Channel coordination with the newsvendor model using asymmetric information

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Abstract

In this paper, after surveying short-term two-echelon supply channel coordination methods, we present an extended version of the newsvendor model in which the supplier has to fulfil all demand of the customer, even if this calls for an additional setup of production. Given uncertain demand forecast, the solution is an optimal production quantity that minimises the expected total cost including setup, inventory holding and obsolete inventory costs. Then, the model is studied in a decentralised setting where the customer has private information about the demand forecast, while the supplier knows the various cost factors. We suggest such a coordination protocol and payment scheme that provides both partners the right incentive for minimising the total cost: the customer is interested in sharing her unbiased demand forecast and uncertainty, while the supplier's rational decision concurs with the overall optimum. Hence, local decisions based on asymmetric information co-

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ordinate the channel in the global sense. The results are also demonstrated by taking some real-life test cases from an industrial study that motivated our work.

Keywords: Newsvendor, channel coordination, asymmetric information, service level, compensation

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1. Introduction

Because of today's continuously changing market conditions, manufacturing enterprises are facing much difficult challenges than before. In spite of the still existing uncertainties of the environment (such as demand fluctuation, resource failures, scrap production, procurement delays) customer expectations are persistently growing. Now, customers seldom accept shortages or backlogs and, in addition, they often want to customise the product characteristics themselves. In the last decades, tighter cooperation between the enterprises along the supply chains appeared to be necessary to respond to this situation. Several recent practical initiatives have taken this approach, like the vendor managed inventory (VMI) or the collaborative planning, forecasting and replenishment (CPFR) programme, to name a few examples (Choi and Sethi, 2010).

One of the most subtle challenges of production is still the appropriate management of inventory. In the last decades of the 20th century, the Just-In-Time (JIT) production paradigm became very popular, since it promised the elimination of inventories, which were considered passive elements of the business creating only expenses and no value (Chikán, 2007). However, this "zero inventory" concept could rarely be realised in practice under the special conditions of JIT production (unvarying demand, negligible setup cost/time, and so forth). In general, inventories are necessary in order to exploit economies of scale or to hedge against various uncertainties. Due to unforeseen changes of demand, stocks of products with short life-cycles may easily become obsolete, which causes not only significant financial losses for the enterprises, but also serious waste of material, labour, energy and environmental resources.

In supply chains, where the decisions are decentralised, the inventory management is even more problematic (Tang, 2006). As previous studies have shown (see Section 2.3), the resultant of the locally optimal decisions usually leads to suboptimal performance, since the objectives of the autonomous decision makers are not aligned with any global objective. This is essentially a distributed planning problem: supply chain members would like to exercise control over some future events based on information what they know at the moment for certain (about products, technologies, resource capabilities, sales histories) and only anticipate (demand, resource and material availability). Hence, the supply chain partners need to collaborate and to take into account some of the other's decisions. However, the issues of resolving conflicts between individual interests as well as of acting for a common goal are far from being resolved (Arshinder et al., 2008).

The theory of *contracting* aims at developing arrangements for aligning the different objectives. Contracts are protocols that control the flows of information, materials (or services) and financial means alike. According to Li and Wang (2007), a contracting scheme should consist of the following components:

- local planning methods which consider the constraints and objectives of the individual partners,
- an infrastructure and protocol for information sharing (see also Váncza et al., 2010), and
- an incentive scheme for aligning the individual interests of the partners.

A contract is said to achieve *channel coordination* if thereby the partners' optimal local decisions lead to optimal system-wide performance. Note that there also exists a weaker definition of coordination aiming only at improving global performance compared to the default baseline solution of decomposed planning (Albrecht, 2010); in this paper however, we regard the former, strong notation.

The motivation of our work comes from an industrial research and development project, which involves a production network with a focal end-product manufacturer and several suppliers. The network produces both standardised and customised consumer goods in a large variety. Customers of the end-products tend to be impatient: the acceptable delivery times are usually much shorter than the actual throughput times. Hence, production of even customised products must be based on demand forecasts, which, in turn, are just due to the nature of the market highly unreliable. The common goal of each network partner is to provide high service level towards the customers of end-products, while, at the same time, keeping production and logistics costs as low as possible. These requirements are conflicting: high service level can only be guaranteed by inventories of components, packaging materials, and end-products. Furthermore, in mass production technology low costs can be achieved only with few setups and large lot sizes. In contrast, the market of customized mass products is volatile: if the demand unexpectedly decreases or ceases, typically due to managerial decisions, then accumulated inventories become obsolete.

The remainder of the paper is organised as follows. In Section 2, we review the most recent results in supply chain coordination theory. We introduce a new model and its analytic solution in Section 3, which guarantees to satisfy all demand with minimal cost. In Section 4, we extend our model to a decentralised two-echelon supply chain with *asymmetric information*, and present a payment scheme with double compensation that can coordinate such a supply chain. Finally, in Section 5, we demonstrate the proposed ideas on some industrial test cases.

2. Literature review

2.1. Classification of channel coordination methods

The general method for studying coordination mechanisms consists of two steps. At first, one assumes a central decision maker with complete information who solves the problem. The result is a so-called *first-best solution* which provides a bound on the obtainable system-wide performance objective. In the second step one regards the *decentralised problem* and designs such a contract protocol that approaches or even achieves the performance of the first-best solution.

An early review of supply chain contracts can be found in Tsay et al. (1999). In this paper supply chain management is defined as the extension of the classic multi-echelon inventory theory with the ideas of decentralisation (multiple decision makers), asymmetric information and new manufacturing and logistic paradigms, such as delayed differentiation and outsourcing. The study also provides a taxonomy for classifying contracts, which consists of eight different contract types. The authors pointed out however, that these classes are not disjoint. Therefore we present a set of aspects, which generalise their taxonomy by allowing classification along multiple viewpoints; then we review the more recent related papers according to this extended classification. The different viewpoints can be classified as follows:

Horizon. Most of the related models consider either *one-period horizon* or *two-period horizon with forecast update.* In the latter, the production can be based on the preliminary forecast with normal production mode or on the updated forecast with emergency production, which means shorter lead-time, but higher cost. These latter models are extensively discussed in Sethi et al. (2005). In addition, the horizon can consist of *multiple periods* and it can be even *infinite.*

Number of products. Almost all models regard only one product. Handling more products in gross is rational in case of technological or financial constraints, like capacity or budget limits.

Demand characteristic. Generally, the demand is considered *stochastic*, although some models assume *deterministic* demand.

Risk treatment. Focus is usually set on models where the players are *risk neutral.* This means that they intend to maximise their expected profit (or minimise their expected costs). However, some studies regard *risk averse* players who also consider risk measures, e.g., standard deviation

(Choi and Chiu, 2010), value-at-risk (Özler et al., 2009) or conditional value-at-risk (Wu et al., 2010).

Shortage treatment. The models differ in their attitude towards stockouts. Most authors consider either *backordering*, when the demand must be fulfilled later at the expense of providing lower price or *lost sales* which also include some theoretical costs (e.g., loss of goodwill, loss of profit, etc.). Some models include a *service level constraint*, which limits the occurrence or quantity of expected stockouts. Even the *100% service level* can be achieved with additional or emergency production (e.g., overtime, outsourcing) for higher costs.

Parameters and variables. This viewpoint shows the largest variations in different models. The main decision variable is quantity-related (*production quantity*, *order quantity*, *number of options*, etc.), but sometimes *prices* are also decision variables. The parameters can be either constant or stochastic. The most common parameters are related to costs: *fixed* (*ordering* or *setup*), *production* and *inventory holding* and *backorder cost*. These are optional; many models disregard fixed or inventory holding costs. There exist numerous other parameters: prices for the different contracts, salvage value, shortage penalty, lead time, etc.

Basic model. Most of the one-period models apply the *newsvendor model*. On a two-period horizon, this is extended with the possibility of two production modes. On multiple period horizon the *base-stock*, or in case of deterministic demand the *EOQ* models are the most widespread.

Technological constraints. Generally, technological constraints are completely disregarded in the coordination literature. However, in real industrial cases *resource capacity* or *budget constraints* can be relevant.

Solution technique. In the basic models—and most papers study these—the optimum of the objective function can be determined with simple algebraic operations (e.g., Grubbström and Erdem, 1999; Cárdenas-Barrón, 2001). However, in case of more complex models and further constraints, more powerful solution techniques may be required, like mathematical programming, dynamic programming, constraint programming, and, in the last resort, heuristics or metaheuristics (e.g., Hop and Tabucanon, 2005; Cárdenas-Barrón, 2010).

Number of players. We focus on the *two-player* case and call the players *supplier* and *customer*. There are also extensions of this simple model: the *multiple customers with correlated demand* and the *multiple suppliers with different production parameters*. Multi-echelon extensions are also conceivable, however sparse in the literature.

Information structure. Some papers study the *symmetric information* case, when all of the players know exactly the same parameters. This approach is very convenient for cost sharing, since all players know the incurring system cost. The *asymmetric* case, when there is an *information gap* between the players is more realistic, but poses new challenges. The asymmetry typically concerns either the cost parameters or the demand forecast. For the sake of simplicity, the demand and the forecast are usually considered to be qualitative, limited to only two possible values: high and low.

Decision structure. The decision making roles of the players depend on the specified decision variables. However, there is a more-or-less general classification in this aspect: forced and voluntary compliance. Under *forced compliance* the supplier is responsible for satisfying all orders of the customer, therefore he[1] does not have the opportunity to decide about the production quantity. Under *voluntary compliance*, the supplier decides about the production quantity and he cannot be forced to fill an order. This latter is more complex analytically, but we agree with the conclusion of Cachon and Lariviere (2001): "[...] forced compliance violates the original premise for studying supply chain contracting: that no one firm controls all supply chain actions. [...] Firm commitments are undesirable because they restrict the system's ability to respond to evolving information." Even so, several papers assume that the supplier decides about the price and then the customer decides the order quantity.

Game theoretic model. From this point of view the models can take *cooperative* or *non-cooperative* approaches. The cooperative approach in most cases studies how the players form coalitions and share the profit. Other typical form of cooperative games involves some *bargaining* framework, e.g., the Nash bargaining model. The non-cooperative approaches usually apply the sequential *Stackelberg game* model, where

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[1] According to the widespread notation in the literature, we refer customer as *she* and the supplier as *he*.

one of the players, the *leader* moves first and then the *follower* reacts. Both cases—the supplier or the customer as the Stackelberg leader—are widely studied in the literature. In case of information asymmetry, a similar sequential model is used and it is called *principal-agent* setting. The study of the long-term supply relationship modelled as a *repeated game* is a promising new research field (Ren et al., 2010).

Contract type. This aspect also provides many possibilities. There are some commonly used contracts, like the *quantity discounts*, *buyback/return policies*, *quantity flexibility*, *revenue sharing*, etc. There is also a generalised *option contract* model, which contains some previously mentioned contracts as special cases. Besides, there exist several combinations and customised approaches, too.

2.2. Theoretical background

The supply chain coordination problems are generally studied by the *economy with asymmetric information*, a discipline spawned from game theory (Salanié, 2005). Following the traditional nomenclature, when the information asymmetry affects a decision variable, it raises a *moral hazard problem* and when it affects an external parameter, we call it *adverse selection problem*.

The main model here is called the *principal-agent model*, where the decision is sequential. When the player with the incomplete information is the leader, it is called *screening model*. In this case, the aim of the leader is to design such a *menu of contracts*, from which the follower's rational choice is optimal for the leader. On the other hand, when the well-informed player is the leader, it is a *signalling model*. Such models are used, when the leader can offer a contract that guarantees the truthful revelation of its private information. This should be in the interest of the leader, because without truthful revelation the adverse selection may cause *market failure* (i.e., no deal at all) which is suboptimal for both players.

The generalisation of screening and signalling models is the terrain of *mechanism design* (or *inverse game theory*). Here the main goal is that given a system-wide optimal strategy tuple, one must design such a game where the given tuple is an equilibrium.

2.3. Literature review

Cachon (2003) gives an extensive discussion of the coordination contracts, mostly focusing on the newsvendor model, considering several extensions of the basic problem. Another, more general and recent survey can be found in Li and Wang (2007). In the following we overview some related works considering asymmetric information structure, and point out the similarities and differences with our model.

Cachon and Lariviere (2001) study the two-period case of the newsvendor model, when the information asymmetry affects the demand forecast, which can be either *high* or *low*. The customer who is better informed, signals the expected demand and the supplier must reserve capacity. Here, the customer has obviously an incentive to inflate its forecast. Option contracts are examined under both forced and voluntary compliances, and although the former one is found to be more efficient in this case, it is not preferred due to its centralisation of decisions.

Çınar and Bilgiç (2005) study within the newsvendor framework the effect of asymmetric information on the inventory handling cost of the customer. The supplier is the leader, who offers a menu of firm order and option contracts. They assume forced compliance, show the existence (but not the uniqueness) of the equilibrium and derive the conditions for channel coordination.

Váncza et al. (2008) consider the problem where the demand forecast is given on a multi-period horizon, but it is only known by the customer. Since the supplier is responsible for the lot-sizing decision, incentive for the truthful information sharing is necessary for coordination. The market uncertainty is proposed to be handled with rolling horizon planning, which makes the model more realistic.

Li et al. (2009) model the case when the demand forecast is known only by the customer, but the supplier is the leader. The supplier offers a menu of contracts consisting of firm orders, options and combined contracts; the authors identify cases when the latter type is dominant. One further speciality of this model is that the price of the end product is stochastic.

Lee and Jeong (2010) assume several customers with constant demand on an infinite horizon. The customers and the supplier have only local information, and their goal is to determine ordering periods that minimize the network-wide cost. The authors present two algorithms that can compute the approximation of the optimal solution. This approach necessitates cooperative attitude from the companies, since they have to follow the steps of the algorithms instead of minimising their own costs.

Esmaeili and Zeephongsekul (2010) also study the infinite horizon case, but they assume that the demand depends on the price and on the marketing efforts. The paper scrutinises several models for the single customer – single supplier case, where the supplier is responsible for the lot-sizing decision. A novel coordination tool is also presented, where the supplier shares the marketing cost with the customer, who in return informs the supplier about the parameters of the demand function.

Albrecht (2010) considers more general master planning problems with deterministic demand for multiple products on a discrete horizon, where the supplier and the customer have their own private information. The author studies iterative coordination mechanisms that improve the overall planing objective, as well as the objective of the supply chain partners.

Egri et al. (2010) present a simple one-loop coordination protocol with features characteristic to the automotive industry that aims to be practically realizable, but also allows improvement compared to the local optimum of the baseline solution. Although the presented protocol itself is general, a dynamic multi-echelon lot-sizing problem is studied in detail for demonstration purposes.

The model presented in this paper is different from all of these previous studies in several aspects. We start from the newsvendor model, but due to the incurring setup cost, we modify it in a non-trivial way. We consider two periods and no allowance of shortage, thus the production must fulfil all demand. Our model assumes fixed setup cost, where the eventual emergency production comes also together with an extra setup in the beginning of the second period. To the best of our knowledge, all previous variants of the newsvendor model disregards such fixed cost.

On the supplier's side of the two-echelon model, infinite capacity is assumed. We also assume that cost and price parameters are constant, only the production quantity is a decision variable, and it should be decided by the supplier. Since the customer does not decide about the order quantity, and the production is decided by the supplier, this model cannot be called forced compliance, although the entire demand must be satisfied. The demand forecast is not a simple qualitative information, but is given by a distribution function which is known exclusively by the customer, who signals it towards the supplier.

We present such a coordination contract, whereby sharing the forecast truthfully is rational for the customer, but neither the private cost parameters of the supplier, nor his production quantity need to be shared. The supplier can arbitrarily determine the production quantity according to the preliminary forecast, but it must setup another production in case of shortage and it bears also the risk of obsolete inventory; these risks are, however, compensated by the proper payment. The proposed contract provides full flexibility of the supply, since it does not bound the service level with ex ante commitments, like for example options.

Table 1 gives an overview of these papers according to the criteria presented in Section 2.1. The horizons include one period, two periods (meaning a forecast update), multiple periods or are infinite. The number of products is one in almost all cases, but there is an exception with multiple items. The demand can be stochastic (S), deterministic (D), rolling horizon (R), two-valued: high or low (T), constant (C) and price-dependent (P). The risk treatment in the reviewed papers are either neutral (N) or not applicable (-) when there are no stochastic variables. The shortages can be backlogged

Table 1: Overview of the related works.

	Cachon and Lariviere (2001)	Çınar and Bilgiç (2005)	Váncza et al. (2008)	Li et al. (2009)	Lee and Jeong (2010)	Esmaeili and Zeephongsekul (2010)	Albrecht (2010)	Egri et al. (2010)	This paper
Horizon length	2	1	n	2	∞	∞	n	n	2
Products	1	1	1	1	1	1	n	1	1
Demand	S,T	S	D,R	D,T	D,C	D,P	D	D	S
Risk	N	N	N	N	-	-	-	-	N
Shortage	L	L	N	N	N	N	L/B	N	N
Players	1-1	1-1	1-1	1-1	1-n	1-1	1-1	1-1	1-1
Asymmetry	F	C	F	F	F,C	F,C	F,C	F,C	F
Decision	F/V	F	V	V	N	V	N	N	V
Leader	C	S	C	S	-	C/S	-	-	C

(B), cause lost sales (L) or not permitted (N). Almost all papers assume one supplier and one customer (1-1), only one permits multiple customers (1-n). The information asymmetry can concern either the demand forecast (F) or some cost parameters (C). Regarding the decision structure, it can be either forced compliance (F), voluntary compliance (V), or negotiation (N)—referring to the cases when the lot-sizes are decided by the customer, the supplier or jointly. If the decisions are done sequentially, the leader can be either the customer (C) or the supplier (S). We separated the characteristic by a comma when they are present simultaneously, and by a slash when they are studied in separate models.

3. Centralised model with no shortage allowed

3.1. Mathematical model

As we mentioned in Section 2.1, the first step in designing a coordination mechanism is to assume a central decision maker with complete information. The solution of this setting provides a lower bound on the achievable system cost, which can be used as a reference in the decentralised model.

We introduce the fixed setup cost into the standard newsvendor model, therefore the elements of our model are the following:

c_s	setup cost,
c_p	production cost per unit,
ϕ, Φ	probability (PDF) and cumulative density functions (CDF) of the demand,
ξ	realized demand, and
q	production quantity (decision variable).

We want to minimise the total cost, since the revenue is independent from the decision variable due to the assumption of 100% service level. The total cost will consist of four parts:

1. the certain setup cost: c_s,
2. the production cost for satisfying the actual demand: $c_p\xi$,
3. the production cost of obsolete leftover products: $c_p \max(q-\xi, 0)$ and

4. the cost of additional setup: $c_s\delta(\xi - q)$, where

$$\delta(\xi - q) = \begin{cases} 0 & \text{, if } \xi - q \leq 0 \\ 1 & \text{, if } \xi - q > 0 \end{cases}$$

Thus the expected total cost in function of the production quantity becomes:

$$\mathbb{E}[TC(q)] = c_s + c_p\mathbb{E}[\xi] + c_p\mathbb{E}[\max(q - \xi, 0)] + c_s\mathbb{E}[\delta(\xi - q)]. \tag{1}$$

Proposition 1. *The derivative of the expected total cost function is*

$$\frac{\mathbf{d}\mathbb{E}[TC(q)]}{\mathbf{d}q} = c_p\Phi(q) - c_s\phi(q). \tag{2}$$

Proof. Using the definition of the expectation value we can express:

$$\mathbb{E}[\max(q - \xi, 0)] = \int\limits_{-\infty}^{q} (q - x)\phi(x)\mathbf{d}x = q\Phi(q) - \int\limits_{-\infty}^{q} x\phi(x)\mathbf{d}x \tag{3}$$

and

$$\mathbb{E}[\delta(\xi - q)] = \int\limits_{q}^{\infty} \phi(x)\mathbf{d}x = 1 - \Phi(q). \tag{4}$$

With these expressions the proposition can be easily proved. □

Determining the root of this derivative function is not as easy as in the standard newsvendor model, because it is not invertible in the general case, therefore we have to focus on a special probability distribution. Most papers assume the normal distribution—which choice is reasoned with the *central limit theorem*—, in spite of its drawbacks: even negative demands have some probability and it forces the PDF to be symmetric.

The normal distribution does not help us in determining the root analytically, hence we regard the *logistic distribution*, whose PDF is similar to the PDF of the normal distribution, but has longer tails. In Fig. 1 the PDFs of the normal and logistic distributions with the same expected value and variance can be seen; with solid and dashed curves respectively.

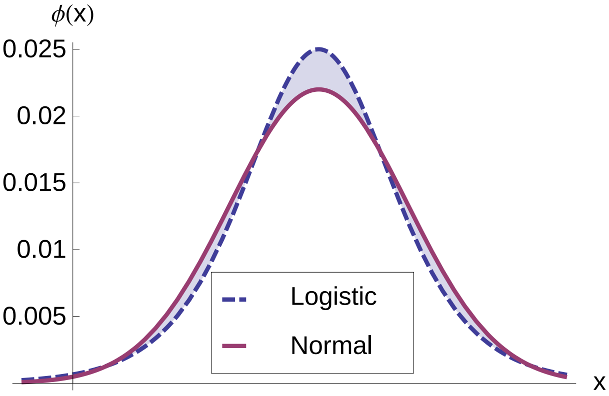

Figure 1: Comparison of the PDFs.

The PDF of the logistic distribution is defined as

$$\phi(x) = \frac{e^{\frac{m-x}{b}}}{b\left(1+e^{\frac{m-x}{b}}\right)^2} \tag{5}$$

and it has the expectation value and variance m and $\sigma^2 = \pi^2 b^2/3$ respectively. It can be seen, that the logistic distribution has a simpler form than the normal distribution, but otherwise they have similar properties.

Theorem 2. *There exists an optimal lot-size q^* which minimises the expected total cost iff $b < c_s/c_p$. In this case, the optimal lot-size is unique:*

$$q^* = m - b\ln\left(\frac{bc_p}{c_s - bc_p}\right). \tag{6}$$

PROOF. Substituting the PDF and CDF of the logistic distribution into Proposition 1 we get:

$$\frac{\mathbf{d}\mathbb{E}[TC(q)]}{\mathbf{d}q} = c_p \frac{1}{1+e^{\frac{m-q}{b}}} - c_s \frac{e^{\frac{m-q}{b}}}{b\left(1+e^{\frac{m-q}{b}}\right)^2}. \tag{7}$$

This should equal to zero, therefore simplifying the equation leads to

$$e^{\frac{m-q^*}{b}} = \frac{bc_p}{c_s - bc_p}. \tag{8}$$

After taking the logarithm of this equation q^* can be expressed as Eq. (6), which has a real solution iff the argument of the logarithm is positive. Since both b and c_p are positive, this yields the condition $b < c_s/c_p$.

This q^* is a minimum place of the expected total cost, since

$$\frac{\mathbf{d}^2\mathbb{E}[TC(q^*)]}{\mathbf{d}q^2} = \frac{c_p(c_s - bc_p)^2}{bc_s^2} > 0. \tag{9}$$

□

This optimal lot size gives a balance between the risk of obsolete inventory and the additional setup. It can be either more or less than the expectation value, depending on the variance and cost parameter. For some examples on this phenomenon see Section 5.

3.2. Discussion of the centralised model

As an illustration, taking a particular industrial example with $m = 65553$, $c_s = 45331$ and $c_p = 3.29$, the shape of the expected total cost function can be seen in Fig. 2.

The percentage numbers express the relative deviation (r) from the expected demand, i.e., we determine the b parameter as $b = \sqrt{3}rm/100\pi$.

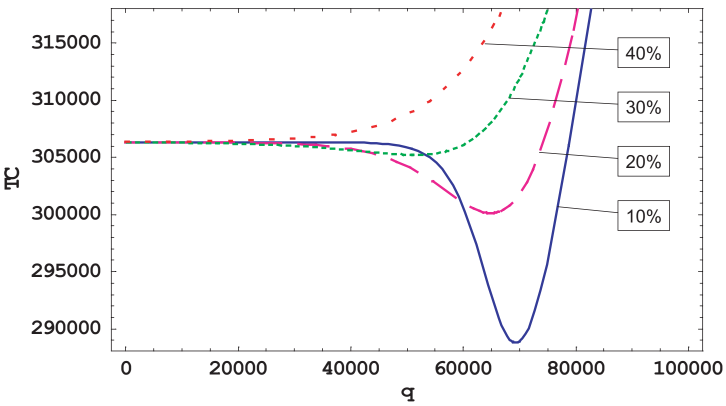

Figure 2: Deterioration of the cost function.

When the deviation is low (e.g., $r = 10\%$)), then an incorrect lot size causes significant raise in the expected total cost. The shape of the curve can be explained in the following way:

- if the condition of Theorem 2 is fulfilled, there is a unique optimum given by Eq. (6),
- decreasing q starting from the optimum increases the probability of the additional setup cost, however, the expected obsolete inventory is decreasing, therefore the function is bounded, and
- increasing q starting from the optimum decreases the expected additional setup cost, but the expected obsolete inventory increases arbitrarily.

As the diagram shows, the minimal expected total cost grows together with the relative deviation. The curve with $r = 40\%$—where $b \geq c_s/c_p$—is *de-*

generated in the sense that it has no positive optimum; our model does not apply.

Using this model one can also express the cost for being present on an uncertain market. If the demand was certain, the total cost would be $c_s + mc_p$, without additional setup and obsolete left over cost. The value of

$$\Delta TC = \mathbb{E}[TC(q^*)] - (c_s + mc_p) \tag{10}$$

thus can be interpreted as the *cost of uncertainty.* Fig. 3 demonstrates this kind of cost, using the same m, c_s and c_p parameters as in the previous example and let r range in the $(0, 30\%]$ interval.

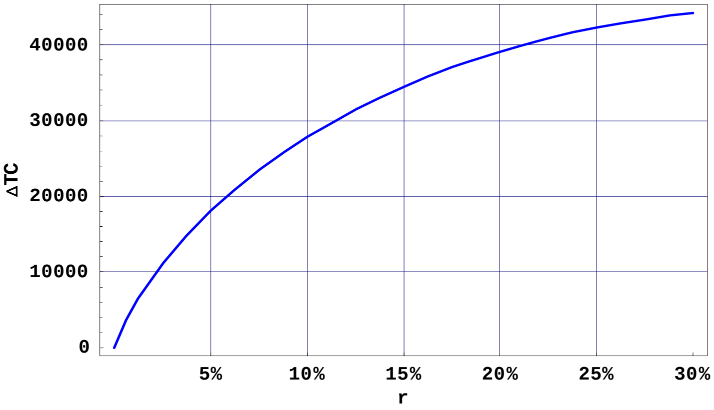

Figure 3: Cost of the uncertain market.

4. Coordinating the decentralised model under asymmetric information assumption

In this section we extend the previous model to a more realistic situation: we consider an end-product manufacturer in the customer's role and a supplier, having asymmetric information. The customer is familiar with the end-product market, thus she makes forecast and estimates the distribution of the demand (m and b parameters of the logistic distribution). A component is produced by the supplier, who knows the actual production and setup costs (c_p and c_s). The decentralisation (let alone the information asymmetry) can lead to suboptimal overall system performance, materialised in more obsolete inventory or unnecessary additional setup. We present such a protocol that guarantees the optimal q^* production quantity as derived in Section 3.

The lot sizing decision should be made by the supplier, who has to plan and schedule his own production, manage the inventory and provide 100% service level towards the customer. The protocol of the supply process is as follows (see also Fig. 4):

1. The customer signals forecast information towards the supplier, but this may differ from the real values, therefore we denote these parameters by m' and b'.
2. The supplier decides about the lot size (q) and produces this quantity.
3. The customer faces the demand (ξ), calls-off this quantity from the supplier.
4. The supplier delivers $\min(\xi, q)$ items instantly. If $\xi < q$, the obsolete inventory remains at the supplier; but if $\xi > q$, the supplier has to start an emergency production for $\xi - q$ items and deliver them as soon as possible.
5. The customer pays according to the payment function described below.

The reasons for the possible information distortion in the first step include

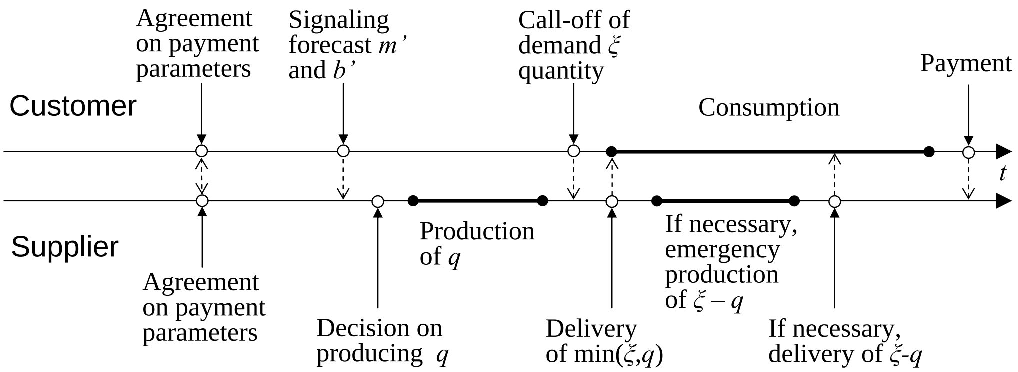

Figure 4: Protocol of the supply process.

inflating forecast in order to surely avoid shortage, deflate forecast in hope of getting bonus for surpassing the target or simply lack of motivation for determining the best available parameters.

We emphasise the assumption that the finally realised demand (ξ) is known by both partners. Note that if there is no information distortion (i.e., $m' = m$ and $b' = b$) and this is a common knowledge, the supplier is facing the centralised newsvendor problem presented in Section 3 with all required information, hence its rational lot sizing decision is also optimal on the system level.

Since the supplier not only offers products, but also *flexibility* as a *service*, we propose a composite payment scheme: the customer must pay not only (i) for the quantity called-off, but also (ii) for the deviation from the forecast, as well as (iii) for the forecast uncertainty. This payment compensates the supplier for the eventual obsolete inventory or the additional setup. The proposed payment scheme is the following:

$$P(m', b', \xi) = c_0\xi + \frac{c_1}{b'}d(m', \xi) + c_2(b'), \tag{11}$$

where c_0 and c_1 are constants: the pre-arranged unit prices for required components and inappropriate demand estimation, respectively. The term $d(m', \xi)$ is the difference between the communicated forecast and the realised demand and $c_2(b')$ is the monotonically increasing compensation term for uncertainty. Note some properties of this payment scheme:

- it depends only on commonly known parameters,
- the first term in the payment is independent from the decisions of the partners and
- if the customer communicates higher uncertainty (larger b'), it will pay less for the deviation (second term), but more for the uncertainty (third term) and vice versa.

Deviation can be measured e.g., by the simple difference, the absolute difference or the squared difference. We have found that the first two measures are inappropriate for channel coordination with the proposed payment scheme, thus we use the latter: $d(m', \xi) = (m' - \xi)^2$. For this case, we have derived such a $c_2(\cdot)$ compensation function for uncertainty wherewith the payment scheme inspires the customer towards truthful information sharing, hence it coordinates the channel.

Theorem 3. *If the demand ξ is a random variable from the logistic distribution with parameters m and b and the payment scheme is*

$$P(m', b', \xi) = c_0\xi + \frac{c_1}{b'}(m' - \xi)^2 + c_1\frac{\pi^2}{3}b', \tag{12}$$

then the expected payment is minimal iff $m' = m$ *and* $b' = b$.

PROOF. The expected payment is the following:

$$\mathbb{E}[P(m', b', \xi)] = c_0 m + \frac{c_1}{b'}\mathbb{E}[(m' - \xi)^2] + c_1 \frac{\pi^2}{3} b'. \tag{13}$$

The expected difference can be computed using the definition of the expected value:

$$\mathbb{E}[(m' - \xi)^2] = \int\limits_{-\infty}^{\infty} \left((m')^2 + x^2 - 2m'x\right) \phi(x)\mathbf{d}x = (m')^2 + \mathbb{E}[\xi^2] - 2m'm. \tag{14}$$

The term $\mathbb{E}[\xi^2]$ can be expressed from the following basic property of the variance: $\sigma^2 = \mathbb{E}[\xi^2] - m^2$ utilising the variance of the logistic distribution: $\sigma^2 = \frac{\pi^2 b^2}{3}$. Then the expected payment becomes:

$$\mathbb{E}[P(m', b', \xi)] = c_0 m + \frac{c_1}{b'}\left((m')^2 + m^2 - 2m'm + \frac{\pi^2 b^2}{3}\right) + c_1 \frac{\pi^2}{3} b'. \tag{15}$$

To minimise the expected payment, the partial derivatives must equal to zero:

$$\frac{\partial \mathbb{E}[P(m', b', \xi)]}{\partial m'} = \frac{c_1}{b'}(2m' - 2m). \tag{16}$$

This equals zero iff $m' = m$, independently from choosing b'. Furthermore

$$\frac{\partial^2 \mathbb{E}[P(m', b', \xi)]}{\partial (m')^2} = 2\frac{c_1}{b'} > 0, \tag{17}$$

i.e., according to the *second derivative test*, this is a minimum. For computing the other partial derivative, we already exploit that $m' = m$ in the optimal case:

$$\frac{\partial \mathbb{E}[P(m', b', \xi)]}{\partial b'} = c_1 \frac{\pi^2}{3} - c_1 \frac{\pi^2 b^2}{3(b')^2}. \tag{18}$$

This equals zero iff $b' = b$, and in this case the second derivative is also positive:

$$\frac{\partial^2 \mathbb{E}[P(m', b', \xi)]}{\partial (b')^2} = c_1 \frac{\pi^2 b^2}{3(b')^3}. \tag{19}$$

□

5. Computational study

In this section we present some experiments that we performed together with industrial partners. The production network involved in this project serves a highly uncertain market with both standardised and customised products. Although the products and most of the components are not perishable (at least not on the short term), their ever-changing variety and short life-cycles justifies the choice of the newsvendor model. The examples presented below are related to packaging material supply, which are typically customised components and therefore high volatility is especially characteristic for them. In this case the setup costs are much larger—almost 15.000 times larger—than the unit production costs, since materials are relatively cheap, but the setup involves washing out the paints from the machines in addition to changing the cutter tool and the offset plate. The customer and the suppliers use VMI contracts, wherewith the suppliers have the responsibility of deciding about the production quantities and fulfilling the entire demand.

5.1. Analysis of the centralised newsvendor

In Fig. 5 we summarise the proposed lot sizes of the newsvendor model presented in Section 3. The parameters c_s, c_p and m presented in Table 2 were taken from the industrial database and relate to components near to the end of their life-cycles.

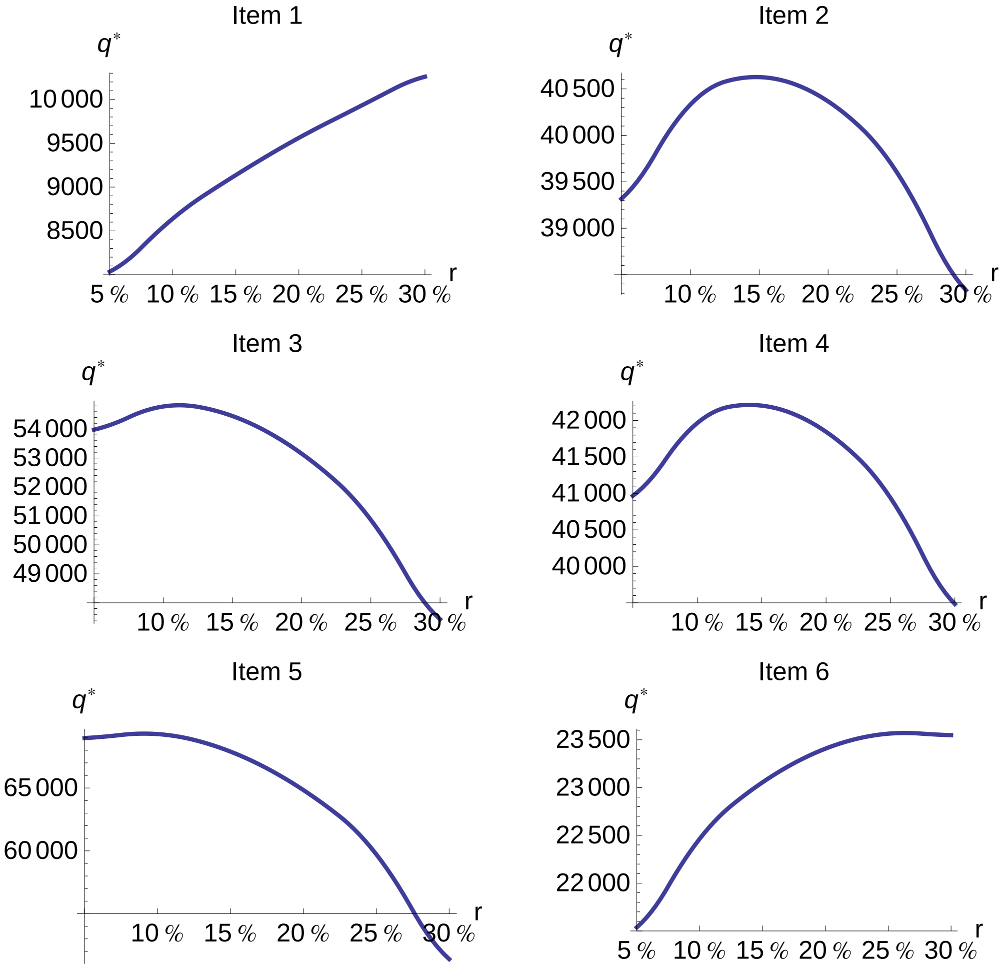

Figure 5: Some results of the newsvendor model.

Since the b parameters were not available due to the deterministic demand planning method applied, we decided to compute the results for a series of values and compare their results. We took certain percentages (r) of the forecasted demand as the standard deviation and derived the b parameters

Table 2: Some industrial examples.

Item	1	2	3	4	5	6
c_s	55269.5	45997.25	46046.5	45892	45331	44541
c_p	3.15	3.29	3.29	3.29	3.29	3.29
m	7152	36733	50899	38323	65553	19807

from them similarly as in Section 3.

The series of the optimal lot sizes can be explained in the following way: if there had been no uncertainty, the lot size would have been equal to the demand. As the uncertainty increases, it is better to increase the lot size in order to avoid the additional setup. However, when the uncertainty reaches a certain threshold, the expected cost of obsolete inventory reaches the expected cost of the additional setup, therefore the optimal lot size starts to decrease. If we increased the uncertainty parameter further until b reaches c_s/c_p, the model would not be able to provide the optimal lot size. Nevertheless, as the uncertainty grows, more attention must be paid by the human experts.

5.2. Analysis of the decentralised supply chain

In the decentralised setting, the profit of the customer is the revenue for the end-products minus the payment for the required components minus further production and logistic costs. Since we do not have all necessary parameters for this profit function for a real case, we focus on the supplier's profit instead, which can be modelled with the payment minus the total cost.

According to the main result of Section 4, the optimal lot size of the decentralised system equals the centralised newsvendor solution, therefore both the total cost and the payment can be computed. Here we present the result of the simulations using the parameters from the first item of Table 2, i.e., $c_s = 55269.5$, $c_p = 3.15$ and $m = 7152$. We set $b = 393.30$ (which means 10% relative deviation), $c_0 = 10$ and simulated the arisen costs and payments for different c_1 compensation parameters by averaging 1000 simulation runs in each case. The results are presented in Fig. 6.

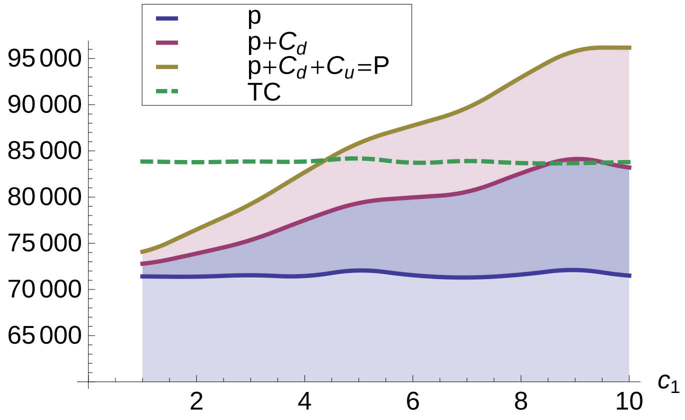

Figure 6: Compensation's effect on supplier's profit.

The optimal lot size $q^* = 8640$ does not change, since the solution of the newsvendor model is independent from the compensation parameter. The p, C_d and C_u mean the three parts of the payment function: the price paid for the components, the compensation for deviation and compensation for forecast uncertainty, respectively. Formally: $p = c_0\xi$, $C_d = c_1(m' - \xi)^2/b'$ and $C_u = c_1 b'\pi^2/3$. The price p is independent of c_1 but fluctuates slightly due to the random demand. The two parts of compensation evidently increase with c_1. The total payment is the sum of the three parts—denoted by P—which can be compared with the total arisen cost (TC) in order to obtain the supplier's profit.

We also performed *ceteris paribus* sensitivity analysis of the other parameters. An interesting example of changing the expected demand can be seen in Fig. 7, where we used the same c_s, c_p and c_0 parameters as in the previous example, furthermore, we set $c_1 = 1$ and the relative deviation to 10%. In this case, both the payment and the cost increase with the expected demand, but due to their different slopes, the larger the demand, the larger the profit of the supplier.

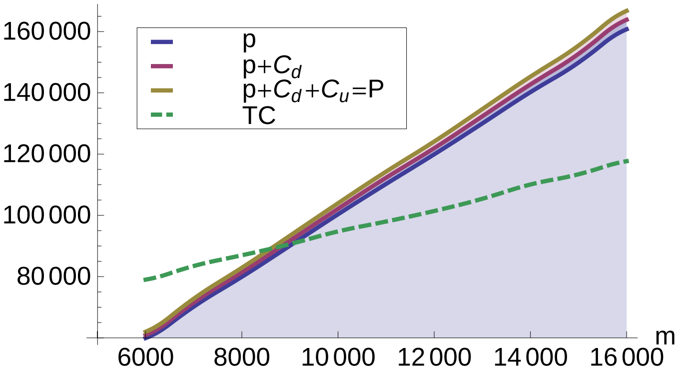

Figure 7: Expected demand's effect on supplier's profit.

One can also notice that $C_d \approx C_u$ in every simulation experiment. This is not accidental: it can be proven that if $m' = m$ and $b' = b$ then $\mathbb{E}[C_d] = C_u$.

A further aim of coordination, the cost and profit sharing seems to be difficult in this framework, because of at least two reasons: (i) we disregarded the customer's profit so far, thus we also cannot express the total profit of the supply chain and (ii) the information about the production and setup cost remains asymmetric in our model, therefore the customer cannot observe the arisen cost. Although the presented payment scheme leads to optimal performance at the system level, solving the problem of fair allocation needs further research and possibly further assumptions towards the model.

6. Conclusions

In the paper, after classifying and discussing short-term channel coordination methods we have presented an extended version of the newsvendor model in which all demand must be fulfilled even if this calls for an additional setup of production. We have derived the optimal lot-size for the case when the distribution of demand can be approximated with the logistic distribution. This solution minimises the overall system-wide cost. Further on, the model has been studied in a decentralised setting where the customer and the supplier have asymmetric information. It was proven that with an appropriate protocol and payment scheme the channel can be coordinated even in this decentralised situation. The suggested coordination mechanism provides the right incentive to the customer to disclose her demand forecast to the supplier: she has no interest in distorting the forecast or its uncertainty. In fact, the customer is motivated to produce as reliable demand forecast as possible, because this would reduce her expected payment for the flexible service of the supplier. In return, the supplier's rational decision results in the minimum expected system-wide cost.

Finally, we summarize the three main novelties of the model presented in this paper.

- We have introduced an additional fixed cost of production to the newsvendor model in case of a shortage situation.

- In the decentralised model, in contrast to standard *games with incomplete information*, no common knowledge about the distribution of the private information is assumed. In this case that would lead to a probability distribution (belief) of the demand probability distribution (forecast), which seems unrealistic.

- We have proposed a coordinating contract that assures the complete fulfilment of the demand, i.e., it does not constrain the flexibility in advance with fixed orders or options.

The protocol is relatively simple and the terms of the payment are conceivable and applicable in practical business processes: the supplier is compensated both for the deviation of the forecasted and actual demand, as well as for the uncertainty of demand.

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