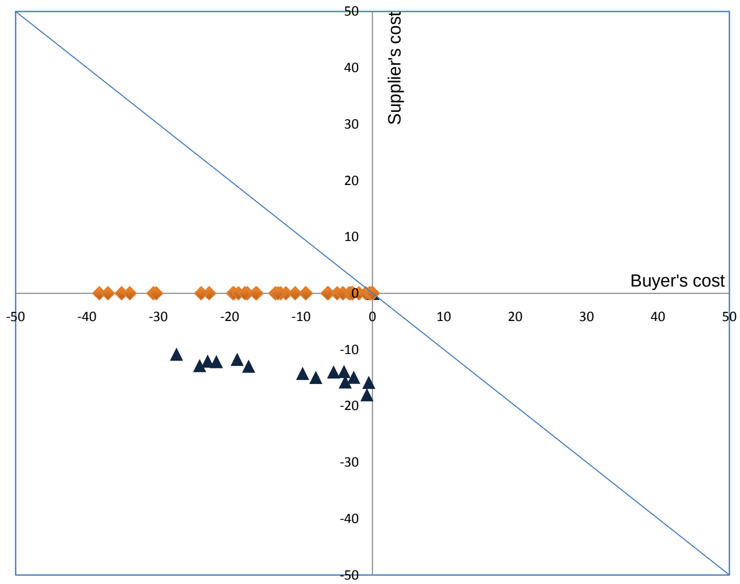

Figure 2: Results of the coordinated approach compared to the decomposed solutions.

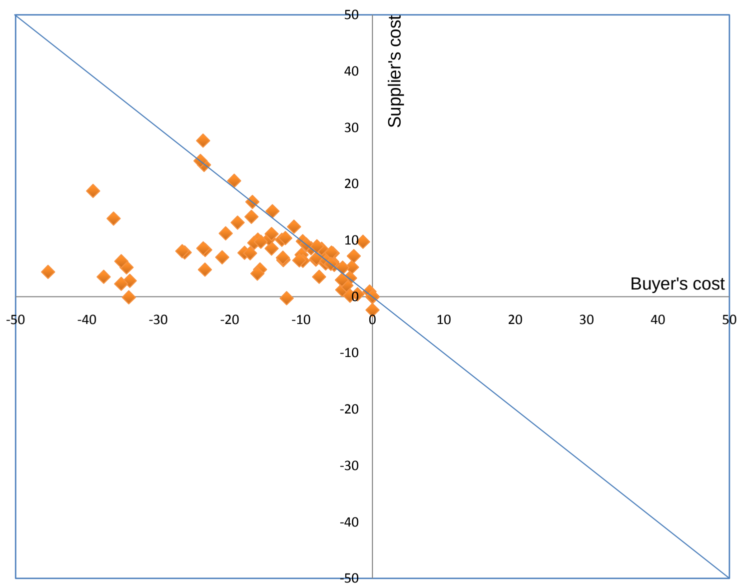

Figure 3: Results of the bilevel approach compared to the decomposed solutions.

buyer could reduce its costs in all cases, by 10.91% on average, compared to the unaware buyer of the decomposition case. Notably, this is a 66.46% reduction, if we take the decomposed buyer's cost as the basis. An extreme outcome has been achieved for one instance, where the total cost of the buyer was negative, i.e., the backlog compensation received from the supplier dominated its actual costs. Bilevel planning is generally not beneficial for the supplier, whose costs increased for 72 instances, by 5.94% on average. Although there is no guarantee that the overall cost decreases in the bilevel case, an average reduction of 4.98% occurred. Surprisingly, this is better result than the one achieved by coordination with the supplier's gain ratio set to 30%. A possible interpretation of this result is that (even asymmetric) information can reduce the overall cost more efficiently than the incentive to cooperate without sufficient information.

7.2 Analytical comparison

After the detailed investigation of the individual approaches, we compare them according to various aspects. The comparison is summarized in Table 5, whereas each of the aspects are explained in detail below.

Information requirements The ranking of the approaches according to increasing information requirements starts with the decomposition approach (the local demand is communicated only), followed by the coordinated approach (multiple alternative plans and compensation requests are dispatched), the bilevel approach (the buyer has complete information of the supplier, but communicates only the demand), and finally, the integrated approach (mutual access to all data of the partners).

Cooperation The integrated approach is applicable in case the business objectives of the parties are completely aligned, and the parties are ready to adopt the centrally generated plans. The other three approaches assume selfish parties chasing their own business objectives within the frames defined by the actual contracts.

Contractual requirements The decomposition and the bilevel approaches do not require specialized contracts among the parties. In contrast, the integrated and the coordinated approaches assume that the rules for information and benefit sharing are precisely laid down in an appropriate contract.

Optimization The decomposition approach optimizes the plan of each party individually. However, the realization can deviate from the plans, and therefore the decomposition plan is unable to provide any kind of performance guarantee. The integrated approach minimizes the total cost in the overall supply chain. The coordination mechanism aims at the same, but it is often hampered by some limitation of the actual coordination mechanism. The bilevel approach minimizes the buyer's cost.

	Decomposition	Integrated	Coordinated	Bilevel
Information requirements	Demand only	Full information	Demand, alternative plans, compensation costs	Full information in one direction
Cooperation	Selfish parties	Full cooperation	Selfish parties	Selfish parties
Contractual requirements	None	Special contract	Special contract	None
Optimization	Ad hoc	Supply chain total cost	Supply chain total cost (approx.)	Buyer's total cost
Computational complexity	Low	High	Moderate	Very high
Extension to multi-level	Yes	Theoretically yes, practically limited	Yes (some limitations)	Theoretically yes, practically limited

Table 5: Comparison of the different approaches.

Computational complexity Models and algorithms for different single-stage problems addressed by the decomposition approach are widely studied in the literature, and various polynomially solvable cases are known. The coordinated approach solves a series of single-stage problems, which is also tractable in most cases. The multi-stage problems faced by the integrated approach are inherently more complicated, though, relevant polynomial cases also exist [27]. In contrast, such results are scarcely available for bilevel problems, and even the simplest problems of interest are NP-hard.

Extension to multi-level In theory, any of the investigated approaches can be extended to any number of levels. In case of the decomposition approach, this extension is part of the industrial practice. For the integrated approach, the practical barrier of multiple levels is the problem size and the sensitivity of the resulting plan to disturbances. In case a coordinated approach is implemented in a multi-level supply chain, a pairwise negotiation scheme must be used in order to prevent extensive response times. Bilevel (or multi-level) programming is widely considered as unpractical for more than two levels, but bilevel optimization can be used in multi-level supply chains for two neighboring levels by isolating the upstream levels with a suitable decomposition.

8 Conclusions

This paper investigated different fundamental approaches to inventory control in supply chains, focusing on how these approaches handle the potentially conflicting objectives and the information asymmetry among the partners. Beyond presenting theoretical considerations, a sample problem was modeled and solved according to each of the approaches, and computational results have also been presented.

The findings of the comparison can be briefly summarized as follows. In case the business objectives of the partners coincide and they are ready to share all relevant, partly sensitive data with each other, the implementation of an *integrated* planning approach may yield significant savings for all parties. When the supply chain consists of autonomous companies with disparate objectives, a *coordination* approach may bring comparable results, if the dynamics of the chain (both in terms of stable network design and non-critical response times) allow for appropriate contracts and communication mechanisms among the parties. On the other hand, an individual party, having access to sufficiently precise data about its upstream partners, may minimize its own cost by implementing a *bilevel* optimization approach. In cases where none of the previous choices are applicable, the current industrial standard decomposition approach remains the default choice.

Acknowledgements

This work has been supported by the OTKA grants K76810 and T73376, the NKTH grant OMFB-01638/2009, as well as the EU grant AC/DC IST-03152. A. Kovács acknowledges the support of the János Bolyai scholarship No. BO/00138/07.

References

[1] B. Abdul-Jalbar, J. Gutiérrez, J. Puerto, and J. Sicilia. Policies for inventory/distribution systems: The effect of centralization vs. decentralization. *International Journal of Production Economics*, 81-82:281–293, 2003.

[2] M. Albrecht. *Supply Chain Coordination Mechanisms.* Springer, 2010.

[3] S. Axsäter. *Inventory Control.* Springer, 2nd edition, 2006.

[4] J. F. Bard. Coordination of a multidivisional organization through two levels of management. *Omega*, 11:457–468, 1983.

[5] G.P. Cachon. *Supply Chain Management: Design, Coordination and Cooperation*, chapter Supply chain coordination with contracts, pages 229–339. Elsevier, 2003.

[6] A. G. de Kok and G. Muratore. Coordinating supply chains: a bilevel programming approach. In *International Workshop on Supply Chain Models for Shared Resource Management*, 2010.

[7] S. Dempe. *Foundations of Bilevel Programming.* Kluwer, 2002.

[8] G. Dudek and H. Stadtler. Negotiations-based collaborative planning between supply chain partners. *European Journal of Operational Research*, 163:668–687, 2005.

[9] P. Egri, A. Döring, T. Timm, and J. Váncza. Collaborative planning with dynamic supply loops. In W. Sihn and P. Kuhlang, editors, *Sustainable*

Production and Logistics in Global Networks, Proc. of the 43rd CIRP International Conference on Manufacturing Systems, pages 844–851, 2010.

[10] R. Ericsson, R. Becker, A. Döring, H. Eckstein, Th. Kopp, I. Poslu, and J. Váncza, editors. *From Build-to-Order to Customize-to-Order: Advancing the Automotive Industry by Collaboration and Modularity*, volume in print. The AC/DC Consortium, 2010.

[11] FICO. Xpress Optimization Suite 7. http://www.fico.com/en/Products/DMTools/Pages/FICO-Xpress-Optimization-Suite.aspx.

[12] B. Fleischmann and H. Meyr. *Supply Chain Management: Design, Coordination and Cooperation*, chapter Planning hierarchy, modelling and advanced planning systems, pages 457–523. Elsevier, 2003.

[13] T. Kis and A. Kovács. On bilevel machine scheduling problems. *OR Spektrum*, in print, 2010.

[14] A. Kovács and T. Kis. Constraint programming approach to a bilevel scheduling problem. *Constraints*, in print, 2010.

[15] M. Leng and M. Parlar. Game theoretic applications in supply chain management: a review. *INFOR*, 43(3):187–220, 2005.

[16] X. Li and Q. Wang. Coordination mechanisms of supply chain systems. *European Journal of Operational Research*, 179(1):1–16, 2007.

[17] G. Perakis and G. Roels. The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Science*, 53(8):1249–1268, 2007.

[18] R. Pibernik and E. Sucky. An approach to inter-domain master planning in supply chains. *International Journal of Production Economics*, 108:200–210, 2007.

[19] E. Roghanian, S. J. Sadjadi, and M. B. Aryanezhad. A probabilistic bi-level linear multi-objective programming problem to supply chain planning. *Applied Mathematics and Computation*, 188:786–800, 2007.

[20] J-H. Ryu, V. Dua, and E. N. Pistikopoulos. A bilevel programming framework for enterprise-wide process networks under uncertainty. *Computers and Chemical Engineering*, 28:1121–1129, 2004.

[21] S.P. Sarmah, D. Acharya, and S.K. Goyal. Buyer vendor coordination models in supply chain management. *European Journal of Operational Research*, 175(1):1–15, 2006.

[22] H. Stadtler. A framework for collaborative planning and state-of-the-art. *OR Spectrum*, 31:5–30, 2009.

[23] J. Tirole. *The Theory of Industrial Organization*. MIT Press, 1988.

[24] A. Wagelmans, S. van Hoesel, and A. Kolen. Economic lot sizing: An $O(n \log n)$ algorithm that runs in linear time in the Wagner-Whitin case. *Operations Research*, 40:S145–S156, 1992.

[25] H. M. Wagner and T. M. Whitin. Dynamic version of the economic lot size model. *Management Science*, 5(1):89–96, 1958.

[26] W. Yang, L. Li, and S. Ma. Coordinating supply chain response-time: a bi-level programming approach. *International Journal of Advanced Manufacturing Technology*, 31:1034–1043, 2007.

[27] W. I. Zangwill. A backlogging model and a multi-echelon model of a dynamic economic lot size production system – a network approach. *Management Science*, 15(9):506–527, 1969.