Universal parameter optimisation in games: SPSA and related algorithms

Csaba Szepesvári

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Games Group Presentation
U. Alberta, 2005

Joint work with Levente Kocsis and Mark H.M. Winands

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Motivation

- Game programs have many parameters
- Hand tuning of parameters is difficult
- Automatic tuning – stochastic search
  - No gradient: Global search (GA, Simulated Annealing)
  - Gradient: Stochastic Gradient Ascent (SGA)
    - Gradient estimation – how?
- Example domains:
  - Omaha Hi-Lo Poker
  - Lines of Actions, MIA++

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Formalism

- Optimization problem:

$$f(\theta) = \mathbb{E}[R(\theta)] \rightarrow \max, \quad \operatorname{argmax}_\theta f(\theta) = ?$$

- Simulation Optimization[1]
- 1950s–present (Metropolis, Rosenbluth, Teller, Robbins, Monro, ..)

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Simulation Optimization

"Simulation optimization can be defined as the process of finding the best input variable values from among all possibilities without explicitly evaluating each possibility. The objective of simulation optimization is to minimize the resources spent while maximizing the information obtained in a simulation experiment."[a]

[a] Yolanda Carson & Anu Maria: "Simulation Optimization: Methods and Applications", Winter-1997

Methods

Methods:

- Gradient-based Method:
  - Infinitesimal Perturbation Analysis (IPA)
  - Likelihood Ratio Method (LR)[2]
  - IPA/LR combined
  - Sample path optimization
- Response Surface Methodology
- Heuristic Methods: GA, ES, SA, Tabu-Search, ...

[2] Also: Score function method

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Connection to Reinforcement Learning (RL)

- Policy-gradient
  - LR-methods
- PEGASUS[3]
  - Sample-path optimization
- TD-methods:
  - No counterpart

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Analytic expression for the gradient available?

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- IPA
- LR
- Sample-path optimization

NO

Finite Difference Methods

- FDSA[a]
- SPSA[b]
- Higher order methods
- One-measurement SPSA[c]

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Why gradient-free?

PRO

- Too expensive to maintain analytic gradient (change-mgmt)
- Analytic gradient computationally intractable

  e.g. features computed using Monte-Carlo
- Easy to implement
- Efficient???

CONTRA

- Inefficient compared to analytic-methods?

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Stochastic Gradient Ascent (SGA)

SGA

Objective:

$$f(\theta) = \mathbb{E}[f(\theta; Y)] \to \max$$

Procedure:

$$\theta_{t+1} = \theta_t + \alpha_t \hat{g}_t(\theta_t)$$

Assumptions:

- Step-size: $\sum_t \alpha_t = +\infty$, $\sum_t \alpha_t^2 < +\infty$
- Gradient-estimate: $\mathbb{E}[\hat{g}_t(\theta_t) \mid \mathcal{F}_t] = \frac{\partial}{\partial \theta} f(\theta_t; Y_t)$.
- Conditional variance: $\mathrm{Var}[\hat{g}_t(\theta_t) \mid \mathcal{F}_t] < K^2(1 + \|\theta_t\|^2)$.

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$$Y \sim p(\cdot),$$

p is not dependent on θ

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Likelihood-Ratio Method – I.

Assumption

$$Y \sim p_\theta(\cdot),$$

p is dependent on θ, but

$$f(\theta; Y) = f(Y),$$

i.e., f is independent of θ.

Likelihood-ratio Method – II.

- Gradient estimate:

$$\begin{aligned} f'(\theta) &= \frac{d}{d\theta}\int f(y)p_\theta(y)dy \\ &= \int f(y)\frac{d}{d\theta}p_\theta(y)dy \\ &= \int f(y)\frac{\frac{d}{d\theta}p_\theta(y)}{p_\theta(y)}p_\theta(y)dy \\ &= \int f(y)\frac{d}{d\theta}\ln p_\theta(y)p_\theta(y)dy, \end{aligned}$$

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Variance of LR estimate

Assumption: $Y = (Y_0, Y_1, \ldots, Y_T)$ is a long trajectory.

$$\begin{aligned}\hat{g}(\theta) &= f(Y)\frac{d}{d\theta}\ln p_\theta(Y) \\ &= f(Y)\sum_{t=0}^{T}\frac{d}{d\theta}\ln p_\theta(Y_t|Y_{0:t-1})\end{aligned}$$

$\Rightarrow$ Variance can be high if T is big

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FDSA

FDSA Gradient Estimate

$$\hat{g}_{ti}(\theta_t) = \frac{f(\theta_t + c_t e_i; Y_{ti}^+) - f(\theta_t - c_t e_i; Y_{ti}^-)}{2c_t}$$

2d evaluations of f

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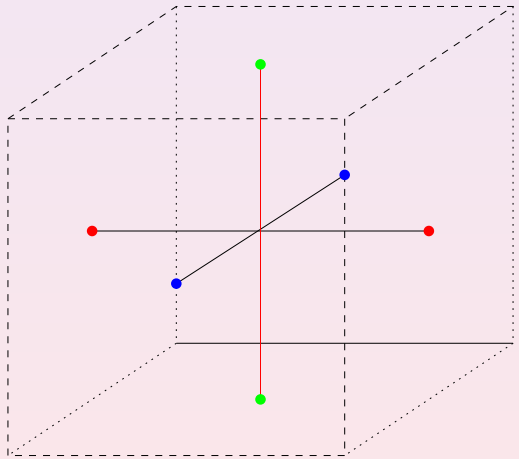

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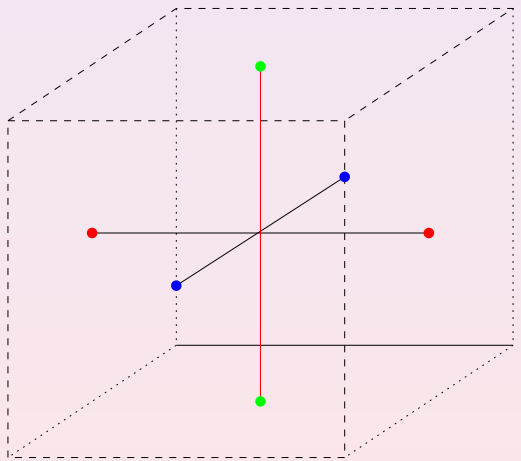

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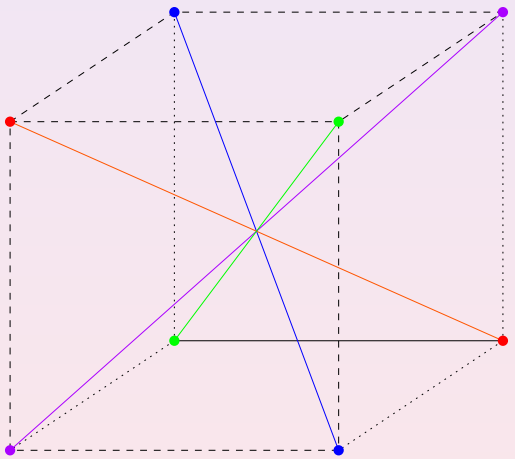

Only 2 evaluations of f

Variants: Discrete-parameter spaces, Non-smooth optimization via noise injection, higher-order methods

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SPSA Gradient Estimate

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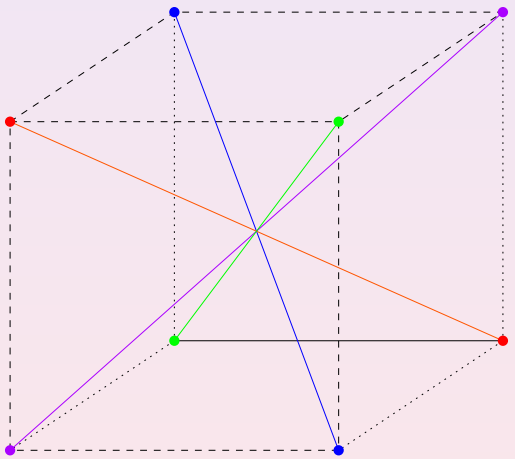

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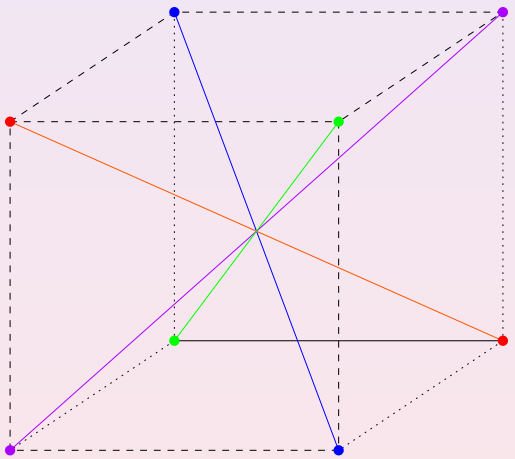

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SPSA Convergence

(A1) Δ_t are independent of the past
(A2) $\{\Delta_{ti}\}_i$ is i.i.d.
(A3) Distribution of Δ_{ti} is symmetric around zero
(A4) $|\Delta_{ti}|$ is bounded w.p.1
(A5) $\mathbb{E}[\Delta_{ti}^{-1}]$ is finite

KW step-size conditions:

- $\alpha_t, c_t > 0$, $\lim_{t\to\infty} c_t = 0$
- $\sum_{t=0}^{\infty} \alpha_t = \infty$ and $\sum_{t=0}^{\infty} \alpha_t^2 / c_t^2 < \infty$

Theorem (Spall, 1992)

If f is sufficiently smooth, the step-sizes satisfy the KW conditions then θ_t converges w.p.1 to a local optimum of f. The bias of the estimate of the gradient is of order $O(c_t^2)$.

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Efficiency

- Asymptotic Performance: SPSA is d-times more efficient than FDSA[4]
- Transient Performance: FDSA is more efficient than SPSA[5]
- Robbins-Monro: $O(t^{-1/2})$
- SPSA and FDSA: $O(t^{-1/3})$

  Optimal rate for randomized KW: $O(t^{-(p-1)/(2p)})$[6]

  $p = 3 \Rightarrow$ optimal rate

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Outline

Common Random Numbers – I.

$$\hat{g}_{ti}(\theta_t) = \frac{f(\theta_t + c_t\Delta_t; Y_t^+) - f(\theta_t - c_t\Delta_t; Y_t^-)}{2c_t\Delta_{ti}}.$$

$$\Delta F_i = F_i^+ - F_i^-$$

$$\mathrm{Var}\left[F_i^+ - F_i^-\right] = \mathrm{Var}\left[F_i^+\right] + \mathrm{Var}\left[F_i^-\right] - 2\mathrm{Cov}\left(F_i^+, F_i^-\right),$$

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$$f(\theta; Y) = \theta Y$$

$$\frac{f(\theta+c\Delta;Y)-f(\theta-c\Delta;Y)}{2c\Delta} = Y$$

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LR or SPSA? – Example[7]

- LR-estimate
  - $Y = Y_\theta \sim \exp(- \cdot /\theta)/\theta$
  - Objective: $f(\theta) = E[Y_\theta]$
  - $f(\theta) = \int y p_\theta(y) dy$.
  - LR-estimate:

$$Y \frac{\partial}{\partial \theta} \ln p_\theta(Y) = \frac{Y}{\theta} \left(\frac{Y}{\theta} - 1 \right).$$

- SPSA-estimate
  - $U \sim \mathcal{U}_{[0,1]}$
  - $Y = \theta \log U \sim p_\theta(\cdot)$
  - Define: $f(\theta; U) = -\theta \log U$.
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$$\frac{(\theta + c\Delta)(-\log U) - (\theta - c\Delta)(-\log U)}{2c\Delta} = -\log U.$$

LR or SPSA? – Example[7]

- LR-estimate
  - $Y = Y_\theta \sim \exp(- \cdot /\theta)/\theta$
  - Objective: $f(\theta) = E[Y_\theta]$
  - $f(\theta) = \int y p_\theta(y) dy$.
  - LR-estimate:

$$Y \frac{\partial}{\partial \theta} \ln p_\theta(Y) = \frac{Y}{\theta} \left(\frac{Y}{\theta} - 1 \right).$$

- SPSA-estimate
  - $U \sim \mathcal{U}_{[0,1]}$
  - $Y = \theta \log U \sim p_\theta(\cdot)$
  - Define: $f(\theta; U) = -\theta \log U$.
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Application to Games

State-space view:

$$\begin{aligned} X_{t+1} &= f(X_t, A_t, U_t) \\ A_t &= \pi(X_t, U'_t) \\ R_t &= R(X_t, A_t) \\ & \mathbb{E}[\sum_{t=0}^{\infty} R_t] \to \max \end{aligned}$$

Application to Games

- Use the same sequence of random variables U_t, U'_t for both evaluations:

$$Y = (U_1, \ldots, U_T, U'_1, \ldots, U'_T)$$

(fix an infinite random sample in advance)

- Easy impl: Manipulate the seed of pseudorandom-generators
- $f = f(\theta; Y)$ – can be complicated!
- ..but the distribution of Y is independent of θ
- $f(\theta; Y)$ might be non-differentiable w.r.t. θ, even when $f(\theta) = \mathbb{E}[f(\theta; Y)]$ is differentiable!

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Outline

Batch-size

$$\hat{g}_{q,i}(\theta) = \frac{1}{q}\sum_{j=1}^{q}\frac{f(\theta + c\Delta; Y_j) - f(\theta - c\Delta; Y_j)}{2c\,\Delta_i}.$$

$$\hat{g}_{q,r,i} = \frac{1}{r}\sum_{k=1}^{r}\hat{g}_{q,i}^{(k)}(\theta)$$

How to select q, r s.t. $q \times r \equiv p$?

Use as many independent samples as possible:

$$q = 1, \quad r = p$$

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RPROP – I.

Problem

How to select α_t, c_t?

- Idea: Use different values for each of the d components of θ
- SGA step-size methods
- RPROP – one of the best performing first-order batch neural network method
- Combine with RPROP

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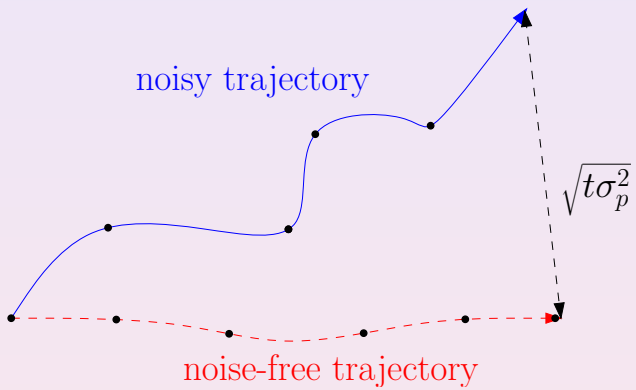
Resiliant backPROPagation (RPROP) – II.

- Individual learning rates δ_{ti}
- Sign-based updates
- $\theta_{t+1,i} = \theta_{t,i} + \delta_{ti}\text{sign}(g_{ti})$
- δ_{ti} increased if $g_{t-1,i}$ and g_{ti} have the same sign, decreased otherwise

RPROP – Update Equations

$$
\begin{aligned}
p_{t,i} &= g_{t-1,i} f_i'(\theta_t), \\
g_{ti} &= \mathbb{I}(p_{t,i} \geq 0) f_i'(\theta_t), \\
\eta_{ti} &= \mathbb{I}(p_{t,i} > 0)\eta^+ + \mathbb{I}(p_{t,i} < 0)\eta^- + \mathbb{I}(p_{t,i} = 0), \\
\delta_{ti} &= P_{[\delta^-,\delta^+]}\left(\eta_{ti}\delta_{t-1,i}\right)
\end{aligned}
$$

Noise suppression

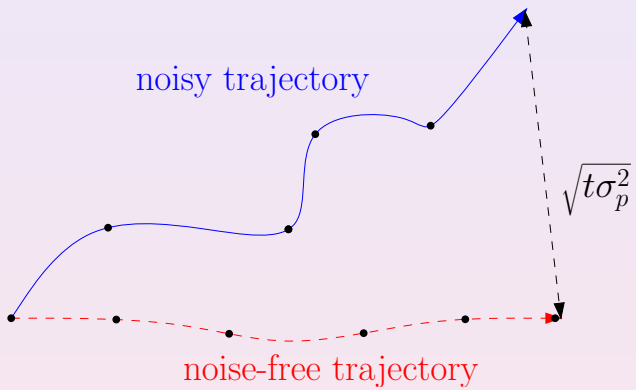


$$\delta_p \sim \exp(-pG^2/(2\sigma_1^2))$$

$$\sigma_p^2 = \frac{1}{p}\sigma_1^2$$

Note: RPROP's equilibrium behaviour is not "ideal"

Noise suppression



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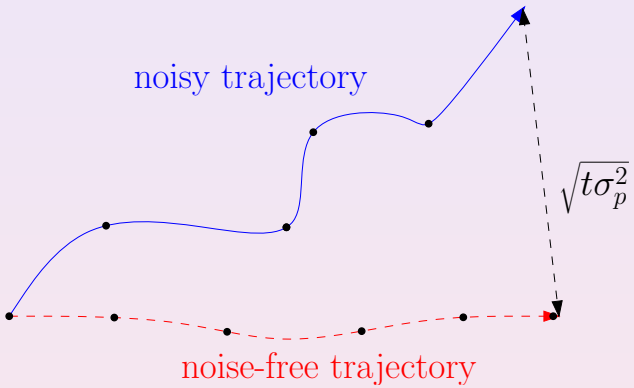

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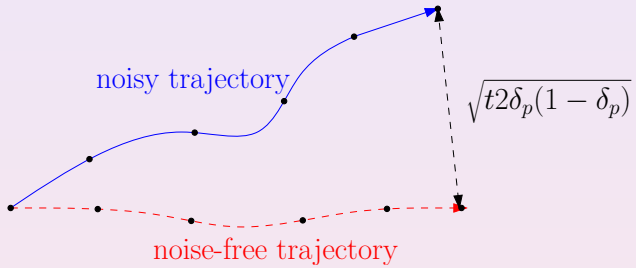

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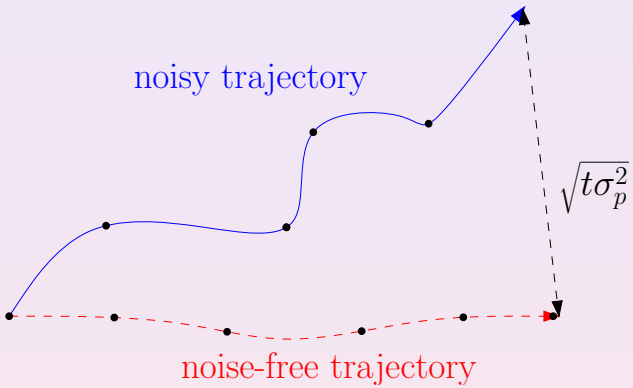

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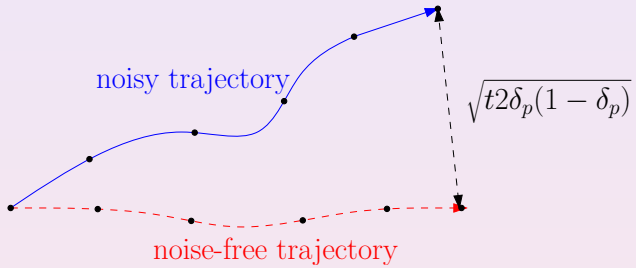

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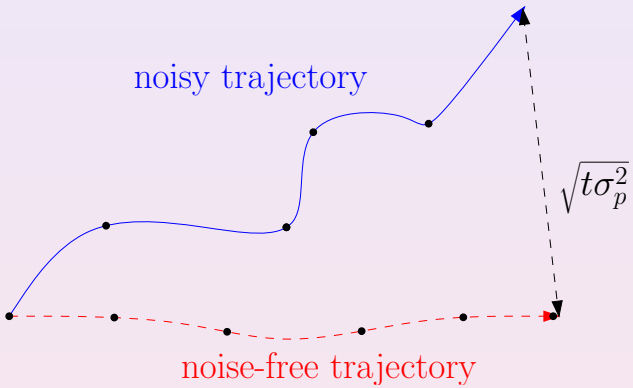

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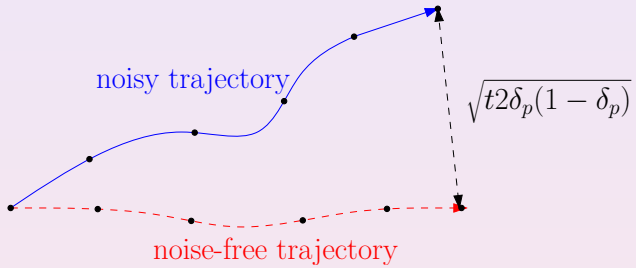

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The RSPSA algorithm

Idea: Combine SPSA and RPROP – couple step-sizes

for (*iteration*)
  $d\theta = 0$
  for (*batch*)
    for (*i*) $p_i = \rho \cdot \delta_i \cdot randb(\pm 1)$
    generate(Y_t)
    $f^+ = \text{eval}(\theta + p, Y_t)$
    $f^- = \text{eval}(\theta - p, Y_t)$
    for (*i*) $d\theta_i$ += $(f^+ - f^-)/(2p_i)$
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Outline

Antithetic Variates

- Problem: Estimate $I = \mathbb{E}[R]$.
- Monte-Carlo:

$$I_n = \frac{1}{n}(R_1 + \ldots + R_n)$$

- Variance is $(1/n)\,\mathrm{Var}\,[R]$
- Assume $n = 2k$, consider

$$I_n^A = \frac{1}{k}\sum_{i=1}^{k} \frac{R_i^+ + R_i^-}{2},$$

- Variance of a pair:

$$\mathrm{Var}\left[\frac{R_i^+ + R_i^-}{2}\right] = \frac{\mathrm{Var}\left[R_i^+\right] + \mathrm{Var}\left[R_i^-\right] + 2\mathrm{Cov}\left(R_i^+, R_i^-\right)}{4},$$

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Antithetic Variates – Example

- $f = f(\theta; Y, W)$
- Y – external choices, W – internal choices of players
- Example (Y) in poker: Cards
- "Antithetic dealing":
  - Y: Original random deal
  - Y': Reverse the hands of the players
- Distributions of $f(Y; W_1) - f(Y'; W_2)$ are aligned:

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Outline

Omeha Hi-Lo: The game

- small blind (5$)/big blind (10$)
- each player (2-10) gets 4 hole cards
- betting (1st round)
- flop: 3 community cards
- betting (2nd round)
- turn: 1 community card
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Betting

- 3 actions: fold, check/call, bet/raise
- raise: +1$ in 1st and 2nd round, +2$ in 3rd and 4th round
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Action-selection

- s – information state of player
- a – action (fold, call, raise)
- $Q(s, a) = \mathbb{E}[w(C, I)R - R_1 \mid s, a]$
  - C – random card configuration
  - I – set of active players at showdown
  - R – final size of the pot
  - R_1 – McRaise's final contribution
- $\pi^*(s) = \text{argmax}_a Q(s, a)$

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- EC $\equiv$ "Everyone's Checking"
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$$p(c|s) = \frac{p(s|c)p(c)}{p(s)}$$

How to compute $p(c|s)$? W.I.S.

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$$Y_i \sim q(\cdot)$$

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Outline

Tuning the opponent model

- 2-players, one is McRaise, other is “tuned”
- Tuning 6 parameters of the opponent model
- Compare non-asymptotic performance of RSPSA and (R)FDSA
- Initialization: original parameter settings
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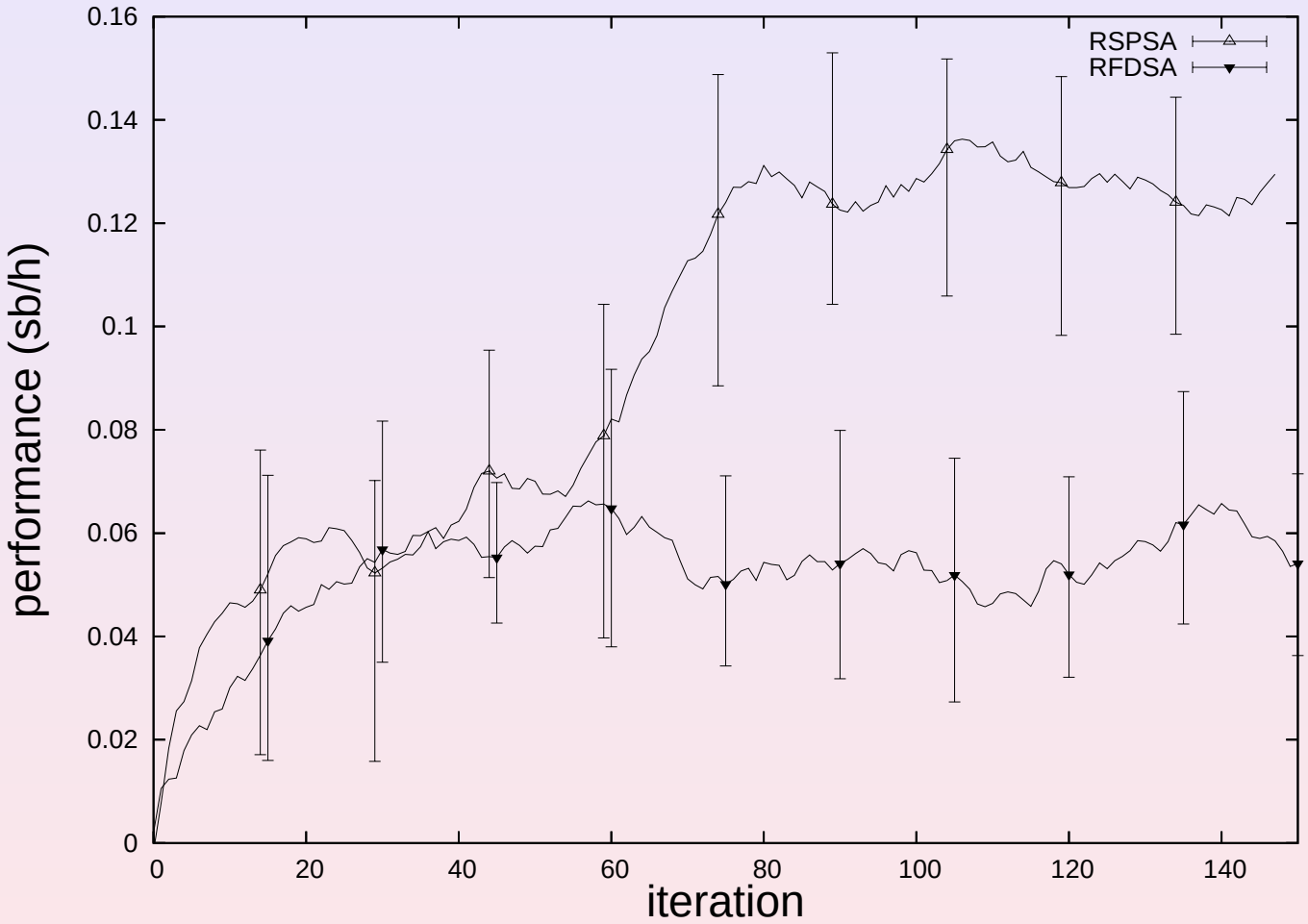
RSPSA
RFDSA
performance (sb/h)
iteration
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0
20
40
60
80
100
120
140

Learning policies and evaluation functions

- RSPSA
  - neural network representing after-state value-function
  - Action-network representing $p(a|s)$
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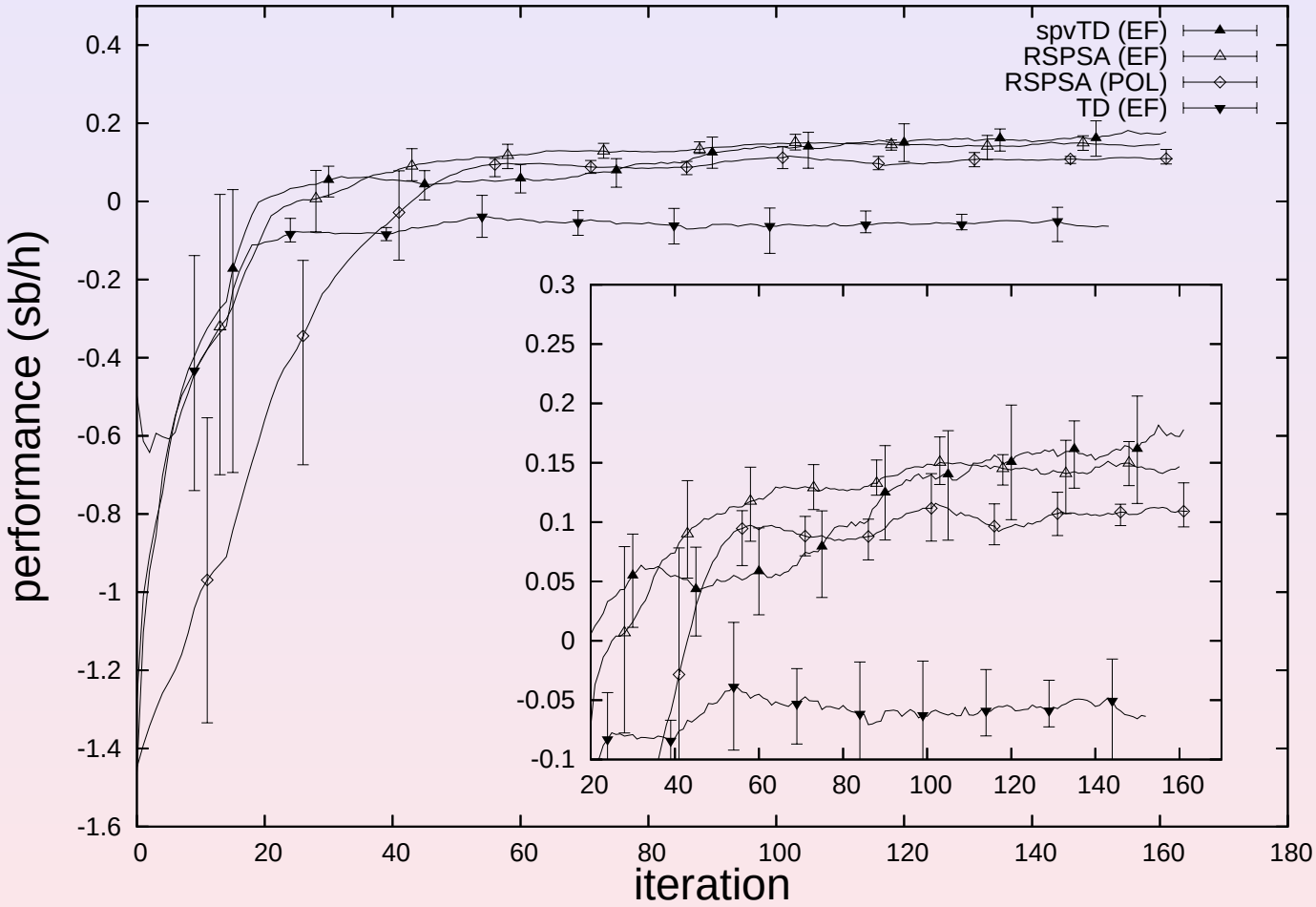
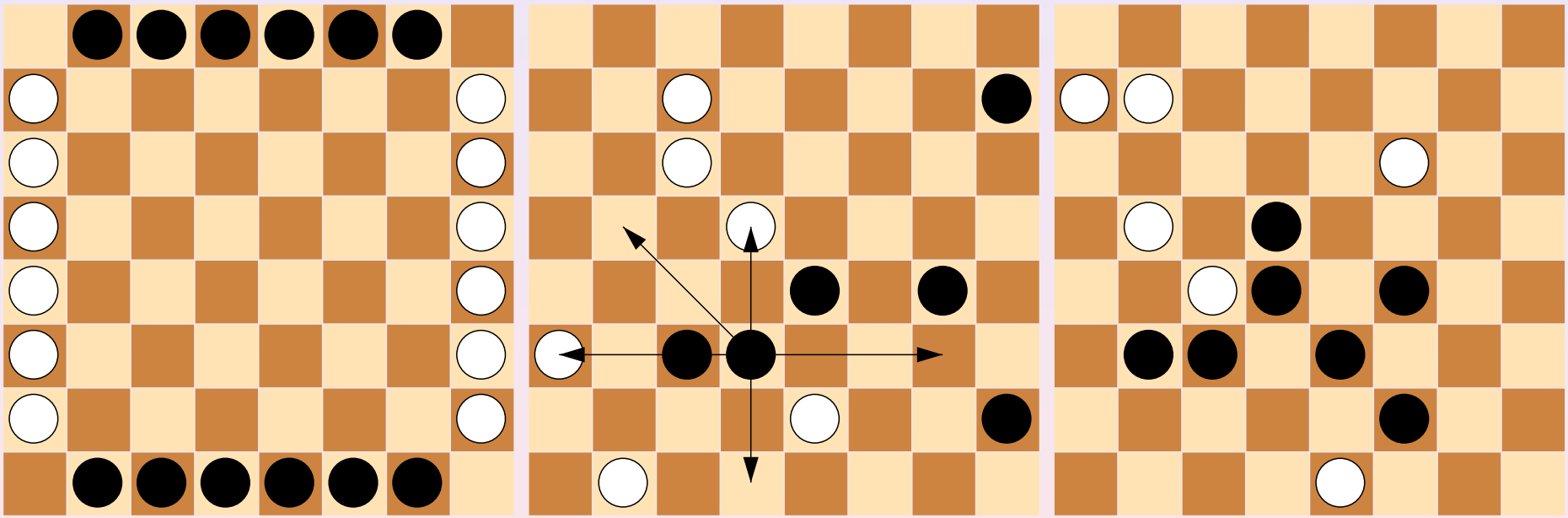

spvTD (EF)
RSPSA (EF)
RSPSA (POL)
TD (EF)
performance (sb/h)
iteration
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0
20
40
60
80
100
120
140
160
180
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
20
40
60
80
100
120
140
160

Outline

Lines of Actions



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- Standard search-based program
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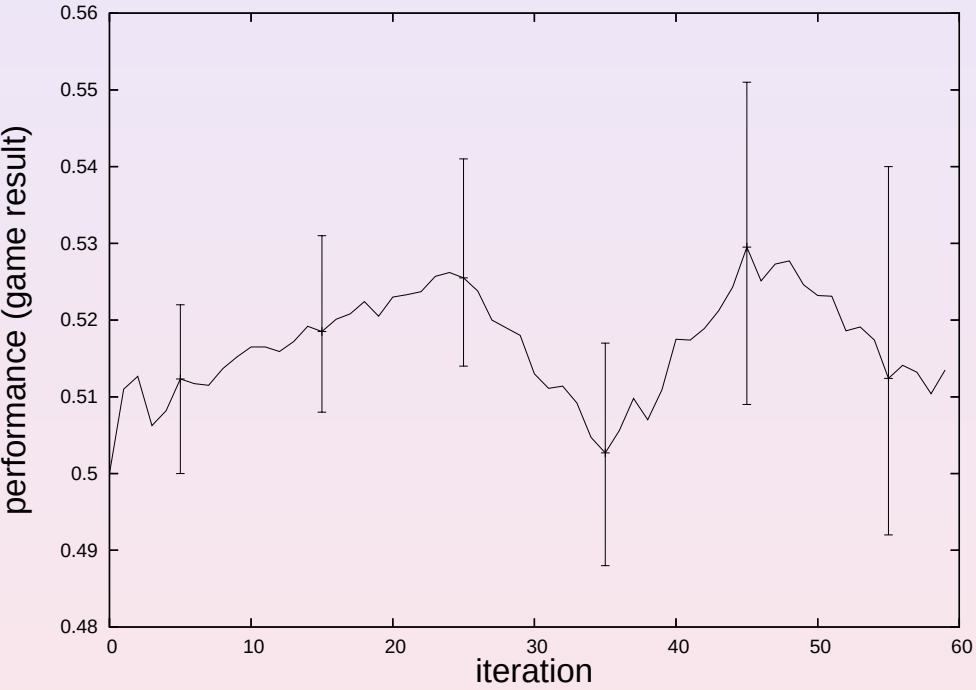
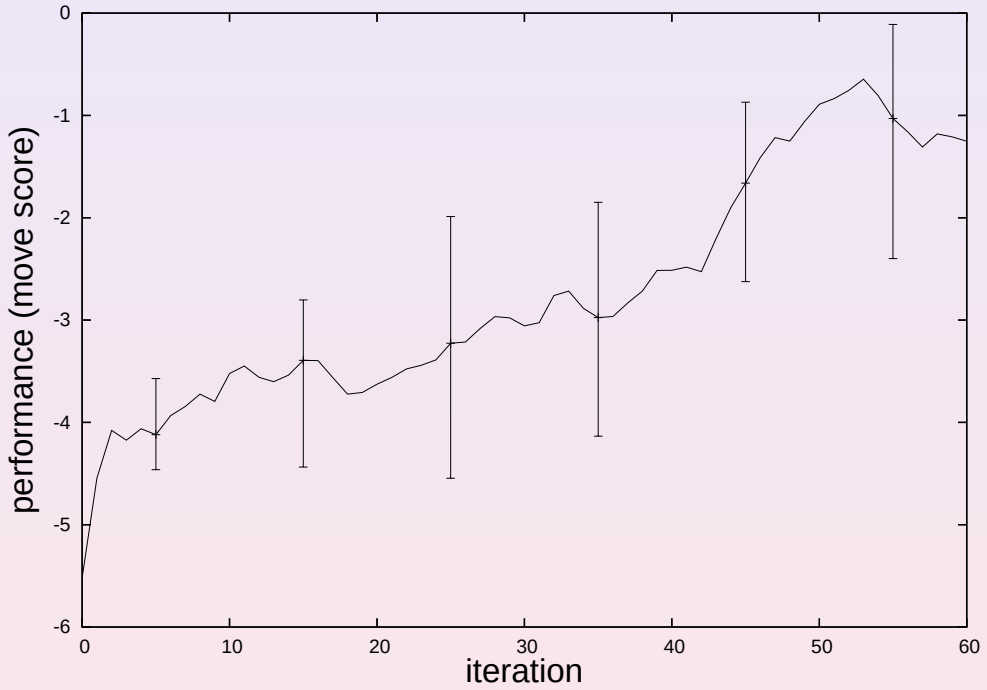
game results



move scores



Conclusions

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