A Repository of Convex Quadratic Programming Problems*

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Abstract

The introduction of a standard set of linear programming problems, to be found in NETLIB/LP/DATA, had an important impact on measuring, comparing and reporting the performance of LP solvers. Until recently the efficiency of new algorithmic developments has been measured using this important reference set. Presently, we are witnessing an ever growing interest in the area of quadratic programming. The research community is somewhat troubled by the lack of a standard format for defining a QP problem and also by the lack of a standard reference set of problems for purposes similar to that of LP. In the paper we propose a standard format and announce the availability of a test set of collected 138 QP problems.

Keywords: *Quadratic programming, QPS format, test problems.*

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1 Introduction

The importance of quadratic optimization is steadily increasing thanks to the many important application areas. There are several solvers, both academic and commercial, available for solving real life convex quadratic programming (QP) problems. The performance characteristics (reliability, efficiency, accuracy, capacity) of the solvers vary quite widely.

In parallel with the above, there is a growing need for a publicly available set of real life quadratic programming problems that can be used by developers of QP algorithms for testing and by potential users for comparing solvers.

When we recently developed our interior point QP solver, BPMPD [5] we encountered some difficulties when we tried to collect good test problems. In a search for such problems, we contacted several sources and, as a result, we have assembled a repository of convex QP problems.

Unfortunately, it turned out that there is no standard file format for QP and different sources indeed used different formats.

After this recognition we set two goals. First, to establish a standard format for presenting QP problems and, second, to satisfy the growing need for publicly available test problems that can be used to evaluate and compare QP solvers. With the kind permission of the problem owners we are making the test set available for the research community.

2 Proposed QP format

Without loss of generality, the convex QP problem is usually assumed to be in the following form:

$$\begin{array}{rl} \min & f(x) = c_0 + c^T x + \frac{1}{2} x^T Q x, \quad Q \text{ symmetric and positive semidefinite} \\ \text{subject to} & Ax = b \\ & l \leq x \leq u \end{array}$$

where A is an $m \times n$ matrix and all the vectors are of appropriate dimensions, in particular, Q is $n \times n$.. Some of the components of l and u may be $-\infty$ and $+\infty$, respectively. This means that components of x can be of any type that is defined in the MPS format for linear programming (LP) problems.

In practice, two QP formulations are common. One of them is the *separable* form where Q contains only diagonal (squared) terms. Sometimes it is natural, other times transformations are performed to bring the problem into this form. The usual reason for doing so is to 'appease' the solver. The other formulation is the general (or non-separable) one where off-diagonal entries also appear (in a symmetric fashion, of course).

In linear programming there is a de facto standard of presenting LP problems. It was developed by IBM (see [4] or [7]) in the early days of computing. It is a fixed record oriented system with 80 character records. While its convenience is questionable, it is the only format accepted by all solvers. This is the main reason why we have decided to use it as a basis for the QP format. Our proposed QP format is an extension to the MPS format and is similar to the one used in the pioneering QP implementations of IBM OSL [8] and Vanderbei [9]. This extension makes it possible that when the QP part of a problem is missing the file is still a legitimate MPS file of the remaining LP problem.

In the QP format, after the BOUNDS section of MPS, a new section is added. It is introduced by the QUADOBJ indicator record. As Q is symmetric, only its lower triangular part has to be given. The elements of Q must appear in columnwise order much the same way as the elements of the A matrix. The only difference is that row and column names that identify the q_{ij} coefficients in Q are the column names of A. Obviously, the row names of A do not play any role here. The 1/2 multiplier of the quadratic part is implied, therefore, the double of the diagonal coefficients has to be entered as shown in the following example.

Original problem:

$$\begin{array}{rcccccccccccc}
\min\ f(x,y) = & 4 & + & 1.5x & - & 2y & + & 4x^2 & + & 2xy & + & 5y^2 \\
\text{s.t.} & & & 2x & + & y & \geq & 2 & & & & \\
 & & & -x & + & 2y & \leq & 6 & & & & \\
 & & & \multicolumn{5}{c}{0 \leq x \leq 20,\ y \geq 0} & & & &
\end{array}$$

After implied rewriting:

$$
\begin{array}{lccccccccccccc}
\min\ f(x,y) = & 4 & + & 1.5x & - & 2y & + & \frac{1}{2}(8x^2 & + & 2xy & + & 2yx & + & 10y^2) \\
\text{s.t.} & & & 2x & + & y & \geq & 2 \\
 & & & -x & + & 2y & \leq & 6 \\
 & & & \multicolumn{3}{c}{0 \leq x \leq 20,\ y \geq 0}
\end{array}
$$

Thus the Q matrix is

$$
Q = \begin{bmatrix} 8 & 2 \\ 2 & 10 \end{bmatrix} \tag{1}
$$

for which the lower triangular part (to be given) is:

$$
Q_L = \begin{bmatrix} 8 & \\ 2 & 10 \end{bmatrix} \tag{2}
$$

In other words, the double of the diagonal (square) terms must be entered while the off-diagonal elements must be given as they appear in the original problem formulation.

The MPS format ([4, 7]) divides the input records into fields. They have a fixed position in each record, namely:

Field	Position
Field-1	2–3
Field-2	5–12
Field-3	15–22
Field-4	25–36
Field-5	40–47
Field-6	50–61

The extended MPS format is referred to as QPS format and the file extension is QPS, accordingly. In QPS format the use of fields is the same. For the above example the QPS file is:

```
NAME          QP example
ROWS
 N  obj
 G  r1
 L  r2
COLUMNS
    c1        r1                 2.0   r2                 -1.0
    c1        obj                1.5
    c2        r1                 1.0   r2                  2.0
    c2        obj               -2.0
RHS
    rhs1      obj               -4.0
    rhs1      r1                 2.0   r2                  6.0
BOUNDS
 UP bnd1      c1                20.0
QUADOBJ
    c1        c1                 8.0
    c1        c2                 2.0
    c2        c2                10.0
ENDATA
```

It is important to note that the relative order of columns in QUADOBJ must be the same as in COLUMNS. This means that the structure of QUADOBJ is the same as that of COLUMNS. It is also to be noted that the constant c_0 in the objective function is given as an RHS value. Therefore, if it is different from zero, it must be entered with the opposite sign.

Additionally, we propose that the indicator of the exponent in the data fields be `e` rather than `d` (for instance, `-0.10036986e-4` and not `-0.10036986d-4`). This is to avoid complications experienced when MPS files were read by programs written in C and the exponent indicator was `d`.

3 Set of problems

Presently, we have 138 problems available. Both formulations of QP problems (separable and non-separable) are well represented in the set. In a separate section we acknowledge the contributors. Below is a list of the problems. Here we give:

NAME	the name of the problem
M	the number of rows in A
N	the number of variables
NZ	the number of nonzeros in A
QN	the number of quadratic variables
QNZ	the number of off-diagonal entries in the lower triangular part of Q
OPT	the solution value obtained by the default settings of BPMPD solver [6].

The separable problems are easily recognized by a 0 entry in the QNZ column.

NAME	M	N	NZ	QN	QNZ	OPT
aug2d	10000	20200	40000	19800	0	1.6874118e+06
aug2dc	10000	20200	40000	20200	0	1.8183681e+06
aug2dcqp	10000	20200	40000	20200	0	6.4981348e+06
aug2dqp	10000	20200	40000	19800	0	6.2370121e+06
aug3d	1000	3873	6546	2673	0	5.5406773e+02
aug3dc	1000	3873	6546	3873	0	7.7126244e+02
aug3dcqp	1000	3873	6546	3873	0	9.9336215e+02
aug3dqp	1000	3873	6546	2673	0	6.7523767e+02
boyd1	18	93261	558985	93261	0	-6.1735220e+07
boyd2	186531	93263	423784	2	0	2.1256767e+01
cont-050	2401	2597	12005	2597	0	-4.5638509e+00
cont-100	9801	10197	49005	10197	0	-4.6443979e+00
cont-101	10098	10197	49599	2700	0	1.9552733e-01
cont-200	39601	40397	198005	40397	0	-4.6848759e+00
cont-201	40198	40397	199199	10400	0	1.9248337e-01
cont-300	90298	90597	448799	23100	0	1.9151232e-01

ctd. →

NAME	M	N	NZ	QN	QNZ	OPT
cvxqp1_l	5000	10000	14998	10000	29984	1.0870480e+08
cvxqp1_m	500	1000	1498	1000	2984	1.0875116e+06
cvxqp1_s	50	100	148	100	286	1.1590718e+04
cvxqp2_l	2500	10000	7499	10000	29984	8.1842458e+07
cvxqp2_m	250	1000	749	1000	2984	8.2015543e+05
cvxqp2_s	25	100	74	100	286	8.1209405e+03
cvxqp3_l	7500	10000	22497	10000	29984	1.1571110e+08
cvxqp3_m	750	1000	2247	1000	2984	1.3628287e+06
cvxqp3_s	75	100	222	100	286	1.1943432e+04
dpklo1	77	133	1575	77	0	3.7009622e-01
dtoc3	9998	14999	34993	14997	0	2.3526248e+02
dual1	1	85	85	85	3473	3.5012966e-02
dual2	1	96	96	96	4412	3.3733676e-02
dual3	1	111	111	111	5997	1.3575584e-01
dual4	1	75	75	75	2724	7.4609084e-01
dualc1	215	9	1935	9	36	6.1552508e+03
dualc2	229	7	1603	7	21	3.5513077e+03
dualc5	278	8	2224	8	28	4.2723233e+02
dualc8	503	8	4024	8	28	1.8309359e+04
exdata	3001	3000	7500	1500	1124250	-1.4184343e+02
genhs28	8	10	24	10	9	9.2717369e-01
gouldqp2	349	699	1047	349	348	1.8427534e-04
gouldqp3	349	699	1047	698	697	2.0627840e+00
hs118	17	15	39	15	0	6.6482045e+02
hs21	1	2	2	2	0	-9.9960000e+01
hs268	5	5	25	5	10	5.7310705e-07
hs35	1	3	3	3	2	1.1111111e-01
hs35mod	1	3	3	3	2	2.5000000e-01
						ctd. →

NAME	M	N	NZ	QN	QNZ	OPT
hs51	3	5	7	5	2	8.8817842e-16
hs52	3	5	7	5	2	5.3266476e+00
hs53	3	5	7	5	2	4.0930233e+00
hs76	3	4	10	4	2	-4.6818182e+00
hues-mod	2	10000	19899	10000	0	3.4824690e+07
huestis	2	10000	19899	10000	0	3.4824690e+11
ksip*	1001	20	18411	20	0	5.7579794e-01
laser	1000	1002	3000	1002	3000	2.4096014e+06
liswet1	10000	10002	30000	10002	0	3.6122402e+01
liswet10	10000	10002	30000	10002	0	4.9485785e+01
liswet11	10000	10002	30000	10002	0	4.9523957e+01
liswet12	10000	10002	30000	10002	0	1.7369274e+03
liswet2	10000	10002	30000	10002	0	2.4998076e+01
liswet3	10000	10002	30000	10002	0	2.5001220e+01
liswet4	10000	10002	30000	10002	0	2.5000112e+01
liswet5	10000	10002	30000	10002	0	2.5034253e+01
liswet6	10000	10002	30000	10002	0	2.4995748e+01
liswet7	10000	10002	30000	10002	0	4.9884089e+02
liswet8	10000	10002	30000	10002	0	7.1447006e+03
liswet9	10000	10002	30000	10002	0	1.9632513e+03
lotschd	7	12	54	6	0	2.3984159e+03
mosarqp1	700	2500	3422	2500	45	-9.5287544e+02
mosarqp2	600	900	2930	900	45	-1.5974821e+03
powell20	10000	10000	20000	10000	0	5.2089583e+10
primal1	85	325	5815	324	0	-3.5012965e-02
primal2	96	649	8042	648	0	-3.3733676e-02
primal3	111	745	21547	744	0	-1.3575584e-01
primal4	75	1489	16031	1488	0	-7.4609083e-01
						ctd. →

NAME	M	N	NZ	QN	QNZ	OPT
primalc1	9	230	2070	229	0	-6.1552508e+03
primalc2	7	231	1617	230	0	-3.5513077e+03
primalc5	8	287	2296	286	0	-4.2723233e+02
primalc8	8	520	4160	519	0	-1.8309430e+04
q25fv47	820	1571	10400	446	59053	1.3744448e+07
qadlittl	56	97	383	17	70	4.8031886e+05
qafiro	27	32	83	3	3	-1.5907818e+00
qbandm	305	472	2494	25	16	1.6352342e+04
qbeaconf	173	262	3375	18	9	1.6471206e+05
qbore3d	233	315	1429	28	50	3.1002008e+03
qbrandy	220	249	2148	16	49	2.8375115e+04
qcapri	271	353	1767	56	838	6.6793293e+07
qe226	223	282	2578	67	897	2.1265343e+02
qetamacr	400	688	2409	378	4069	8.6760370e+04
qfffff80	524	854	6227	278	1638	8.7314747e+05
qforplan	161	421	4563	36	546	7.4566315e+09
qgfrdxpn	616	1092	2377	54	108	1.0079059e+11
qgrow15	300	645	5620	38	462	-1.0169364e+08
qgrow22	440	946	8252	65	787	-1.4962895e+08
qgrow7	140	301	2612	30	327	-4.2798714e+07
qisrael	174	142	2269	42	656	2.5347838e+07
qpcblend	74	83	491	83	0	-7.8425409e-03
qpcboei1	351	384	3485	384	0	1.1503914e+07
qpcboei2	166	143	1196	143	0	8.1719623e+06
qpcstair	356	467	3856	467	0	6.2043875e+06
qpilotno	975	2172	13057	94	391	4.7285869e+06
qptest	2	2	4	2	1	4.3718750e+00
qrecipe	91	180	663	20	30	-2.6661600e+02
						ctd. →

NAME	M	N	NZ	QN	QNZ	OPT
qsc205	205	203	551	11	10	-5.8139518e-03
qscagr25	471	500	1554	28	100	2.0173794e+08
qscagr7	129	140	420	8	17	2.6865949e+07
qscfxm1	330	457	2589	56	677	1.6882692e+07
qscfxm2	660	914	5183	74	1057	2.7776162e+07
qscfxm3	990	1371	7777	89	1132	3.0816355e+07
qscorpio	388	358	1426	22	18	1.8805096e+03
qscrs8	490	1169	3182	33	88	9.0456001e+02
qscsd1	77	760	2388	54	691	8.6666667e+00
qscsd6	147	1350	4316	96	1308	5.0808214e+01
qscsd8	397	2750	8584	140	2370	9.4076357e+02
qsctap1	300	480	1692	36	117	1.4158611e+03
qsctap2	1090	1880	6714	141	636	1.7350265e+03
qsctap3	1480	2480	8874	186	861	1.4387547e+03
qseba	515	1028	4352	96	550	8.1481801e+07
qshare1b	117	225	1151	18	21	7.2007832e+05
qshare2b	96	79	694	10	45	1.1703692e+04
qshell	536	1775	3556	405	34385	1.5726368e+12
qship04l	402	2118	6332	14	42	2.4200155e+06
qship04s	402	1458	4352	14	42	2.4249937e+06
qship08l	778	4283	12802	940	34025	2.3760406e+06
qship08s	778	2387	7114	538	11139	2.3857289e+06
qship12l	1151	5427	16170	2023	60205	3.0188766e+06
qship12s	1151	2763	8178	1042	16361	3.0569623e+06
qsierra	1227	2036	7302	122	61	2.3750458e+07
qstair	356	467	3856	66	952	7.9854528e+06
qstandat	359	1075	3031	138	666	6.4118384e+03
s268	5	5	25	5	10	5.7310705e-07
						ctd. →

NAME	M	N	NZ	QN	QNZ	OPT
stadat1	3999	2001	9997	2000	0	-2.8526864e+07
stadat2	3999	2001	9997	2000	0	-3.2626665e+01
stadat3	7999	4001	19997	4000	0	-3.5779453e+01
stcqp1	2052	4097	13338	4097	22506	1.5514356e+05
stcqp2	2052	4097	13338	4097	22506	2.2327313e+04
tame	1	2	2	2	1	0.0000000e+00
ubh1	12000	18009	48000	6003	0	1.1160008e+00
values	1	202	202	202	3620	-1.3966211e+00
yao	2000	2002	6000	2002	0	1.9770426e+02
zecevic2	2	2	4	1	0	-4.1250000e+00

*Problem `ksip` shows considerable numerical sensitivity. The value of the objective functions varies rapidly as solution approaches the feasible domain and the final solution value depends on the feasibility tolerance used. The standard stopping criterion of BPMPD is: If for the primal infeasibility $||Ax - b||/(1.0 + ||b||) < 10^{-8}$ and for the dual infeasibility $||A^T y + z - Qx - c||/(1.0 + ||Qx + c||) < 10^{-8}$ (where z is the vector of dual slack variables) relations hold then the current solution is considered optimal. The reported optimal solution of `ksip` was achieved by a tolerance value of 10^{-11} instead of the default 10^{-8}.

4 Availability

The problems are freely available to the academic community. They can be found on the attached CD ROM in three different compressed (zipped) files, `QPDATA1`, `QPDATA2` and `QPDATA3`. There is also a file, called `00readme.qp`, describing the files. The same set of problems is also available and can be downloaded from `http://www.doc.ic.ac.uk/~im/` by visiting the title A Selection of Data Files and clicking on each of the three zip files. They are organized in the following way. `QPDATA1` contains 76 problems from the CUTE library [1]. 46 problems provided by the Brunel optimization group [3] are in `QPDATA2`, while the other 16 problems from miscellaneous sources are located in `QPDATA3`. Data in some problem files contain exponents. The exponent indicator is always `'e'`, in compliance with our proposal (see section 2). All the files have the .qps extension.

There is another web site holding the same set of problems in a slightly different structure: `ftp://ftp.sztaki.hu/pub/oplab/QPDATA` is an anonymous ftp site with three subdirectories corresponding to the above three zip files. Here, the subdirectories contain individual zip files of the problems. The zipped files can be unzipped by using either `pkunzip`, or the freely available `unzip`. The latter can be downloaded from Info-ZIP: `http://www.cdrom.com/pub/infozip/UnZp.html`.

Important note: The `.qps` files are DOS type text files. Therefore, under UNIX it is advisable to use `unzip -aa ...` command to get the correct UNIX text format of end-of-line.

Both web sites of QPDATA problems are maintained and new problem instances may be added. We keep the library open to further contributions. Anybody with such an intent should contact any of the current authors by Email.

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CUTE [1] library provided by Ingrid Bongartz and Andy Conn (IBM T.J. Watson Research Center), Nick Gould (Rutherford Appleton Laboratory, UK), and Philippe Toint (Facultes Universitaires Notre-Dame de la Paix):

aug2d aug2dc aug2dcqp aug2dqp aug3d aug3dc aug3dcqp aug3dqp cvxqp1_l cvxqp1_m cvxqp1_s cvxqp2_l cvxqp2_m cvxqp2_s cvxqp3_l cvxqp3_m cvxqp3_s dtoc3 dual1 dual2 dual3 dual4 dualc1 dualc2 dualc5 dualc8 genhs28 gouldqp2 gouldqp3 hs118 hs21 hs268 hs35 hs35mod hs51 hs52 hs53 hs76 hues-mod huestis ksip liswet1 liswet10 liswet11 liswet12 liswet2 liswet3 liswet4 liswet5 liswet6 liswet7 liswet8 liswet9 lotschd mosarqp1 mosarqp2 powell20 primal1 primal2 primal3 primal4 primalc1 primalc2 primalc5 primalc8 qpcblend qpcboei1 qpcboei2 qpcstair s268 stcqp1 stcqp2 tame ubh1 yao zecevic2

Helen Jones and Gautam Mitra (Mathematical Programming Group, Brunel University, London) [3]:

q25fv47 qadlittl qafiro qbandm qbeaconf qbore3d qbrandy qcapri qe226 qetamacr qfffff80 qforplan qgfrdxpn qgrow15 qgrow22 qgrow7 qisrael qpilotno qrecipe qsc205 qscagr25 qscagr7 qscfxm1 qscfxm2 qscfxm3 qscorpio qscrs8 qscsd1 qscsd6 qscsd8 qsctap1 qsctap2 qsctap3 qseba qshare1b qshare2b qshell qship04l qship04s qship08l qship08s qship12l qship12s qsierra qstair qstandat

Piet Groeneboom (University of Washington, USA):

stadat1 stadat2 stadat3

Hans D. Mittelmann (Arizona State University, USA):

cont-050 cont-100 cont-101 cont-200 cont-201 cont-300

Athanassia Chalimourda (Ruhr University, Bochum, Germany):

exdata values

Don Boyd (Rensselaer Polytechnic Institute):

boyd1 boyd2

James McNames (Stanford University, USA):

laser

Henry Wolkowitz (University of Waterloo, Canada):

dpklo1

We also included the small QP example shown in section 2 in a separate file

qptest

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